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► **To cite this version:**

Ahmed Chemori, Nicolas Marchand. A prediction based controller for trajectories tracking and stabilization of a non-minimum phase PVTOL aircraft. NMPCFS: Nonlinear Model Predictive Control for Fast Systems, Oct 2006, Grenoble, France. hal-00108453

**HAL Id: hal-00108453**

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Submitted on 11 Sep 2019

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# A PREDICTION BASED CONTROLLER FOR STABILIZATION OF A NON-MINIMUM PHASE PVTOL AIRCRAFT

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Abstract: In this paper a nonlinear prediction-based control approach is proposed for stabilization of Planar Vertical Take-Off and Landing (PVTOL) aircraft. This system has fewer control inputs than degrees of freedom (i.e. under-actuated) and has unstable zero dynamics (i.e. non-minimum phase). The proposed control approach is based on partial feedback linearization, which allows the emergence of a completely linearized sub-system and internal dynamics. Then prediction based optimal trajectories are proposed for the linearized variables, where the optimization objective is to enhance the behavior and the stability of the internal dynamics. Stability analysis of the closed-loop system is performed using a graphical tool based on Poincaré's section. The performance of the proposed scheme is illustrated through simulations.

Keywords: PVTOL aircraft, non-minimum phase, internal dynamics, nonlinear predictive control, stabilization, robustness.

## 1. INTRODUCTION

Vertical Take-off and Landing (VTOL) describes flying machines that can lift off and land up vertically. This class of machines includes helicopters, balloons and few aircrafts. PVTOL aircrafts have the same property but projected in the plane.

The simplified PVTOL is a mathematical model of flying aircraft that evolve in the vertical plane with three degrees of freedom. These coordinates correspond to its position and orientation in the frontal plane. However this system is equipped with only two thrusters that produce a force and a moment on the flying machine.

PVTOL stabilization represents a challenging and a very interesting nonlinear control problem since it includes difficulties such as under-actuation and non-minimum phase features. Consequently it gains more and more interest within the control community. In the last few years, many control strategies have been proposed to control the sim-

plified PVTOL aircraft, which has a minimum number of states, but that retains the main features that must be considered when designing controls for a real aircraft.

The existing works can be subdivided into two main classes: the first one addresses the trajectory tracking problem, whereas the second one aims to solve the stabilization problem. The first set of results dealing with trajectory tracking is chronologically earlier. Hauser et al. (1992a) proposed an approximate input-output linearization which results in bounded tracking (and stabilization) for the V/STOL. This approach was then extended by Martin et al. (1996) with a flat output approach. In contrast to the approximate linearization proposed by Hauser, the Martin's approach enables to take into account non-minimum phase flat systems as well as a non zero coupling parameter. A multirate digital approach was initially proposed in Di Giamberardino and Djemai (1994)

and then further extended. It is worth to emphasize that these works assume that the rolling angle remains within the interval  $]-\frac{\pi}{2}, \frac{\pi}{2}[$ .

The efforts brought to achieve global stabilization are however more recent. In Fantoni et al. (2002), taking first the rolling angle directly as control and after a coordinate change, the system is transformed to a double integrator chain. Using a result of Teel et al. (1995), this first subsystem is stabilized with a positive and bounded thrust. A tracking strategy is then used to force the roll angle to converge to the trajectory required to stabilize the first subsystem. This strategy that initially enabled only bounded thrust was extended in Zavala et al. (2003) to take care also of a bounded angular acceleration. It should be emphasized that the result obtained is global contrary to most preceding results. Beside this important work, a robust stabilization can be found in Lin et al. (1999). The robust stabilization objective is formulated as an optimal control. The obtained Hamilton-Jacobi-Bellman (HJB) problem can then be explicitly solved for a particular parametrization of the controller, based on a transformation that often appears in the trajectory tracking framework, assumes that the rolling angle remains in the strict positive half plane  $]-\frac{\pi}{2}, \frac{\pi}{2}[$ . Furthermore, a small displacement assumption is made on the angle  $\theta$  in order to simplify the HJB problem formulation.

In this paper a recently developed control scheme (cf. Chemori and Alamir (2005)) is applied to control the PVTOL aircraft. Simulation results are given to state the efficiency of the proposed control approach.

The outline of the paper is as follows. In section 2 the PVTOL aircraft is described and its dynamic model is given. Section 3 is devoted to partial feedback linearization of the dynamic model. While in section 4 the proposed feedback controller is summarized. Stability analysis of the resulting closed-loop system is discussed in section 5. Section 6 is then dedicated to simulation results, and the paper ends with a conclusion.

