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# Identification of rock mass properties in elasto-plasticity

Desheng Deng\*, Duc Nguyen-Minh

*Laboratoire de Mécanique des Solides, École Polytechnique, 91128 Palaiseau, France*

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## Abstract

A simple and effective back analysis method has been proposed on the basis of a new criterion of identification, the minimization of error on the virtual work principle. This method works for both linear elastic and nonlinear elasto-plastic problems. The elasto-plastic rock mass properties for different criteria of plasticity can be well identified based on field measurements.

*Keywords:* Identification; Back analysis; Virtual work principle; Rock mass; Elasto-plasticity

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## 1. Introduction

In geomechanical engineering, the complexity of rock masses makes the identification of their properties very difficult. Even if many rock samples are collected in situ to determine the mechanical parameters of rock masses in laboratory tests, the data obtained from these tests with small samples cannot be representative of the large structure scale [1]. In situ tests seem to be more convenient in obtaining actual mechanical parameters of rock masses. But in situ tests are generally very costly, and the results obtained may depend on the location of instruments, especially when a large underground structure is concerned. These difficulties provoke uncertainties in the input parameters for a numerical model, which often produce a discrepancy between the prevision and the observation of the real structure. In order to overcome these difficulties, the observational design method is introduced [2] where back analysis techniques are employed.

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\* Corresponding author at Département CGM—Mines, École Polytechnique de Montréal, C.P. 6079, Succ. Centre-ville, Montréal Qc, Canada H3C 3A7. Tel.: +1-514-340-4711 x5189; fax: +1-514-340-4477.  
*E-mail address:* desheng.deng@polymtl.ca (D. Deng).

Back analysis, or parameter identification, is frequently used in geomechanical engineering [1–5] since the 1970s, for determining the mechanical parameters of rock masses on the basis of field measurements performed during in situ tests, or during excavation or construction works. In general terms, a back analysis requires the choice of an objective function, (cost function, or error function), representing the discrepancy between the quantities  $u_i^*$  measured in the field and the corresponding data  $u_i$  obtained by the stress analysis of a numerical model [1,3,5]:

$$\mathbf{F} = \left\{ \sum_{i=1}^n [u_i(\mathbf{P}) - u_i^*]^2 \right\}^{1/2} \quad (1)$$

where  $n$  is the number of measurements,  $\mathbf{P}$  is the vector of unknown parameters. This function is minimized with respect to the unknown parameters  $\mathbf{P}$  that govern the mechanical problem, thus leading to optimal values of the mechanical properties for the rock masses [6].

The objective function defined by Eq. (1) is a highly nonlinear function of parameters  $\mathbf{P}$ , even when linear elastic structure behavior is assumed. Since an analytical expression of  $\mathbf{F}$  cannot be defined, it is very difficult to obtain an analytical evaluation of the function gradients. These properties of the objective function make the back analysis procedure very costly, especially in elasto-plasticity [7,8].

Indeed, the behavior of the objective function significantly affects the efficiency of a nonlinear optimization procedure [1]. In recent years, many efforts are made [9,10] for new objective functions which make the characterization problem more efficient with the explicit gradients of objective functions. In the present work, a new criterion of identification is established, based on minimization of error on the virtual work principle, and a back analysis method based on this criterion is proposed. Attempts are made, in particular, to identify the elasto-plastic rock mass properties.

## 2. Direct and modified problems

To present the back analysis method, let consider first the direct and modified problems.

### 2.1. Direct problem

A structure occupies a domain  $\Omega$ , with border  $\Gamma$  (Fig. 1). Let suffix  $i$  be the three orthogonal directions in space, and exponent  $d$  represent given components. The limit boundary conditions of  $\Omega$  can be defined as follows:

$$\begin{cases} \sigma_{ij} \cdot \mathbf{n}_j = \mathbf{T}_i^d & \text{on } \mathbf{S}_{T_i} \\ \mathbf{u}_i = \mathbf{u}_i^d & \text{on } \mathbf{S}_{u_i} \end{cases} \quad (2)$$

with  $\mathbf{S}_{T_i} \cup \mathbf{S}_{u_i} = \Gamma$  and  $\mathbf{S}_{T_i} \cap \mathbf{S}_{u_i} = \emptyset$ ,  $i = 1, 2, 3$ . Where  $\mathbf{u}$  and  $\sigma$  are displacement and stress respectively;  $\mathbf{n}$  is the external normal vector of  $\Gamma$ ;  $\mathbf{T}$  is surface force. Let

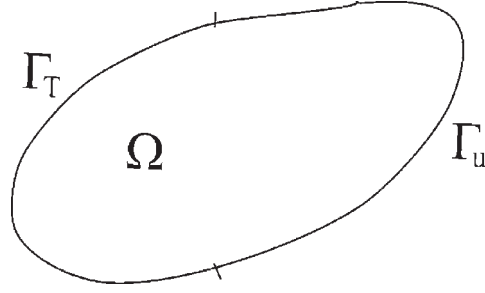


Fig. 1. Direct problem.

$\Gamma_T = \mathbf{S}_{T_i}$  ( $i = 1, 2, 3$ ) be the border with force prescribed and  $\Gamma_u = \mathbf{S}_{u_i}$  ( $i = 1, 2, 3$ ) the border with displacement prescribed.

