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# Elastoplastic analysis of repeated moving contact

## Application to railways damage phenomena

K. Dang Van <sup>a</sup>, M.H. Maitournam <sup>a</sup>, B. Prasil <sup>b</sup>

<sup>a</sup> *Laboratoire de Mécanique des Solides, CNRS URA 317, Ecole Polytechnique, 91128 Palaiseau, Cedex, France*

<sup>b</sup> *SNCF, Direction des Installations Fixes, 18 rue d'Amsterdam, 75008 Paris, France*

### Abstract

A quantitative theory for the prediction of contact damage is presented. It relies on:

1. thermo-elastoplastic computational methods ('stationary methods') (Dang Van and Maitournam, *J. Mech. Phys. Solids*, 41 (1993) 1691–1710) used to evaluate the stress, strain and temperature fields in structures subjected to repeated moving loads. These methods, based on the steady state assumption, comprise two numerical procedures for the calculations of stress and strain either for an arbitrary number of loading passes or directly for the stabilized state;
2. a multiaxial fatigue criterion (Dang Van, *ASTM STP*, 1191 (1993) 120–130; Dang Van, in: Beevers and Blom (eds.), *Theoretical Concepts and Numerical Analysis of Fatigue*, ESIS, 1992; Ballard et al., *Fatigue Fract. Eng. Mater. Struct.*, 18 (1995) 397–411) based on a macro-micro analysis, and evaluating micro-stresses in disoriented elements from the macroscopic loading path. Ratchetting rate, temperature rise and fatigue failure are investigated in the case of line contacts.

**Keywords:** Multiaxial fatigue; Ratchetting rate; Moving loads; Thermo-elastoplasticity; Finite element analysis

### 1. Introduction

Fatigue and fracture phenomena induced by repeated rolling or sliding contacts represent a very large class of problems in railways. Many attempts have been made to understand and model the different damage processes as 'kidney shape' cracking, shelling, squats, head-checking, .... For this purpose it is necessary first to evaluate the mechanical quantities in the vicinity of the contact region, and second to use appropriate damage criteria. Many studies have been devoted to this subject. However, these efforts have not been fully successful because contact between solids generates multiaxial non-proportional stress and strain cycles which are particularly difficult to model. Most of the existing theories are based on elastic analysis which is not realistic enough; because, as shown by measuring, worn rails exhibit important compressive residual stresses (up to  $-800$  MPa was measured on SNCF rails) induced by the severe plastic deformation observed near the surface. Existing elastoplastic analyses are either semi-analytical (so they cannot take into account any kind of material, geometry, etc.) or use a classical finite element procedure which is very time-consuming. The lack of reliable multiaxial fatigue criteria is another major difficulty in studying the damage due to contact phenomena.

In this paper, we present an approach based on the two following features.

The first one is a specific thermo-elastoplastic computational method [1–3] which is used to evaluate the stress, strain and temperature fields in the vicinity of the moving contact area. This method relies on the steady state assumption in the moving contact reference. It leads to the two following numerical procedures for the calculations of stress and strain either for an arbitrary number of loading passes or directly for the stabilized state (elastic shakedown, plastic shakedown or ratchetting): (i) the pass-by-pass stationary method (PPSM) for a single load pass; this method is exact and does not suffer from the limitations of previous rolling contact finite element analyses [7–9]. The calculation time for one pass is approximately the same as that obtained in classical plasticity for one step of translation of the pressure distribution; (ii) the direct stationary method (DSM) for a repeated moving load. This method goes straight to the steady state in this case and is rather approximate. These procedures were extended in the case of thermomechanical coupling, realized by the presence of thermal expansion in the mechanical problem, and of heat sources (due to inelastic deformation, thermo-elastic coupling and interfacial dissipation mechanisms such as friction and micro-slip) in the thermal

problem [3]. They led to the evaluation of the temperature field in the vicinity of the contact area.

The second feature is a multiaxial fatigue criterion [4–6] relying on a macro–micro analysis, and on the evaluation, from the macroscopic loading path, of micro-stresses in dis-oriented elements. This criterion is already widely used in mechanical industries.

The combination of both methods gives a good tool for reasonable quantitative analysis of contact damage phenomena. Illustrative examples, through the calculations of ratchetting rate, temperature rise and prediction of fatigue failure, are considered.