## 2. DESCRIPTION & DYNAMICS OF THE PVTOL AIRCRAFT

Consider the nonlinear simplified dynamics of the PVTOL aircraft initially proposed in Hauser et al. (1992b), and widely used since:

$$\ddot{x} = -\sin(\theta)u_1 + \varepsilon \cos(\theta)u_2 \quad (1)$$

$$\ddot{y} = \cos(\theta)u_1 + \varepsilon \sin(\theta)u_2 - 1 \quad (2)$$

$$\ddot{\theta} = u_2 \quad (3)$$

where  $x$  and  $y$  represent respectively the horizontal and vertical position of the aircraft center

of mass as shown in figure 1.  $\theta$  is the roll angle that the aircraft makes with the horizon. The control inputs  $u_1$  and  $u_2$  represent the normalized quantities related to the vertical thrust directed upwards with respect to the aircraft and the angular acceleration (rolling moment).

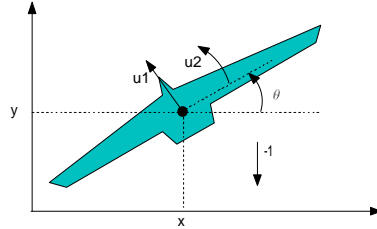


Fig. 1. View of the PVTOL aircraft

The parameter  $\varepsilon$  is a coefficient which characterizes the coupling between the rolling moment and the lateral acceleration of the aircraft. For usual aircrafts, this parameter is small but on recent applications like Unmanned Air Vehicles (UAV) or flapping robots, this parameter may have a non negligible influence. The coefficient  $'-1'$  in equation (2) corresponds to the normalized gravitational acceleration. The different model parameters listed above are illustrated in figure 1.

## 3. FEEDBACK LINEARIZATION

Consider the simplified dynamic model (1)-(3) of the PVTOL aircraft, and the two outputs  $y_1 = x, y_2 = y$ . Let us now derive the outputs until at least one of the inputs appear. The two outputs derived twice give:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -\sin(\theta) & \varepsilon \cos(\theta) \\ \cos(\theta) & \varepsilon \sin(\theta) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (4)$$

where the matrix  $D = \begin{bmatrix} -\sin(\theta) & \varepsilon \cos(\theta) \\ \cos(\theta) & \varepsilon \sin(\theta) \end{bmatrix}$  is called the decoupling matrix. It is non singular since  $\det(D) = -\varepsilon$ .

*Remark 1.* In the case where  $\varepsilon = 0$ , it exists a bijective coordinate transformation that brings the dynamics with  $\varepsilon = 0$  to the dynamics (1)-(3) Olfati-Saber (2000). Then, even though  $\varepsilon = 0$ , with this transformation the decoupling matrix is always non singular.

Consider now the control inputs chosen as :

$$\begin{cases} u_1 = -\sin(\theta)v_1 + \cos(\theta)(1 + v_2) \\ u_2 = \frac{1}{\varepsilon} \cos(\theta)v_1 + \frac{1}{\varepsilon} \sin(\theta)(1 + v_2) \end{cases} \quad (5)$$

These controls injected in the dynamics (1)-(3) give the following partial linearized system:

$$\ddot{x} = v_1 \quad (6)$$

$$\ddot{y} = v_2 \quad (7)$$

$$\ddot{\theta} = \frac{1}{\varepsilon}(\sin(\theta) + \cos(\theta)v_1 + \sin(\theta)v_2) \quad (8)$$

which could be written in the standard form of partially linearized systems (Khalil (1996))

$$\dot{\zeta} = A\zeta + Bv \quad ; \quad \zeta \in \mathbb{R}^{n_\zeta} \quad (9)$$

$$\dot{\eta} = Z(\zeta, \eta) \quad ; \quad \eta \in \mathbb{R}^{n_\eta} \quad (10)$$

with  $\zeta = [x \ \dot{x} \ y \ \dot{y}]^T$   $\eta = [\theta \ \dot{\theta}]^T$ . The completely linearized part is controllable, that is  $\text{rank}(\mathcal{C}(A, B)) = n_\zeta$ . Equation (10) defines the internal dynamics of the PVTOL system. Since the internal dynamics is unstable, then the system is called *non-minimum phase*.

