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# The testing and behaviour modelling of sheet metals at strain rates from $10^{-4}$ to $10^4$ s $^{-1}$

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Uniaxial quasi-static tests and dynamic compression tests at high strain rates have been performed to determine the mechanical behaviour of sheet metals widely used in the automotive industry. The quality of experimental data at high strain rates obtained with a split Hopkinson pressure bar (SHPB) depended on signals processing. Improvements in this field are discussed: the wave dispersion correction, an exact delay setting in the shifting of the waves and the use of an inverse calculation technique. A constitutive elastic-plastic model for strain rates in the range  $10^{-4}$ – $10^4$  s $^{-1}$  is then proposed for sheet metals. The thermal softening due to the adiabatic conditions of the dynamic test was taken into account in creating this model.

*Keywords:* Strain rates; Quasi-static testing; Dynamic compression testing; Sheet metal

## 1. Introduction

Steel and aluminium sheets are widely used in the automotive industry. In order to take into account passive safety requirements of cars at the early stage of the design, numerical modelling of crash tests is needed. Recent developments in computer technology make such numerical simulations with explicit FEM codes possible at a reasonable cost. However, the constitutive models available for materials involved in such an impacting situations are generally quite simple. In order to obtain more accurate simulations, developments of suitable constitutive laws that can be implemented easily in FEM codes are necessary. For this purpose, testing of sheet metals under both quasi-static and dynamic loading are performed (the material properties not only depend on their metallurgical composition but also on their processing during manufacture). On the basis of experimental data, a constitutive model is proposed.

## 2. Experimental procedure

Quasi-static tests were performed using a classical hydraulic testing machine. Dynamic tests were performed with a split Hopkinson pressure bar (SHPB), also called Kolsky's apparatus [1]. Extensive reviews of this technique have been given by Kolsky [2] and Lindholm [3]. Improvements of the accuracy of SHPB concerning the corrections of inertia and friction effects can be found in the works of Davies and Hunter [4], Bertholf and Karnes [5] and Malinowski and Klepaczko [6].

The experimental arrangement was composed of 16.5-mm diameter steel bars. These bars were 3 m long to allow for significant measurements at lower strain rates of about 50 s $^{-1}$ . A special "sandwich" specimen was used. It was made from four pieces of sheet cemented altogether (Fig. 1). No buckling effect was observed before the strain reached a significant value (see Fig. 6, Section 4).

The three basic waves (incident, reflected and transmitted) recorded by the gauges cemented on the input and output bars allow for the determination of the forces and the speeds at the bar-specimen interfaces. As the waves are recorded at the measuring points, they

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have then to be shifted, in time and space, from the strain gauges to the specimen faces in order to calculate the forces and speeds. This shifting leads to two main problems. First, the waves change shape while propagating along the bar because of wave dispersion effects [7–9]. Second, it is difficult to obtain the exact delay in time needed to ensure that the beginnings of the three waves correspond to the same instant. These problems make the precise determination of Young’s modulus and the yield stress difficult.

In this work, a more accurate data processing method was used. The correction of the wave dispersion based on the harmonic wave solution in an infinite rod was applied. The correction procedure is the same as that shown in previous works [7–9]. However, we calculated directly the dispersive relation instead of using Bancroft’s data [10], where error due to the interpolation is unavoidable. This permits a more accurate correction [11]. A method to determine the exact delays is proposed. It is based on the realistic assumption that the materials tested demonstrate quasi-elastic behaviour during the early moments of the test. This is an iterative method, which consists of simulating the reflected and transmitted waves for the given incident wave in a fictitious elastic specimen and comparing them with the real waves during the early stages of the tests, and which permits a precise delay setting [11,12].

To obtain the stress–strain curve from the measurements of forces and speeds at the two specimen faces, the conventional analysis assumes a homogeneous state of stresses and strains in the specimen. This assumption leads to the classical two-waves or three-waves formulae [2,3]. However, the assumption is not exact, especially in the early stages of the test, because of the transient effect in the specimen. For example, when the input side of the specimen is loaded, the output side is not loaded until the wave goes through the specimen. To eliminate the error due to the assumption of homogeneous stress and strain fields, an identification process based on an inverse calculation has been developed. Assuming that the material behaviour can be described by a simple model with some parameters to be determined, a set of parameters which allows for the best simulation of the measured forces and speeds can be found by means of a transient numerical analysis [13]. The model used for the simulation of one test is not considered as the constitutive model of the material. It is only used to give a good transient description of the test with no need for the assumption of homogeneity. When the best set of parameters is identified, the

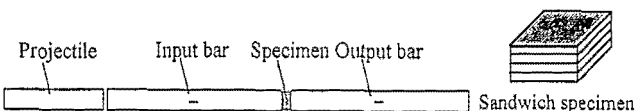


Fig. 1. SHPB setup and “sandwich” specimen.