## 2. Thermomechanical state due to rolling

### 2.1. Recall of the principle of the 'stationary methods'

Full details of the 'stationary methods' used in this paper and their finite elements implementation can be found in [1]. Here, we just recall the principle of the analysis. Let us consider a prismatic structure made of an elastoplastic material (let's say a rail) subjected to loads (let's say normal and tangential pressures) moving with a velocity  $V = Ve_x$  (Fig. 1).

We assume that there is a steady state in a reference frame moving with the loads; for any material quantity  $B$ , it leads to the following relation:

$$\dot{B} = VB_{1x} \quad (1)$$

#### 2.1.1. The mechanical problem

The first idea consists of writing the equations governing the mechanical problem in the moving loads reference. Thus one obtains:

##### 1. equations of motion

$$\text{div } \sigma = \rho V^2 u_{1xx} \quad (2)$$

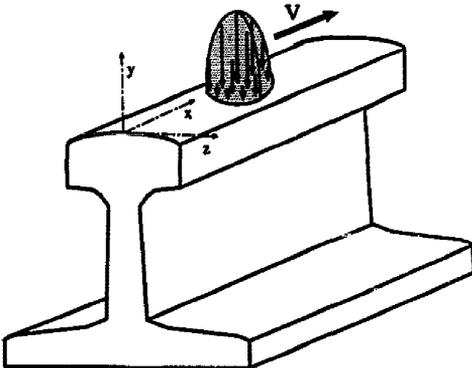


Fig. 1. Problems under consideration.

##### 2. constitutive laws

$$\sigma = L:\epsilon^e + \sigma_0 \quad (3)$$

$$A_k = Z:\alpha_k \quad (4)$$

$$\epsilon = \epsilon^e + \epsilon^p \quad (5)$$

$$-\epsilon_{1x}^p = \Lambda \frac{\partial f}{\partial \sigma} \quad f \leq 0 \quad \Lambda \geq 0 \quad \Lambda f = 0 \quad (6)$$

$$\alpha_{k,x} = \Lambda l(\sigma, A_k) \quad f \leq 0 \quad \Lambda \geq 0 \quad \Lambda f = 0 \quad (7)$$

$$f(\sigma, \epsilon^p) = \sqrt{\frac{1}{2}(\sigma - A_k) : (\sigma - A_k)} - k_e \quad (8)$$

with

$$\Lambda = \frac{1}{H} \zeta \left( -\frac{\partial f}{\partial \sigma} : L : \epsilon_{1x} + \alpha \theta_{1x} \left( \frac{\partial f}{\partial \sigma} \right) \right) \quad (9)$$

$$\zeta = 1 \text{ if } f = 0 \quad \zeta = 0 \text{ if } f < 0 \quad (10)$$

$\langle \cdot \rangle$  meaning positive part, and

$$H = \frac{\partial f}{\partial \sigma} : L : \frac{\partial f}{\partial \sigma} + \frac{\partial f}{\partial A_k} : Z : l \quad (11)$$

$\Lambda$  is the plastic multiplier, ( $f=0$ ) defines the elastic domain,  $k_e$  is the shear yield stress,  $\sigma_0$  initial stress,  $\alpha_k$  is internal parameters and  $A_k$  their associated forces,  $L$  is the tensor of elastic coefficients,  $Z$  and  $l$  are assumed to be known.

In these equations, time derivatives have been replaced by space derivatives, according to Eq. (1). Their integrations are performed in space, along the direction of the motion of the loads; so that, numerically (using the finite element method), neither the load nor the structure is translated. The plastic behaviour can be very general; in the case of rail, besides the Von Mises isotropic and/or linear kinematic hardening material, the non-linear kinematic hardening law proposed by Bower [10] has been used.

The second idea is to use the stationarity of the limit state (the stress field is periodic, and the plastic strain also in the absence of ratchetting) in order to find its value directly.

The two resulting numerical procedures for the determination of the stabilized state are derived [1].

1. The pass-by-pass stationary method (PPSM) for the calculation of a single pass; so by computing the successive passes by this method, one can find all the features of the 'stabilized' state (numbers of cycles before reaching it, residual stresses, plastic deformations, etc). This 'stabilized' state can of course be ratchetting; in such a case, the ratchetting rate is immediately deduced as the increment of plastic strains caused by a pass.

2. The direct stationary method (DSM) for the direct determination of the 'stabilized' state if it is a shakedown (elastic or plastic). The ratchetting is indicated by a non-convergence of the algorithm.