#### 4. THE PREDICTION BASED FEEDBACK CONTROLLER

The control approach used to control the PVTOL aircraft is that proposed in Chemori and Alamir (2005) for systems with jumps, but it remains applicable for classical jump-free nonlinear systems, insofar as any classical  $n$ -dimensional jump-free system may be embedded in an  $(n + 1)$ -dimensional system with virtual jump (cf. lemma 1 in Chemori and Alamir (2005)). In this section the basic principle of the approach is summarized, while the forthcoming section is devoted to the stability analysis of the resulting closed-loop system. Roughly speaking, the basic idea of the proposed nonlinear predictive control scheme is to use the completely linearized sub-states  $\zeta$  to enhance the stability of the internal dynamics  $\eta$  of the system. This principle is illustrated on the block diagram of figure 2. The algorithm of the

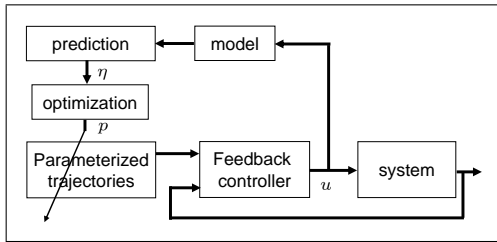


Fig. 2. The control approach principle

proposed scheme basically consists in the following steps :

**Step 1 :** Definition of B-spline  $p$ -parameterized trajectories on the directly controlled variables  $\zeta$  (cf. figure 3). These trajectories should link the two points  $\zeta^0, \zeta^f$  in the space  $\mathbb{R}^\zeta$ .  $\zeta^0$  is the initial sub-state, whereas  $\zeta^f$  ( $= 0$  for stabilization) is the desired final one. The sampling time is denoted by  $T$ , whereas the virtual jumps are  $\tau_c$ -equispaced, that is  $t_k = k\tau_c$ . The decision

instants over each interval  $[t_k, t_{k+1}]$  are defined by  $\tau_k^j = t_k + j\frac{\tau_c}{m}$  ;  $j = 0, \dots, m - 1$ . Where  $m$  is a design parameter which denotes the number of times that the optimal trajectories are updated between each two successive virtual jumps (for the PVTOL aircraft it is fixed to  $m = 2$ , cf. simulations later on).

**Step 2 :** Computation of the optimization parameter  $p$  based on a prediction of the internal dynamics, this is done by optimization of the following performance index:

$$\hat{p}_k := \text{Arg} \min_{p \in \mathcal{P}} \|\eta(t_{k+1}^-) - \eta^f\|_Q^2 \quad (11)$$

where  $\eta^f$  defines some desired sub-states on the internal dynamics. The optimization parameter  $\hat{p}_k$  is that invoked in step 1 which parameterizes the reference trajectories. It is chosen scalar in order to reduce the computation time. In doing so, the proposed control approach could easily be applied to fast systems control.

$\eta(t_{k+1}^-)$  that appears in (11) can be expressed by the following relation :

$$\eta(t_{k+1}^-) = F(\eta(t_k^-), p_k, \zeta^f) \quad (12)$$

$F$  is a prediction function of the internal dynamics over the interval  $[t_k, t_{k+1}]$ . The times  $t_k^-$  and  $t_{k+1}^-$  that appear in the relations (11),(12) represent instants just before the virtual jumps occurring at times  $t_k, t_{k+1}$  (respectively).

**Step 3 :** Tracking of the optimal reference trajectories until next decision instant  $\tau_k^{j+1}$  by means of the following feedback :

$$\begin{cases} v_1 = \ddot{x}_d + k_d(\dot{x} - \dot{x}_d) + k_p(x - x_d) \\ v_2 = \ddot{y}_d + k'_d(\dot{y} - \dot{y}_d) + k'_p(y - y_d) \end{cases} \quad (13)$$

where the subscript  $d$  denotes the reference trajectory,  $k_d, k_p, k'_d, k'_p$  are positive feedback gains. Then consider the actual state of the system as initial state and go to **step 1**.

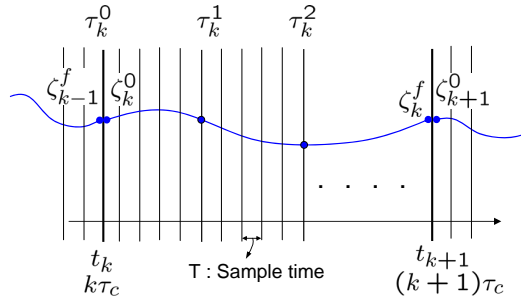


Fig. 3. Trajectories illustration

These three steps of the algorithm are to be executed at each decision instant  $\tau_k^j$ . Even though the second step includes an optimization problem to resolve, a real time application of the approach is possible, since the computation time is significantly reduced (cf. simulations).