The initial state (at time  $t=0$ ) is given as:

$$\begin{cases} \mathbf{u} = \mathbf{0} \\ \boldsymbol{\sigma} = \boldsymbol{\sigma}_0 \end{cases} \quad (3)$$

During a certain time interval  $[0, t_f]$ , the structure is submitted to a known load history, with

$$\begin{cases} \boldsymbol{\sigma}_0 \cdot \mathbf{n}|_{\Gamma_T} = \mathbf{T}_0 \\ \boldsymbol{\sigma}_{t_f} \cdot \mathbf{n}|_{\Gamma_T} = \mathbf{T}_{t_f} \end{cases} \quad (4)$$

In this time interval, the displacement field has to be kinematically admissible with the limit condition on border  $\Gamma_u$ :

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \frac{1}{2}(\nabla \dot{\mathbf{u}} + \nabla^T \dot{\mathbf{u}}) \\ \dot{\mathbf{u}}|_{\Gamma_u} = \dot{\mathbf{u}}^d \end{cases} \quad (5)$$

and the stress field has to be statically admissible with the limit condition on the border  $\Gamma_T$ :

$$\begin{cases} \mathbf{div} \dot{\boldsymbol{\sigma}} + \rho \dot{\mathbf{f}} = 0 \\ \dot{\boldsymbol{\sigma}} \cdot \mathbf{n}|_{\Gamma_T} = \dot{\mathbf{T}}^d \end{cases} \quad (6)$$

where  $(\dot{\phantom{x}})$  is derivative with respect to time,  $\rho$  is the mass density and  $\mathbf{f}$  is the massive force. To simplify the writing, let suppose

$$\begin{cases} \dot{\mathbf{u}}^d = \mathbf{0} \\ \dot{\mathbf{f}} = \mathbf{0} \end{cases} \quad (7)$$

which is a general case met in geomechanical engineering.

With the constitutive law of material and corresponding parameters known, the displacement  $\mathbf{u}_r$ , strain  $\boldsymbol{\varepsilon}_r$  and stress  $\boldsymbol{\sigma}_r$  in domain  $\Omega$  at time  $t = t_f$  can be calculated by a stress analysis method, for example the finite element method. According to the virtual work principle, the following relation is obtained:

$$\int_{\Omega} (\boldsymbol{\sigma}_r - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon}_r \, d\mathbf{x} - \int_{\Gamma_T} (\mathbf{T}_{T_f} - \mathbf{T}_0) \cdot \mathbf{u}_r \, d\mathbf{x} = \mathbf{0} \quad (8)$$

This equation is always verified for solutions of the direct problem,  $\mathbf{u}_r$ ,  $\boldsymbol{\varepsilon}_r$  and  $\boldsymbol{\sigma}_r$ , whatever the constitutive law of the structure is.

## 2.2. Modified problem

In the modified problem, the structure domain  $\Omega$  and its border is defined as hereinabove, with the initial state (3) and the load history known. But there are unknown parameters in the constitutive law of the material. Suppose that displacement measurements are obtained on the border  $\Gamma_T^u \subset \Gamma_T$  (Fig. 2) during the time interval  $[0, t_f]$ ,

$$\dot{\mathbf{u}}|_{\Gamma_T^u} = \dot{\mathbf{u}}^m \quad (9)$$

Thus, in comparison with the direct problem, the limit boundary conditions of the modified problem are modified. The border with displacement prescribed is augmented as  $\Gamma_u \cup \Gamma_T^u$ , whereas the border with force prescribed is reduced as  $\Gamma_T/\Gamma_T^u$ . The modified problem can be presented as follows:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \frac{1}{2}(\nabla \dot{\mathbf{u}} + \nabla^T \dot{\mathbf{u}}) \\ \dot{\mathbf{u}}|_{\Gamma_T^u} = \dot{\mathbf{u}}^m \end{cases} \quad (10)$$

$$\begin{cases} \mathbf{div} \, \dot{\boldsymbol{\sigma}} = \mathbf{0} \\ \dot{\boldsymbol{\sigma}} \cdot \mathbf{n}|_{\Gamma_T/\Gamma_T^u} = \dot{\mathbf{T}}^d \end{cases} \quad (11)$$

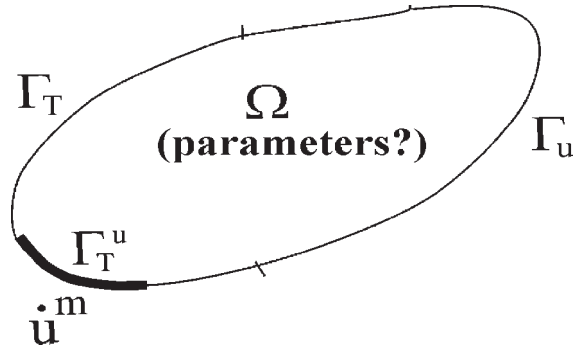


Fig. 2. Modified problem.

### 3. Back analysis method

The objective of the back analysis is to identify the unknown parameters  $\mathbf{P}$  which lead to previsions as close as possible to the measurements. Given any values for unknown parameters  $\mathbf{P}$ , the solutions of the modified problem are compatible with the measurement  $\dot{\mathbf{u}}^m$  on  $\Gamma_T^u$  and  $\dot{\mathbf{T}}^d$  on  $\Gamma_T/\Gamma_T^u$ , but not necessary with the limit conditions of the direct problem on  $\Gamma_T$ . In order to tend towards the results of the direct problem, the solutions of the modified problem should also satisfy at best the limit conditions of the direct problem.