Thus, these methods allow the description of strains and stresses due to cyclically and alternatively moving contacts,

the quick determination of the nature of the stabilized state (elastic shakedown, plastic shakedown or ratchetting). Such analyses of repeated contacts using shakedown principles were carried out with semi-analytical methods by Johnson and others [11–16].

### 2.1.2. Thermomechanical coupling

The thermomechanical coupling is realised (i) in the thermal equations by volumetric heat sources (intrinsic dissipation  $\sigma:\dot{\epsilon}^p - A_i\dot{\alpha}_i$ , reversible heat source  $\rho\theta\dot{s}$ ) and by interfacial dissipation; (ii) in the mechanical equations by thermal expansion [3]. Thus, the thermal equation is written as

$$\text{div}(-k \text{grad } \theta) - \rho c V \theta_{,t} = r - V\sigma:\epsilon_{,t}^p + VA_i\alpha_{i,t} + V\alpha\theta \text{tr}(\sigma)_{,t} \quad (12)$$

Details of the solution procedure of the coupled thermo-mechanical problem can be found in [3].

### 2.2. Fatigue analysis

The present paper focuses on high cycle fatigue of the rail, due to repeated contact. The theory used is that developed by Dang Van and others [4–6]. The criteria they proposed are based on a ‘macro–micro’ analysis distinguishing two scales: (i) the engineers’ scale, called the macroscopic scale, in which the elementary volume contains many crystals with different shapes and orientations. On this scale, as far as high cycle fatigue is concerned, the material behaviour is elastic, at least after a certain number of load cycles (elastic shakedown); (ii) the microscopic scale of the order of the grain size, where fatigue occurs. The high cycle fatigue criteria are therefore expressed in terms of stresses evaluated at this last scale and called microscopic stresses. They generally differ from the macroscopic stresses, and a correction is needed [4]. The simplest criterion (Dang Van’s criterion) assumes first, that elastic shakedown occurs at all scales and second, that fatigue rupture occurs in a critically oriented grain if, at a certain time  $t$  of the load cycle, one has

$$f(\sigma) = \tau(t) + ap(t) - b \geq 0 \quad (13)$$

where  $p = (1/3)\text{tr}\sigma$  is the microscopic hydrostatic pressure,  $\tau(t)$  the microscopic Tresca shear stress,  $t$  the time, and  $a$  and  $b$  are two material constants. Thus the Dang Van criterion (for no fatigue crack occurrence) is expressed as

$$\max_t \{\tau(t) + ap(t)\} < b \quad (14)$$

The two constants  $a$  and  $b$  are, for instance, related to classical experimental fatigue strengths  $S_f$  (in alternate bending) and  $S_t$  (in alternate twisting) by

$$a = \frac{S_t - S_f/2}{S_f/3} \quad b = S_t \quad (15)$$

The loading path is usually represented in the  $(p, \tau)$  plane. If the loading curve remains beneath Dang Van’s line no

fatigue crack will appear, otherwise rupture is presumed (see Section 3.3).

## 3. Numerical results

The examples of numerical results given in this paper are two-dimensional. They treat the case of a Hertzian pressure ( $P_0$  maximum pressure,  $a$  half contact width) and a tangential pressure distribution ( $\mu$  local friction coefficient,  $Q/P$  traction coefficient) moving over a half space.

### 3.1. Ratchetting rate

In this section we aim to evaluate numerically the ratchetting rate using the non-linear kinematic hardening law proposed by Bower [10] for rail steel. The material constants are:  $E = 208$  GPa,  $\nu = 0.3$ ,  $k_c = 231$  MPa,  $C = 33.9$  GPa,  $\gamma_1 = 8.3$ ,  $\gamma_2 = 0.41$ .

The computations are done using the PPSM method. The number of load passes is about 1000. The results are presented in Figs. 2 and 3. Fig. 2 shows an example of a deformed mesh for  $P_0/k_c = 4.76$  and  $\mu = 0$ . Fig. 3 shows for different values of  $P_0$  and  $\mu$  the evolution of the accumulated surface displacement  $\delta$ . The rates of accumulation of this displacement

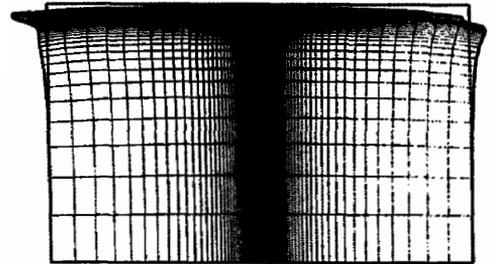


Fig. 2. Residual deformed mesh ( $\times 100$ ) after 1000 passes, for  $P_0/k_c = 4.76$ ,  $\mu = 0$ . The total length is  $26a$ .