## 5. STABILITY ANALYSIS

The stability of the closed-loop system (composed of completely linearized part and internal dynamics) depends on the stability of the internal dynamics under the chosen time varying feedback. To do that, a graphical tool based on the Poincaré's section is used (cf. Chemori and Alamir (2005)). The basic idea consists in plotting the following multi-step function for increasing values of  $r = \|\eta - \eta^f\|_Q^2$ :

$$\Psi_{k_0}^Q(r) := \left[ \sup_{\|\eta - \eta^f\|_Q^2 = r} \|F_{cl}^{k_0}(\eta, \zeta^f, \eta^f) - \eta^f\|_Q^2 \right] \quad (14)$$

where  $F_{cl}^{k_0}$  is obtained by two operations. The first one consists in injecting (11) in (12) which gives  $\eta(t_{k+1}^-) = F(\eta(t_k^-), \hat{p}(\eta(t_k^-), \zeta^f, \eta^f), \zeta^f)$ , then the later is expressed in multi-step from by  $k_0$  successive compositions as  $\eta(t_{k+k_0}^-) = F_{cl}^{k_0}(\eta(t_k^-), \zeta^f, \eta^f)$ .

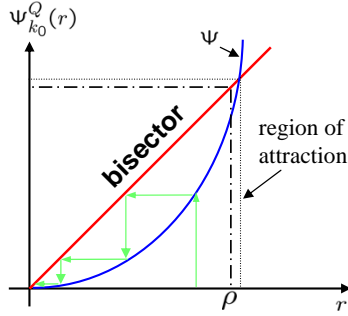


Fig. 4. Stability analysis tool : convergence to a stable limit cycle

According to the obtained curve, two cases are possible, they are illustrated in figures 4 and 5.

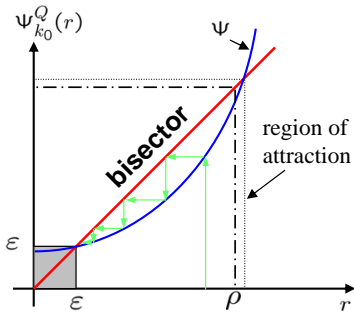


Fig. 5. Stability analysis tool : convergence to a neighborhood of a stable limit cycle

In the first case (cf. figure 4), a convergence to a stable limit cycle is obtained (in the case of stabilization this limit cycle should degenerate to a point on the phase portrait). However in the second case (cf. figures 5), a convergence to a neighborhood of a limit cycle is obtained. It is worth noting that the proposed graphical tool, besides stability analysis, enables the estimation of a region of attraction (cf. figures 4, 5). For the

PVTOL aircraft, the stability analysis falls in the first case (cf. next section, especially figure 6).

## 6. SIMULATION RESULTS

Consider the PVTOL aircraft dynamic model (1)-(3) with the coupling parameter  $\varepsilon = 0.5$ , the different parameters of the proposed control approach are summarized in table 1.

Parameter	Significance	value
$t_f$	The horizon length	1.85sec
$Q$	Weighting matrix	$Diag(1, 0.3)$
$p$	Optimization parameter	$x(t_f/4)$
$m$	Trajectories updates	2

Table 1. Control approach parameters

Two simulation case studies are proposed in the sequel. In the first simulation, the objective is to stabilize the PVTOL aircraft. The second one aims to show the robustness of the proposed control strategy against parameters uncertainties.

### 6.1 Stabilization control

In this simulation the stabilization problem of the aircraft is considered from the initial condition defined by  $x_0 = [1 \ 1 \ 30 \ 0.2 \ -0.5 \ 20]^T$ . The optimal trajectories on the directly controlled variables are updated twice over each interval  $[t_k, t_{k+1}]$ , that is  $m = 2$ .

#### 6.1.1. Stability analysis and computing time

The stability analysis of the closed-loop system is carried out using the graphical tool based on the Poincaré section, explained in section 5. Figure 6 displays the multi-step function  $\Psi$ . According to the obtained curve, the sufficient conditions of point 1 of proposition 1 (cf. Chemori and Alamir (2005)) are satisfied for  $k_0 = 5$ . The evaluation of the computing time is plotted in figure 7. The maximum computing time is  $t_{max} = 0.091sec$ .