To achieve the back analysis, an error on the virtual work principle is defined. At instant  $t_f$ , the error is

$$\mathbf{F}(\mathbf{P}, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}) = \left[ \int_{\Omega} (\boldsymbol{\sigma} - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon} \, d\mathbf{x} - \int_{\Gamma_T} (\mathbf{T}_{t_f} - \mathbf{T}_0) \cdot \mathbf{u} \, d\mathbf{x} \right]^2 \quad (12)$$

where  $\mathbf{u}$ ,  $\boldsymbol{\varepsilon}$ , and  $\boldsymbol{\sigma}$  are solutions (function of  $\mathbf{P}$ ) of the modified problem at instant  $t_f$ . Remind that  $\mathbf{T}_{t_f} - \mathbf{T}_0$  is the external force of the direct problem on  $\Gamma_T$ .

The criterion of the identification is to minimize the error  $\mathbf{F}$ , which is the cost function of back analysis. The principle of back analysis is to look for parameters satisfying the following equation:

$$\int_{\Omega} (\boldsymbol{\sigma} - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon} \, d\mathbf{x} - \int_{\Gamma_T} (\mathbf{T}_{t_f} - \mathbf{T}_0) \cdot \mathbf{u} \, d\mathbf{x} = 0 \quad (13)$$

which makes the gradients of the cost function  $\mathbf{F}$  vanish and ensures the minimum of  $\mathbf{F}$ . Eq. (13) is the basic equation of the back analysis.

It can be noted that  $\mathbf{W} = \int_{\Omega} (\boldsymbol{\sigma} - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon} \, d\mathbf{x}$  is the work of internal force of the modified problem and

$\Phi = \int_{\Gamma_T} (\mathbf{T}_{t_f} - \mathbf{T}_0) \cdot \mathbf{u} \, d\mathbf{x}$  is the work of the external force of the direct problem on displacement field of the modified problem.

Suppose that the basic Eq. (13) can be presented in the explicit form of parameters  $\mathbf{P}$ :

$$\tilde{\mathbf{F}}(\mathbf{P}, \mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}) = 0 \quad (14)$$

As  $\mathbf{u}$ ,  $\boldsymbol{\varepsilon}$  and  $\boldsymbol{\sigma}$  are function of parameters  $\mathbf{P}$ , that expression provides in effect a relationship between updated parameter values  $\mathbf{P}^{n+1}$  and the precedent ones  $\mathbf{P}^n$ :

$$\tilde{\mathbf{F}}[\mathbf{P}^{n+1}, \mathbf{u}(\mathbf{P}^n), \boldsymbol{\varepsilon}(\mathbf{P}^n), \boldsymbol{\sigma}(\mathbf{P}^n)] = 0 \quad (15)$$

The solution of this equation reminds a fixed-point problem that needs in general an iterative procedure, which is supposed to converge. As the back analysis is based on the virtual work principle, this method is valid not only for linear elastic problems but also for nonlinear elasto-plastic ones. The principle of minimization using gradients of the cost function ensures an efficient back analysis. But as the analytical

expression of these gradients is not needed, the method is very simple. However, as one set of measurements (under the same load at a given time interval) gives only one basic equation, whatever the number of measurements, only one parameter can be characterized at the same time. Thus, to identify  $N$  parameters simultaneously,  $N$  sets of measurements (under different loads at different time intervals) are needed. If the parameters to be identified cannot be expressed explicitly in Eq. (14), this identification procedure is not applicable.

## 4. Identification of parameters

### 4.1. Elastic moduli

Supposing the domain  $\Omega$  is linear elastic isotropic homogeneous, according to the constitutive law, we have

$$\boldsymbol{\sigma} - \boldsymbol{\sigma}_0 = \frac{\nu \mathbf{E}}{(1 + \nu)(1 - 2\nu)} (\text{tr } \boldsymbol{\varepsilon}) \mathbf{I} + \frac{\mathbf{E}}{1 + \nu} \boldsymbol{\varepsilon} \quad (16)$$

with  $\mathbf{E}$  the Young's modulus,  $\nu$  the Poisson's ratio, and  $\mathbf{I}$  the unit tensor. Thus, the basic Eq. (13) becomes

$$\frac{\nu \mathbf{E}}{(1 + \nu)(1 - 2\nu)} \int_{\Omega} (\text{tr } \boldsymbol{\varepsilon})^2 \mathbf{d}\mathbf{x} + \frac{\mathbf{E}}{1 + \nu} \int_{\Omega} \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} \mathbf{d}\mathbf{x} - \Phi = \mathbf{0} \quad (17)$$

From Eq. (17), an iterative procedure can be established for back analysis of the Young's modulus  $\mathbf{E}$  or the Poisson's ratio  $\nu$ . For example, if  $\mathbf{E}$  has to be identified, Eq. (17) gives

$$\mathbf{E} = \frac{\Phi}{\frac{\nu}{(1 + \nu)(1 - 2\nu)} \int_{\Omega} (\text{tr } \boldsymbol{\varepsilon})^2 \mathbf{d}\mathbf{x} + \frac{1}{1 + \nu} \int_{\Omega} \boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} \mathbf{d}\mathbf{x}} \quad (18)$$

If  $\nu$  has to be identified, it is the positive solution of (17) as a second degree equation of  $\nu$ .