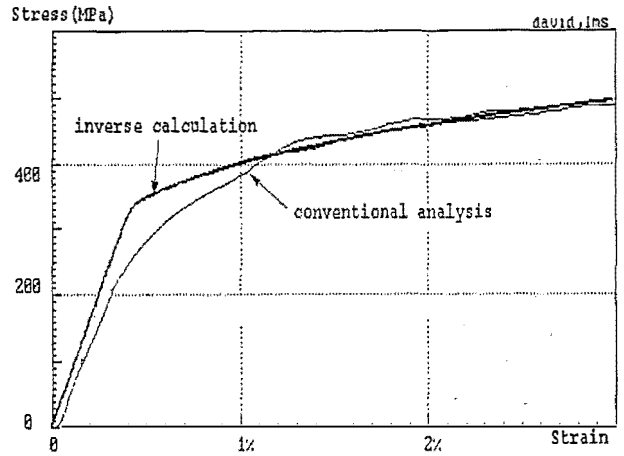


Fig. 2. Comparison between inverse analysis and conventional analysis (three-waves formula) where the axial uniformity of stress and strain fields are assumed.

stress–strain curve at a constant strain rate can be derived from the model. In this way, a more accurate experimental result than that found by conventional analyses can be obtained. It is shown in Fig. 2 that the stress–strain curve obtained by this method is close to that arrived at by conventional analyses, but the apparent elastic modulus and yield stress are more easily determined.

### 3. Phenomenological modelling of experimental data

In order to find a constitutive law suitable for FEM code, phenomenological modelling on the basis of experimental data was performed. Physical models were discarded because they are normally quite complicated and depend on a lot of parameters [14]. Simple phenomenological models are considered in this paper.

In this field, we first examined the over-stress elastic–viscoplastic model introduced by Sokolovsky [15] and Malvern [16], that has been generalised to a three-dimensional form by Perzyna [17].

For reasons of simplicity, we discuss the strain rate sensitivity of elastic viscoplastic models in a one-dimensional form with some common rate sensitivity functions (Eq. (1))

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{E} \frac{\partial \sigma}{\partial t}, \quad \text{if } \sigma \leq \sigma_s$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{E} \frac{\partial \sigma}{\partial t} + \phi(\sigma, \varepsilon), \quad \text{if } \sigma > \sigma_s \quad (1)$$

where the rate sensitivity is implicitly described by the function  $\phi$  and  $\sigma_s$  is the quasi-static yield stress.

Some widely used rate sensitivity formulas are shown as follows:

$$\phi(\sigma, \varepsilon) = \varepsilon_0 \exp\left(\frac{\sigma - f(\varepsilon)}{Cf(\varepsilon)}\right), \quad \text{Exponential formula}$$

$$\phi(\sigma, \varepsilon) = \varepsilon_0 \left( \frac{\sigma}{f(\varepsilon)} \right)^{1/m}, \quad \text{Power formula}$$

$$\phi(\sigma, \varepsilon) = \frac{K}{E} (\sigma - f(\varepsilon)), \quad \text{Malvern formula} \quad (2)$$

where  $f(\varepsilon)$  is quasi-static stress-strain curve and  $C$ ,  $K$ ,  $E$ ,  $m$  and  $\varepsilon_0$  are coefficients.

We have tried to identify the parameters using those functions and their linear combination as the rate sensitivity model. They did not give an adequate description of experimental results in the range of strain rate ( $1-1000 \text{ s}^{-1}$ ) in which we are interested (Fig. 3).

We also examined the so-called empirical models which are valid over a large range of strain rates. Those models have been provided by Johnson and Cook [18], Zerilli and Armstrong [19] and Steinburg [20]. For example, the Steinburg yield stress model is written as follows:

$$\sigma = [Y_T(\dot{\varepsilon}_p, T) + Y_A f(\varepsilon_p)](G(p, T)/G_0) \quad (3)$$

where  $Y_A(\varepsilon_p)$  is the quasi-static yield stress;  $Y_T(\dot{\varepsilon}_p, T)$  describes the increase due to rate sensitivity and to the temperature; and  $G(p, T)$  is a function of the hydrostatic pressure and the temperature.