**6.1.2. Other simulation results** The obtained simulation results are plotted in figures 8-11. Because of the normalized nature of the system variables, their artificial units are omitted.

In figure 8, the evolution of the position and orientation of the aircraft is plotted versus time, where it could be seen that the aircraft is stabilized over 3.5sec. The evolution of the internal dynamics of the system is shown in figure 9, which displays the phase portrait  $(\theta, \dot{\theta})$ . Figure 10 displays the movement of the aircraft in the vertical plane during the stabilization. The control actions generated by the proposed controller are shown in figure 11. According to this figure it could be

clearly seen that the constraints on the control inputs are satisfied, namely the thrust is directed upwards the aircraft (i.e.  $u_1 > 0$ ) throughout the stabilization operation.

### 6.2 Robustness towards parameters uncertainties

To attest the efficiency of the proposed control approach, let us consider uncertainties on the system model parameters, and especially on the coupling parameter  $\varepsilon$ , since it is often unknown and could not be measured.

$$\varepsilon_u = \varepsilon + \Delta_\varepsilon$$

Let the uncertainty  $\Delta_\varepsilon$  be 10% of the nominal value. The system will be simulated with the same parameters of control approach as the above simulation. The obtained results are illustrated through the simulation curves of the nominal system superimposed on those of the uncertain system. This will enable us to see the effect of the introduced uncertainty. Figure 12 displays the evolution of the aircraft coordinates  $(x, y, \theta)$  versus time. It is worth noting that the uncertain system is stabilized over 3.5sec too, despite the considered uncertainty on  $\varepsilon$ . The phase portrait of the system's internal dynamics is depicted in figure 13, where the evolution of the velocity of the roll angle is plotted versus its position. The generated control inputs are shown in figure 14 for both nominal and uncertain system. It could be concluded that, despite the uncertainty considered, the controller is able to stabilize the system. This fact attests the robustness of the proposed control scheme against parameters uncertainties.

## 7. CONCLUSION

Planar Vertical Take-Off and Landing (PVTOL) aircraft is an interesting illustration example of nonlinear under-actuated non-minimum phase systems that gains increasing interest within the control community. In this paper a nonlinear prediction based control approach is proposed to resolve the stabilization problem. The proposed control approach is based upon partial feedback linearization, and optimal trajectories generation to enhance the behavior and the stability of the system's internal dynamics. The stability analysis of the resulting closed-loop system is performed using a graphical tool based on the Poincaré's section. The approach performance and robustness are illustrated through simulation case studies.

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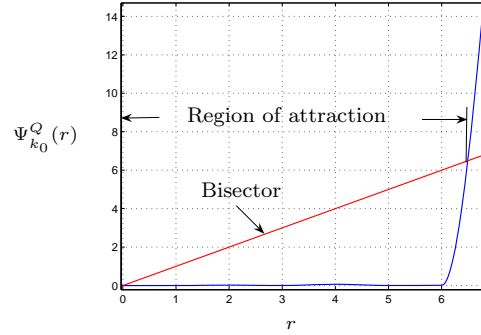