Anisotropic and non-homogeneous cases can also be handled in the similar way by putting the corresponding constitutive law in the basic equation of back analysis.

### 4.2. Plastic parameters

Consider the material in domain  $\Omega$  as elasto-plastic. After resolving the modified problem, the plastified zone  $\Omega^p$  can be distinguished from the elastic one  $\Omega^e$  in domain  $\Omega$ , at time  $t = t_f$ . Thus,

$$\mathbf{W} = \mathbf{W}^e + \mathbf{W}^p \quad (19)$$

with

$$\mathbf{W}^e = \int_{\Omega^e} (\boldsymbol{\sigma} - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon} \, d\mathbf{x}, \quad \mathbf{W}^p = \int_{\Omega^p} (\boldsymbol{\sigma} - \boldsymbol{\sigma}_0) : \boldsymbol{\varepsilon} \, d\mathbf{x} \quad (20)$$

Considering the plastic criterion in  $\mathbf{W}^p$ , it is possible to obtain a relationship between the plastic parameters  $\mathbf{P}$  and the solutions of strain and stress of the modified problem by the basic Eq. (13), so that the parameters  $\mathbf{P}$  can be identified. Solutions for von Mises and Mohr-Coulomb plastic criteria in elastic perfect plasticity are presented bellow. The same procedure also holds for other criteria of plasticity.

- **Von Mises criterion**

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\mathbf{c}^2 \quad (21)$$

where  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are ordered principal stresses ( $\sigma_1 \geq \sigma_2 \geq \sigma_3$ ),  $\mathbf{c}$  is the yield stress for simple shearing. Let  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_3$  be the components of strain in the principal stress coordinates. In order to simplify the writing, let us suppose  $\boldsymbol{\sigma}_0 = \mathbf{0}$ . From Eq. (21), we have

$$\sigma_1 = \frac{1}{4} \left\{ 2(\sigma_2 + \sigma_3) + \sqrt{4(\sigma_2 + \sigma_3)^2 - 8[\sigma_2^2 + \sigma_3^2 + (\sigma_2 - \sigma_3)^2 - 2\mathbf{c}^2]} \right\} \quad (22)$$

Thus

$$\begin{aligned} \mathbf{W}^p &= \int_{\Omega^p} \boldsymbol{\sigma} : \boldsymbol{\varepsilon} \, d\mathbf{x} = \int_{\Omega^p} (\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3) \, d\mathbf{x} \\ &= \frac{1}{2} \int_{\Omega^p} [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 + \sigma_3)^2 - 2[\sigma_2^2 + \sigma_3^2 + (\sigma_2 - \sigma_3)^2 - 2\mathbf{c}^2]}] \epsilon_1 \, d\mathbf{x} \\ &\quad + \int_{\Omega^p} (\sigma_2 \epsilon_2 + \sigma_3 \epsilon_3) \, d\mathbf{x} \end{aligned} \quad (23)$$

From the basic Eq. (13), the following equation is derived:

$$\begin{aligned} \frac{1}{2} \int_{\Omega^p} [(\sigma_2 + \sigma_3) + \sqrt{(\sigma_2 + \sigma_3)^2 - 2[\sigma_2^2 + \sigma_3^2 + (\sigma_2 - \sigma_3)^2 - 2\mathbf{c}^2]}] \epsilon_1 \, d\mathbf{x} \\ + \int_{\Omega^p} (\sigma_2 \epsilon_2 + \sigma_3 \epsilon_3) \, d\mathbf{x} + \mathbf{W}^e - \Phi = \mathbf{0} \end{aligned} \quad (24)$$

The parameter  $\mathbf{c}$  can then be obtained from Eq. (24) by dichotomy.



- **Mohr-Coulomb criterion**

$$\frac{1}{2}(\sigma_1 - \sigma_3) = c \cos\varphi - \frac{1}{2}(\sigma_1 + \sigma_3)\sin\varphi \quad (25)$$

with  $c$  the cohesion and  $\varphi$  friction angle of material. Then we obtain

$$\sigma_1 = \frac{2c \cos\varphi + \sigma_3(1 - \sin\varphi)}{1 + \sin\varphi} \quad (26)$$

$$\mathbf{W}^p = \int_{\Omega^p} \frac{2c \cos\varphi + \sigma_3(1 - \sin\varphi)}{1 + \sin\varphi} \varepsilon_1 \, d\mathbf{x} + \int_{\Omega^p} (\sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) d\mathbf{x} \quad (27)$$

The basic Eq. (13) then gives:

$$\int_{\Omega^p} \frac{2c \cos\varphi + \sigma_3(1 - \sin\varphi)}{1 + \sin\varphi} \varepsilon_1 \, d\mathbf{x} + \int_{\Omega^p} (\sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3) d\mathbf{x} + \mathbf{W}^e - \Phi = \mathbf{0} \quad (28)$$

Back analysis of the friction angle  $\varphi$  needs to solve Eq. (28) by dichotomy. Whereas, for cohesion  $c$ , we have an explicit solution from (28):

$$c = \frac{\Phi - \mathbf{W}^e - \int_{\Omega^p} [\sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3 + \frac{\sigma_3(1 - \sin\varphi)}{1 + \sin\varphi} \varepsilon_1] d\mathbf{x}}{\frac{2 \cos\varphi}{1 + \sin\varphi} \int_{\Omega^p} \varepsilon_1 \, d\mathbf{x}} \quad (29)$$

This back analysis method works also for identifying elasto-plastic joint parameters and loading parameters [11,12].