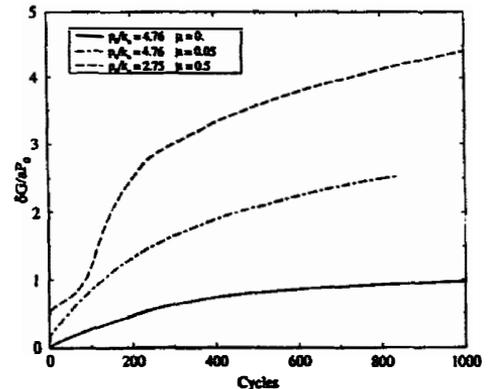


Fig. 3. Accumulated surface displacement for different values of  $P_0/k_c$  and  $\mu$ .

are significantly greater than those calculated by Bower and Johnson [11]. One can observe that in the case of high friction coefficient ( $\mu = 0.5$ ) the ratchetting rate has a period of increase before decreasing.

### 3.2. Temperature rise due to thermomechanical coupling

In this section, our objective is to evaluate the temperature rise generated by the volumetric heat sources and the interfacial dissipation, and also to quantify the effect of the normal load and the friction coefficient on the distribution of this temperature. This evaluation is performed only for the first pass of the load moving along the rail. Adiabatic conditions are assumed.

The characteristics of the considered material are:  $E = 210 \text{ GPa}$ ,  $\nu = 0.3$ ,  $k_c = 237 \text{ MPa}$ ,  $C = 32 \text{ GPa}$ ,  $\alpha = 1.5 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ ,  $\rho = 7800 \text{ kg m}^{-3}$ ,  $c = 449 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ,  $k = 48 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ .  $C$  is the hardening modulus,  $\alpha$  the coefficient of thermal expansion,  $k$  the thermal conductivity,  $c$  the specific heat,  $\rho$  the density. A line contact is considered moving with a velocity  $V = 500 \text{ km h}^{-1}$ . Its characteristics are:  $a = 8.75 \text{ mm}$ , the half contact width,  $P_0/k_c = 3.75$ ,  $\mu = 0.7$ , the ratio  $Q/P$  ( $Q$  total traction, and  $P$  normal force) varies from zero to 0.5 and takes three values (0, 0.2 and 0.5). Fig. 4 shows the variation in temperature rise at depth at the centre of the moving contact. Fig. 5 shows the variation of the surface temperature. The elevation of temperature is in this case not important (maximum =  $6.4 \text{ }^\circ\text{C}$ ). This result shows first that heat elevation could not be responsible for the metallurgical transformation (white layer) observed near the surface, and second that a thermomechanical analysis is not necessary for this problem.

### 3.3. Fatigue prediction

Calculations of the thermomechanical state for repeated rolling/sliding contacts are performed with the 'stationary methods'. The natures of the stabilized state, the actual thermomechanical state and the residual thermomechanical state are obtained. If the stabilized state is elastic (elastic shake-

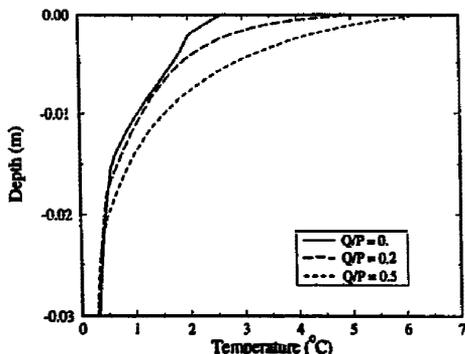


Fig. 4. Variation of temperature at depth at the centre of the contact for various  $Q/P$ .

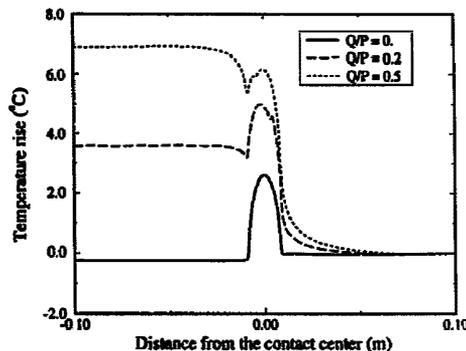


Fig. 5. Variation of temperature at the surface of the contact for various  $Q/P$ .