Fig. 6. A stability analysis tool : The curve  $\Psi_{k_0}^Q(r)$  for  $k_0 = 5$

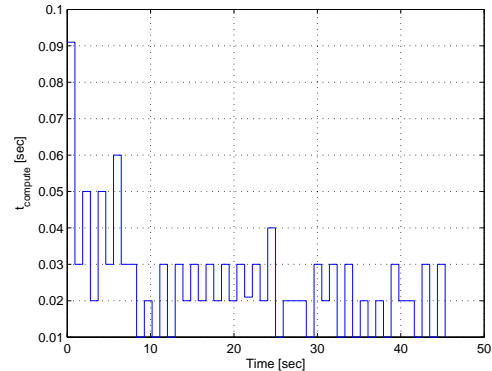


Fig. 7. Evaluation of computing time

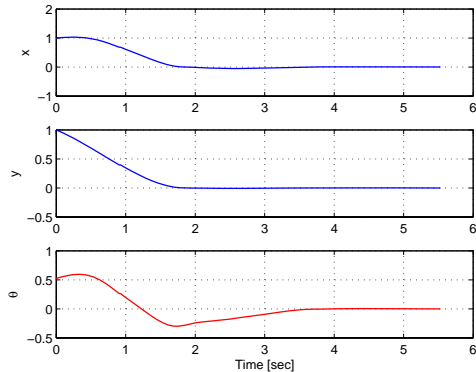


Fig. 8. Evolution of the PVTOL position and orientation

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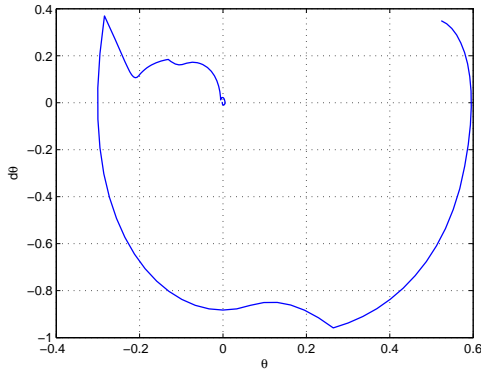


Fig. 9. Phase portrait of internal dynamics

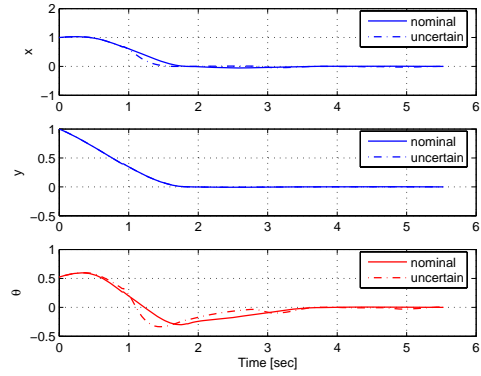


Fig. 12. Evolution of the PVTOL position and orientation

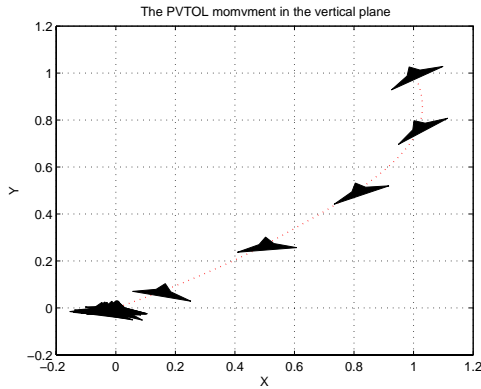


Fig. 10. Stick figures of the PVTOL in the vertical plane

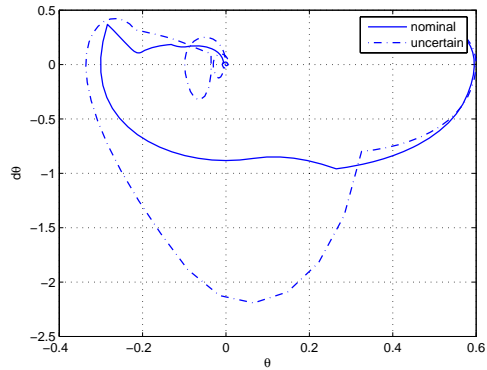


Fig. 13. Phase portrait of internal dynamics

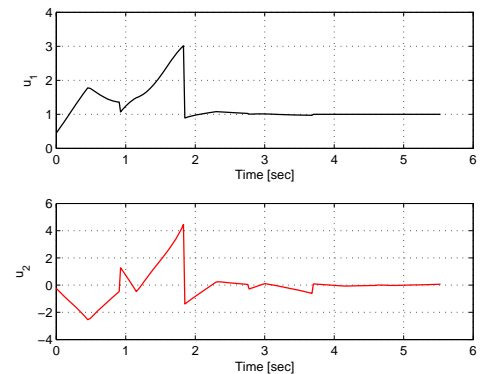


Fig. 11. Evolution of the control inputs

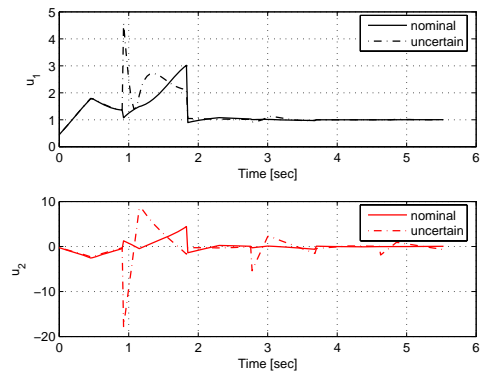


Fig. 14. Evolution of the control inputs

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