## 5. Example

Let us consider a deep tunnel without support (Fig. 3) driven in a continuous rock mass, with homogeneous isotropic initial stress  $\sigma_o = 20$  kPa, and let this problem be considered as a plane strain problem. For symmetry reason, we consider a quarter model, representing a tunnel of radius  $R$  in the middle of a square zone of  $20R \times 20R$ .

At first, a direct calculation is performed with a given constitutive law and corresponding parameters, in order to obtain “measurement” which will be used for the identification problem.

### 5.1. Identification of Young's modulus

Suppose the rock mass is elastic linear, with Young's modulus  $\mathbf{E} = 100$  MPa and Poisson's ratio  $\nu = 0.2$  for the direct calculation. The tunnel excavation is

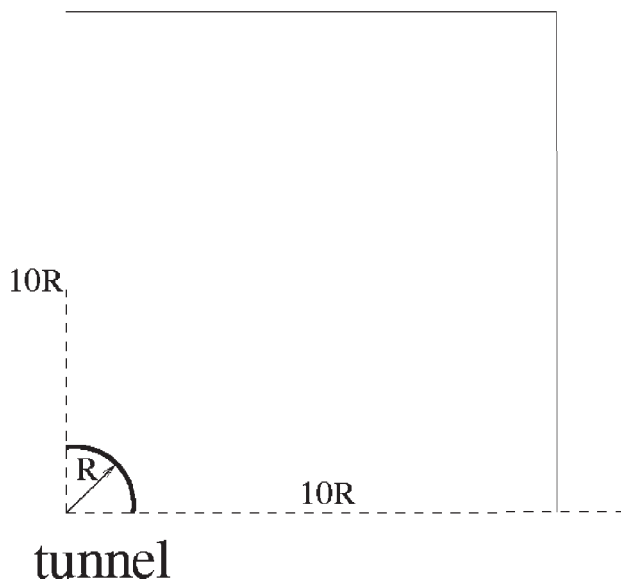


Fig. 3. Example, a deep tunnel.

simulated by the principle of convergence confinement method. The displacements on half the tunnel wall in the direct calculation after full excavation are selected as measurements. Then the Young's modulus is back analysed by an iterative process. Fig. 4 shows a good iteration convergence with less 5% discrepancy after 4 iterations with different initial guess values of Young's modulus  $E$ .

Fig. 5 illustrates the evolution of error  $F$  on the virtual work principle with modulus to be identified, and Fig. 6 shows the fast decrease of the error during iteration process of the back analysis for the initial guess value of  $E = 50$  MPa.

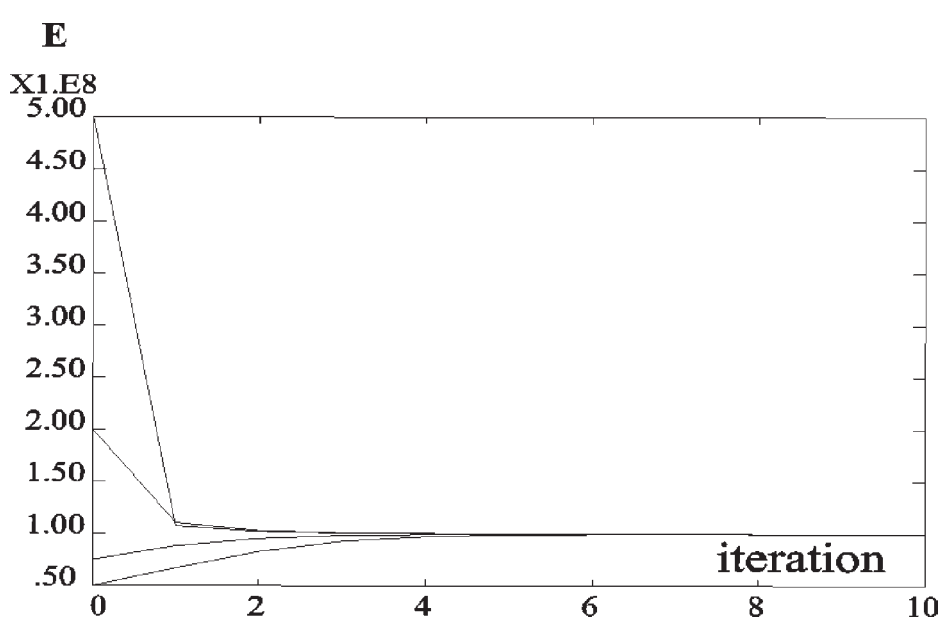


Fig. 4. Iteration convergence of  $E$  with different initial values.