These models were been initially developed to describe high-speed impact tests, such as Taylor's test. In this test the response of a cylindrical specimen impacting at high speed against a rigid target is studied [21]. The models give satisfactory results in this case; they have been used frequently for military applications under conditions of very high strain rates, high temperatures and high hydro-pressures [18–20]. However, the sheet metals involved in a crash test are submitted to much lower strain rates at lower pressures and temperatures. The most realistic strain rate is situated in the range  $10-300 \text{ s}^{-1}$ . Thus, it would be worthwhile to develop a specific model based on experimental data which gives more accurate results in this range. Keeping the concept of an empirical model [22], a Ludwig-type

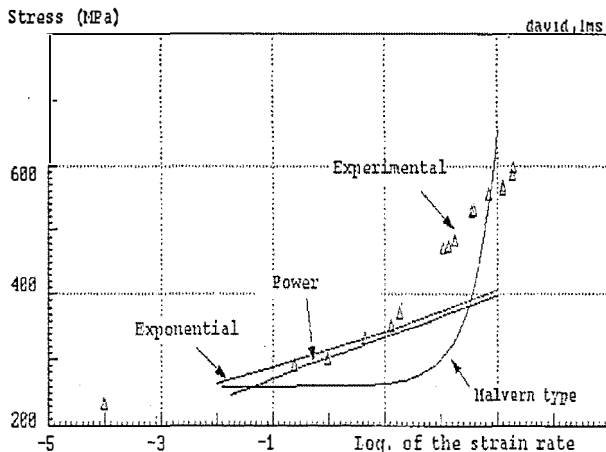


Fig. 3. Comparison between experimental results and those calculated with different models; the stress associated with a given 3% strain are shown as a function of the log of the strain rate.

Table 1

Rate sensitivity of elastic limit and stress for carbon steel sheets

Strain rates ( $\text{s}^{-1}$ )	Yield stress (MPa)	Stress <sup>a</sup> (MPa)
116	460	470
144	466	474
186	498	503
366	521	527
408	520	529
755	545	553
1309	541	562
1320	545	565
1901	560	584
1999	565	596

<sup>a</sup> Associated with a given 3% strain.

model with the coefficients depending on the strain rate is proposed as follows:

$$\sigma = (A(\dot{\varepsilon}) + B(\dot{\varepsilon})\varepsilon_p^{n(\dot{\varepsilon})})(1 - \mu\Delta T) \quad (4)$$

with  $A = a_1 + a_2 \log(\dot{\varepsilon}/\dot{\varepsilon}_0) + a_3 (\log(\dot{\varepsilon}/\dot{\varepsilon}_0))^3$ ;  $B = b_1 - b_2 \log(\dot{\varepsilon}/\dot{\varepsilon}_0) + b_3 (\dot{\varepsilon}/\dot{\varepsilon}_0)^{0.5}$ ;  $n = n_1 + n_2 \log(\dot{\varepsilon}/\dot{\varepsilon}_0) + n_3 (\log(\dot{\varepsilon}/\dot{\varepsilon}_0))^3$ , and where  $\sigma$ ,  $\varepsilon_p$  and  $\varepsilon$  are stress, plastic strain and strain rate;  $\Delta T$  denotes the temperature difference with respect to the room temperature ( $23 \text{ }^\circ\text{C}$ ); and  $\dot{\varepsilon}_0$ ,  $\dot{\varepsilon}_t$ ,  $\mu$ ,  $a_i$ ,  $b_i$  and  $n_i$  are coefficients.

#### 4. Results and discussion

Some sheet metals used in automotive industry are tested under quasi-static and dynamic loading with the method presented in Section 2. Experimental results are accurate and may be reproduced with the use of precise data processing techniques. As an example, the rate sensitivities of the elastic limit and of the stress at a given 3% strain obtained with SHPB are shown in Table 1 for carbon steel sheets.

As the model given by Eq. (4) is also sensitive to temperature, quasi-static iso-temperature tests at  $110 \text{ }^\circ\text{C}$  and at room temperature ( $23 \text{ }^\circ\text{C}$ ) were performed to determine the thermal softening coefficient. In Fig. 4 it is shown that a relatively strong thermal effect was observed for these steel sheets. The coefficient in Eq. (4) was then determined ( $\mu = 0.11$ ).

It was then possible to take account of the thermal softening owing to the adiabatic condition of the impact tests, using the assumption that all the plastic work is transformed into heat (Eq. (5)).

$$\Delta T = \frac{1}{\rho C_\rho} \int_0^{\varepsilon_p} \sigma d\varepsilon_p \quad (5)$$

where  $C_\rho$  is the specific heat and  $\rho$  is the mass density.