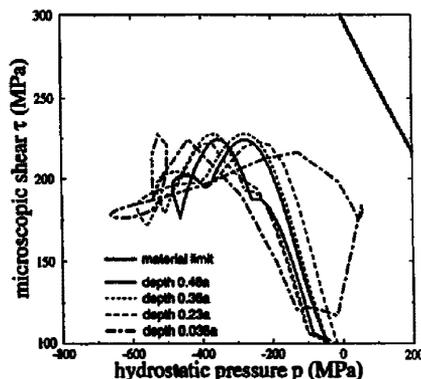


Fig. 6. Loading paths at different depths.

down), Dang Van's criterion is used to predict the initiation of fatigue cracks. In this paper we present one example of application for  $P_0/k_c = 3.75$ ,  $\mu = 0.7$  and  $Q/P = 0.1$

The stabilized state reached after six passes of the load is an elastic shakedown.

The loading path in the  $(p, \tau)$  plane is represented in Fig. 6. The material's limit is represented by the line  $(ap + \tau = b, a = 0.638 \text{ and } b = 300 \text{ MPa})$ . One can see that loading curves remain beneath the material's limit, thus no fatigue crack will appear. This kind of procedure is systematically used for the study of rail fatigue.

## 4. Conclusion

The numerical procedure (PPSM) presented in this paper is exact within the finite element approximation. It permits the investigation of a very large class of problems involved in the rail/wheel contact: stabilized state, ratchetting rate, temperature rise. By coupling this method with appropriate fatigue criteria, a good tool for the prediction of (high cycle or low cycle) fatigue crack appearance can be obtained. In this paper an illustration only for high cycle fatigue has been done. Nevertheless, three-dimensional and dynamic struc-

tural effects, real contact characteristics and more sophisticated processes, such as the experimentally observed metallurgical transformation, must be taken into account to achieve a realistic model for the prediction of fatigue failure.

## Appendix A. Nomenclature

$a$	half-contact width
$a$	constant of Dang Van criterion = $(S_t - S_r/2) / (S_r/3)$
$A_k$	associated forces
$b$	constant of Dang Van criterion = $S_t$
$c$	the specific heat
$C$	hardening modulus
$E$	Young's modulus
$f$	yield function
$S_t$	alternate bending fatigue strength
$k_o$	shear yield stress
$k$	thermal conductivity
$l$	tensor defining $\alpha_{k,x}$ via $\Lambda$
$L$	tensor of elastic coefficients
$p$	microscopic hydrostatic pressure
$P$	normal force
$P_0$	maximum Hertzian pressure
$Q$	traction force
$r$	volumetric heat source
$s$	entropy
$t$	time
$S_t$	alternate twisting fatigue strength
$u$	displacement vector
$V$	moving load velocity
$Z$	tensor relating $A_k$ and $\alpha_k$
$\alpha$	coefficient of thermal expansion
$\alpha_k$	internal parameters
$\gamma_1, \gamma_2$	material constants of the non-linear kinematic hardening proposed by Bower
$\epsilon$	strain tensor
$\epsilon^e$	elastic strain tensor
$\epsilon^p$	plastic strain tensor
$\theta$	temperature
$\Lambda$	plastic multiplier
$\mu$	local friction coefficient
$\nu$	Poisson ratio
$\rho$	density
$\sigma$	stress tensor
$\sigma_0$	initial stress tensor
$\tau$	microscopic Tresca shear stress

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## Biographies

Prof. Ky Dang Van is Doctor-es-Science of the University of Paris 6, Engineer of the Ecole Nationale des Ponts et Chaussées (Paris) and Research Director of the CNRS (French National Centre for Scientific Research). He is in charge of research in fatigue, fracture and numerical simulation of processes in the Laboratoire de Mécanique des Solides of the Ecole Polytechnique (Palaiseau, France); his fatigue multiaxial criterion is widely used.

Dr. M. Habibou Maitournam obtained both his engineering degree and his PhD at the Ecole Nationale des Ponts et Chaussées (Paris). He is Research Engineer at the Laboratoire de Mécanique des Solides of the Ecole Polytechnique (Palaiseau, France) and is working in the field of numerical analysis in thermo-elastoplasticity and fatigue of structures.

Mr. Bernard Prasil is Ingénieur Principal at the SNCF (French National Railways Company). He is in charge of the Rails Subdivision at the Equipment Office of the SNCF.