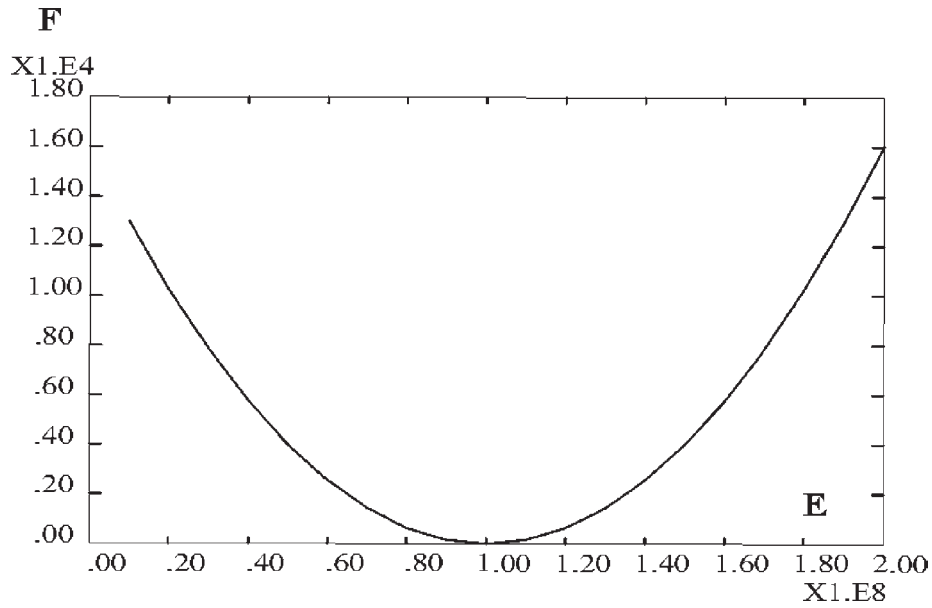


Fig. 5. Evolution of error  $F$  with Young's modulus.

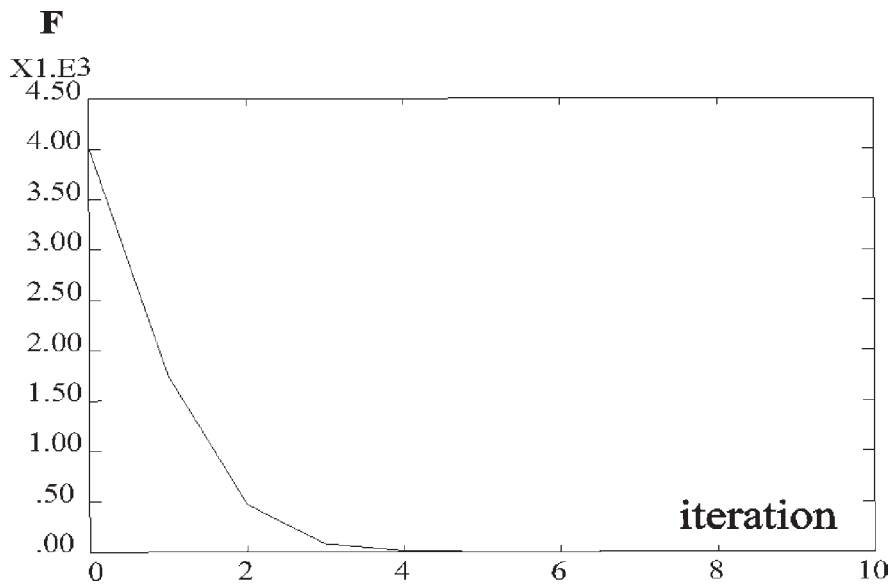


Fig. 6. Decrease of error during back analysis.

### 5.2. Identification of plastic parameter for Von Mises criterion

Suppose the rock mass is an elastic-perfect plastic material with Von Mises criterion. For direct calculation, let the plastic parameter be  $c = 10$  kPa. After full excavation, a plastic zone around the tunnel is developed with a plastic radius  $R_p = 3.5 R$ , and the corresponding tunnel convergence  $U_p$  is 4 times  $U_e$ , the convergence of the tunnel if supposed elastic. Using the displacements on half the tunnel wall of the direct calculation, the parameter  $c$  is back analysed with a fast iteration convergence (see Fig. 7). Due to the large plastic zone, the back calculated

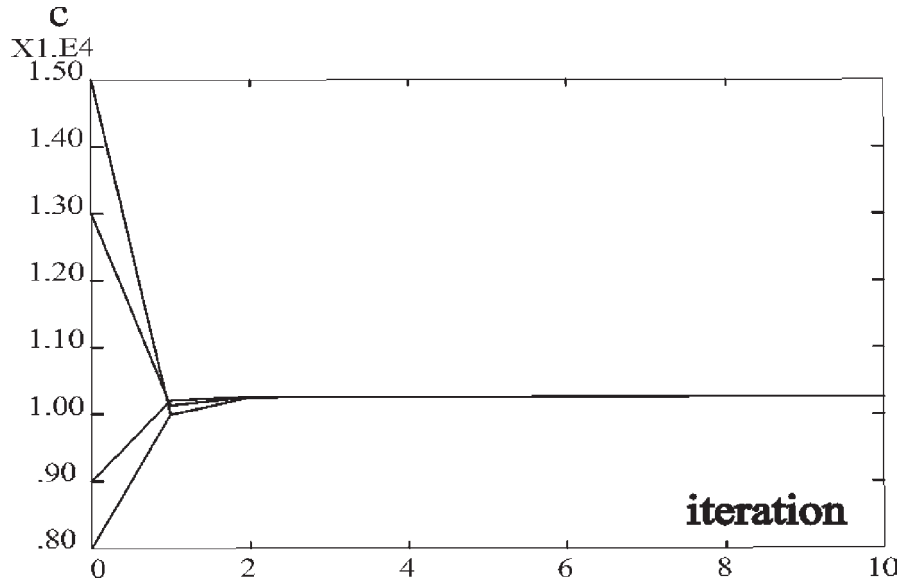


Fig. 7. Iteration convergence of  $c$  (Von Mises criterion) with different initial values.