The coefficients of models described by Eq. (4) were identified and Fig. 5 shows that a good agreement with experimental results was obtained for the strain rate in

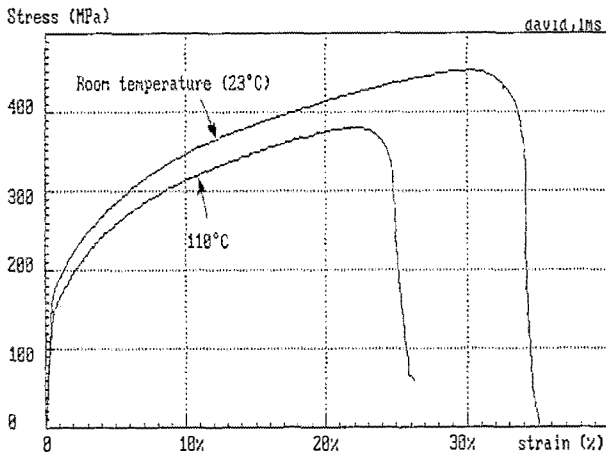


Fig. 4. Thermal softening effects; influence of temperature on the stress-strain curve at a quasi-static strain rate.

the range  $10^{-4}$ – $10^4$   $s^{-1}$ . This model not only describes the rate sensitivity at a given constant strain level but also the stress evolution as a function of strain for a constant strain rate. For instance, the stress-strain curves given by the present model with identified coefficients fit with both quasi-static and dynamic experimental data (Fig. 6). This empirical model gives an accurate description of the behaviour of the sheet steels studied, for a large range of strain rates.

The accuracy of the proposed phenomenological model is, in our opinion, due to its simplicity. Indeed, the essential goal was to accomplish a precise description of experimental data using the concepts of an empirical approach. It must be noted that such an approach can miss some physical aspects, such as strain rate history effects. However, this formulation allows for an easy application to engineering numerical calculations.

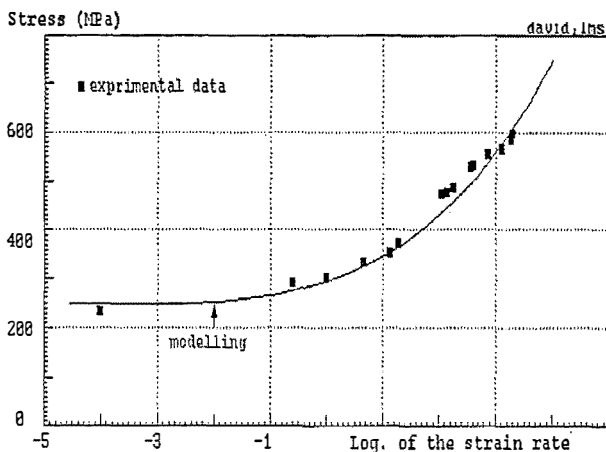


Fig. 5. Comparison between experimental results and those calculated with proposed model; the stress associated with a given 3% strain is shown as a function of the log of the strain rate.

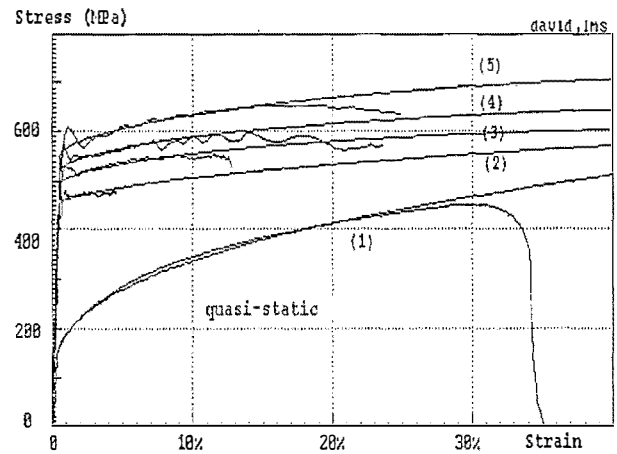


Fig. 6. Comparison between experimental results and those calculated with the proposed model. Stress-strain curves are shown at following strain rates: (1) quasi-static; (2)  $116 s^{-1}$ ; (3)  $186 s^{-1}$ ; (4)  $755 s^{-1}$ ; (5)  $1901 s^{-1}$ .

## 5. Conclusion

Dynamic tests have been performed on steel sheets using the SHPB apparatus. To avoid buckling, sandwich specimens were used successfully. The signals were processed taking account of the wave dispersion correction and using a special technique for an exact delay setting. The accuracy of the determination of the behaviour at a constant strain rate has been improved by means of an inverse calculation method.

Both the physical and empirical modelling have been discussed. The empirical modelling is simpler and better suited to numerical calculation. Furthermore, the constitutive elastic-plastic description seems to be simpler than the over-stress elastic-viscoplastic model. A Ludwig-type empirical constitutive elastic-plastic model has been proposed. The model contains a thermal softening term and the adiabatic condition in the dynamic tests is taken into account. For the steel sheets studied, the proposed model is applicable for strain rates in the range  $10^{-4}$ – $10^4$   $s^{-1}$ .

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