parameter bears an error of 2.5%. To verify the stability of the back analysis, a noise of 10% is allowed to the measurements, which results in an error of 3.5% on the identified parameter.

### 5.3. Identification of plastic parameters for Druck–Prager criterion

Suppose the rock mass is elastic-perfect plastic with Druck–Prager plastic criterion. In the direct calculation, the plastic parameters are given as  $c = 10$  kPa and  $\varphi = 30^\circ$ . In order to identify two parameters, two sets of “measurements” are calcu-

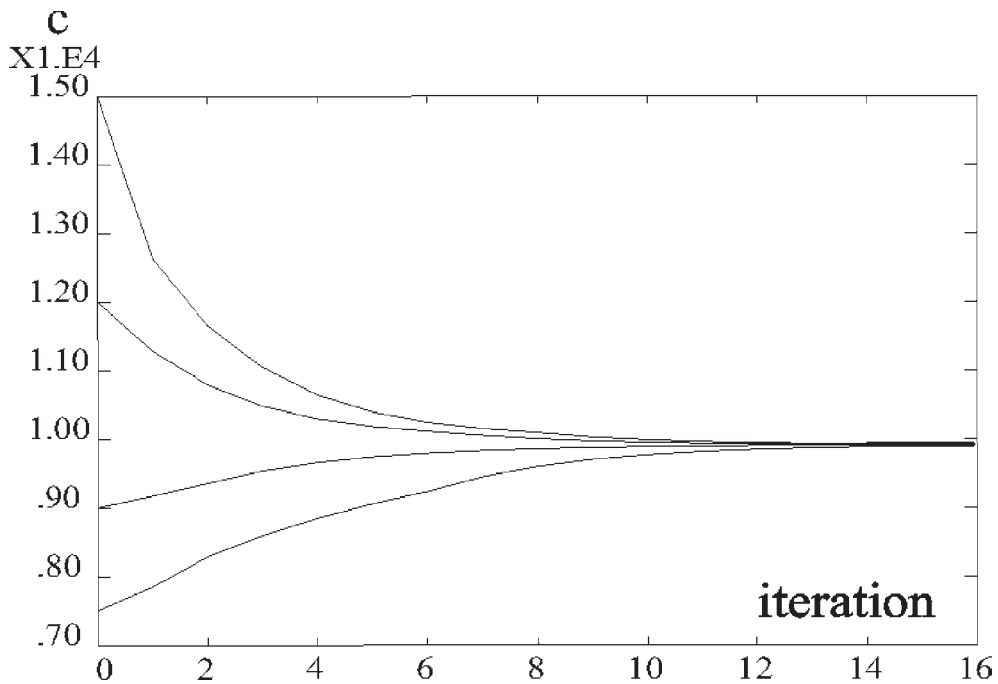


Fig. 8. Iteration convergence of  $c$  (Druck–Prager criterion) with different initial values.

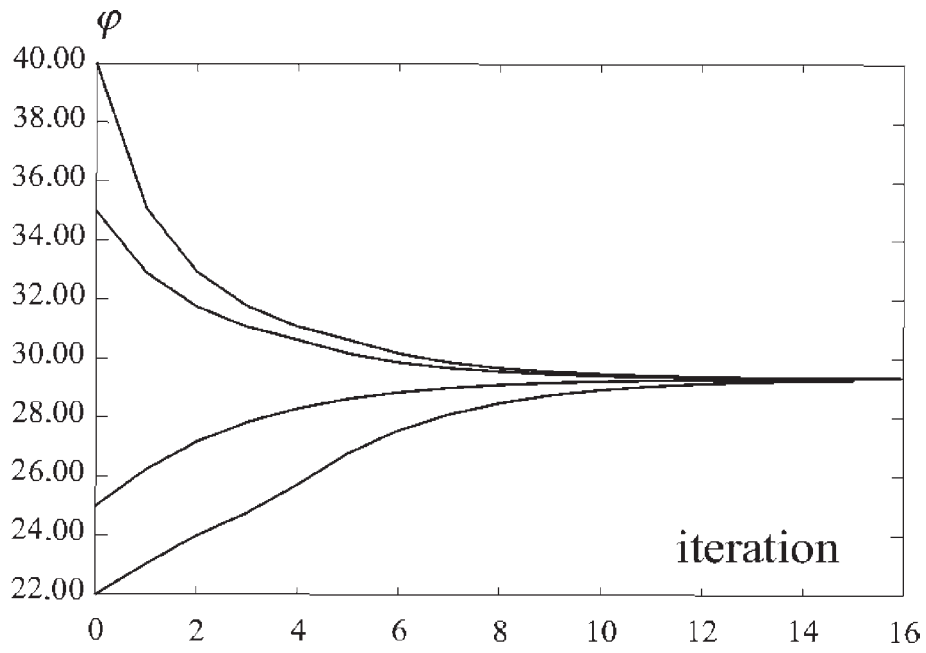


Fig. 9. Iteration convergence of  $\varphi$  (Druck-Prager criterion) with different initial values.

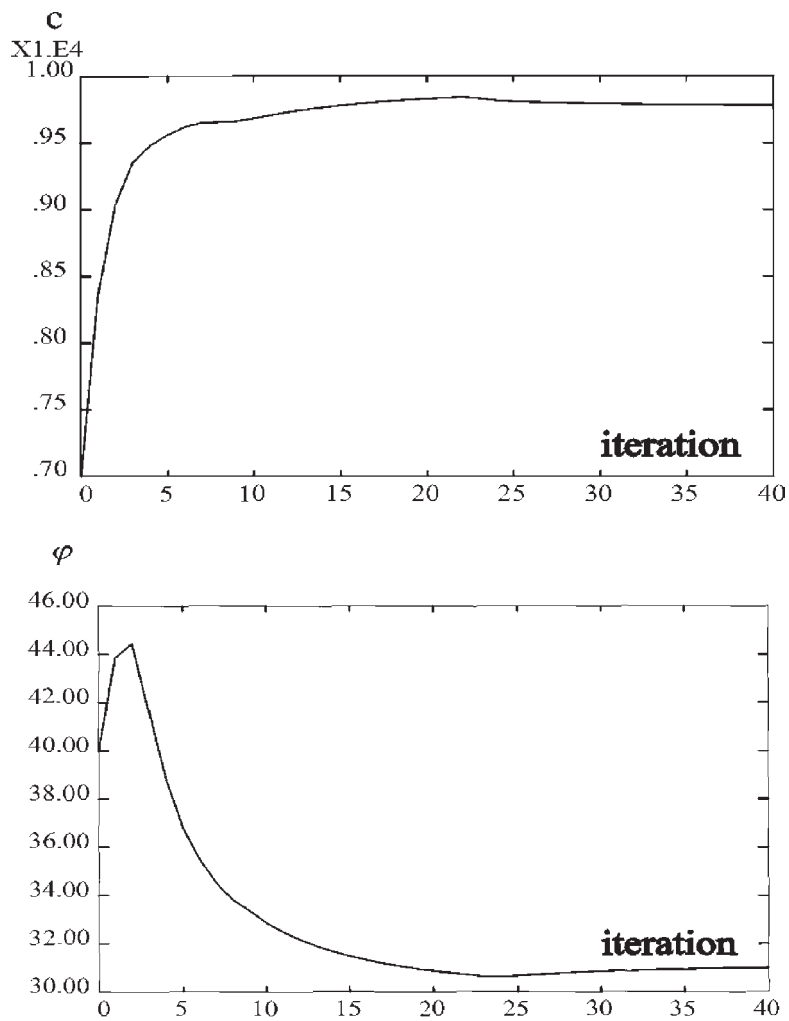


Fig. 10. Iteration convergence of  $c$  and  $\varphi$  (Druck-Prager criterion).

lated. One set corresponds to 60% tunnel excavation with  $R_p = 1.5 R$  and  $U_p = 1.2 U_e$ , and the other one corresponds to whole excavation with  $R_p = 2.7 R$  and  $U_p = 2.0 U_e$ . At first, only one parameter ( $\mathbf{c}$  or  $\varphi$ ) is back analyzed, and the other one is supposed known, using the displacements on half the tunnel surface of 60% excavation. Fig. 8 is the back analyzed cohesion  $\mathbf{c}$ , and Fig. 9 is the back analyzed friction angle  $\varphi$ . After 8–9 iterations, these parameters are identified within 5% error.

Then two parameters are simultaneously back analyzed, using two sets of “measurements”, the displacements on half of the tunnel wall corresponding to 60% and full tunnel excavation, the parameters  $\mathbf{c}$  and  $\varphi$  are back analysed simultaneously. The iteration convergence for identifying  $\mathbf{c}$  and  $\varphi$  are shown in Fig. 10, with the initial parameter guess values as  $\mathbf{c} = 7$  kPa and  $\varphi = 40^\circ$ . After 20 iterations, these parameters are identified with an error less than 5%.

## 6. Conclusion

A back analysis method based on the virtual work principle is presented with the establishment of a new criterion of identification, the minimization of error on the virtual work principle. The method is valid for both linear elastic and nonlinear elasto-plastic problems. In the back analysis, the minimization principle using the gradients of a cost function is employed, but it is not necessary to obtain the gradients. Thus, the method is very simple and easy to be programmed, and shows to be efficient and robust. If several sets of measurements are given, the identification of several parameters can be done simultaneously. This paper demonstrates its use in identification of elasto-plastic material parameters, such as elastic moduli, friction angle and cohesion of rock mass. The numerical example illustrates the identification of rock mass in a tunnel excavation problem in elasticity and in elasto-plasticity. The Young’s modulus and plastic parameters for Von Mises and Druck–Prager criteria are well characterized. The iterative process converges very quickly and stably, whether the rock mass is modelled by linear elastic or nonlinear behaviour.

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