Ultracold Bose Gases in 1D Random Potentials: From Lifshits Glasses to Bose-Einstein Condensates

Pierre Lugan, David Clément, Philippe Bouyer, Alain Aspect, Maciej Lewenstein, Laurent Sanchez-Palencia

Ultracold Bose Gases in 1D Random Potentials: From Lifshits Glasses to Bose-Einstein Condensates

P. Lugan,[1] D. Clément,[1] P. Bouyer,[1] A. Aspect,[1] M. Lewenstein,[2] and L. Sanchez-Palencia[1]

[1]*Laboratoire Charles Fabry de l'Institut d'Optique, CNRS and Université Paris-Sud XI, Campus Polytechnique, RD 128, F-91127 Palaiseau cedex, France*[*]

[2]*ICREA and ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, E-08860 Castelldefels (Barcelona), Spain*

(Dated: 14th October 2006)

We study an ultracold Bose gas in the presence of 1D disorder for repulsive inter-atomic interactions, ranging from zero to the Thomas-Fermi regime. We show that for very small interactions, the Bose gas populates a finite number of localized single particle Lifshits states, while for large interactions, it forms a delocalized disordered Bose-Einstein condensate. We discuss the phase diagram and compute the equations of state in various phases.

PACS numbers: 05.30.Jp,03.75.Hh,64.60.Cn,79.60.Ht

Disorder is present in nearly all condensed matter systems, due to unavoidable defects of the sustaining media. This is known, not only to strongly alter quantum flows, but also to lead to spectacular effects, such as, for instance, Anderson localization [1, 2, 3, 4]. In contrast to condensed matter, controlled disorder (or pseudo-disorder) can be realized with ultracold gases [5], opening possibilities for quantitative investigations of localization effects [6] (for a review, see [7]). Recently, several groups have initiated the experimental study of localization in Bose-Einstein condensates (BEC) [8, 9, 10, 11], and of strongly correlated Bose gases [12] in controlled disorder.

One of the most fundamental issues in this respect concerns the interplay between localization and interactions in many body quantum systems at zero temperature. In the absence of interactions, a quantum gas populates localized states [1], either a single one (case of bosons), or a collection of these (fermions). Weak repulsive interactions delocalize, but strong ones in confining traps lead to Wigner-Mott-like localization [13]. This competition may lead to the appearance of new metallic or superfluid phases [7]. Surprisingly, even for weakly interacting Bose gases, where the mean field Hartree-Fock-Gross-Pitaevskii-Bogolyubov-de Gennes (HFGPBdG) description is expected to be valid, there exists no clear picture of the localization-delocalization scenario. The experiments and theory of [9, 10] point towards stronger localization effects in the transport of the BEC. The results of [11] clearly indicate that the BEC wavefunction at low densities mimics a superposition of localized, and practically non-overlapping states. The natural consequence of this observation is to seek after the true ground state of the system in the form of generalized HFGPBdG states, consisting of coexisting BECs in various single atom orbitals from the low energy tail of the spectrum. In the presence of a random potential, this corresponds to the so-called *Lifshits states* (LS) [14].

In this Letter, we address this question, and consider a d-dimensional Bose gas, placed in a 1D random potential at zero temperature with repulsive interactions. No *a priori* restriction is imposed either on the amplitude or on the correlation length of the random potential. We show that generalized HFGPBdG states indeed provide a very good description of the many body ground state for interactions ranging from zero to the Thomas-Fermi (TF) regime. Our theoretical treatment provides us with a novel, physically clear, picture of disordered weakly interacting ultracold Bose gases. This is the main result of this work. We stress that the solution that we find is very different from that of non-interacting fermions, which at zero temperature, form a *Fermi glass*, and occupy a large number of localized single particle levels [7]. In contrast, many bosons may occupy the same state, and thus the bosons populate only a finite number of LSs, forming, what we term a *Lifshits glass*. We discuss the possible quantum phases of the system versus the strength of interactions and the amplitude and correlation length of the disorder, and we derive the corresponding schematic phase diagram (see Fig. 1). In the limit of very weak interactions, the Bose gas is in the *Lifshits glass* phase, whereas for stronger interactions, the gas condenses, forming a (possibly screened) delocalized *disordered BEC* [18]. Analytic results for the phase boundaries and for the equations of state in the various regimes are derived. We illustrate our results in the case of a speckle random potential, similar to that used in Refs. [8, 9].

Consider a d-dimensional ultracold Bose gas with repulsive atom-atom interactions in the weak coupling regime $n^{1-2/d} \ll \hbar^2/mg$, where m is the atomic mass, n is the density and g is the coupling constant in dimension d. Here, the gas is assumed to be axially confined in a box of length

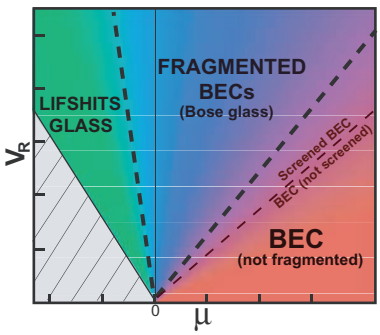

Figure 1: (color online) Schematic phase diagram of an ultracold Bose gas in a 1D random potential. The dashed lines represent the boundaries (corresponding to *crossovers*) identified in this work and are controlled by the parameter $\alpha_R = \hbar^2/2m\sigma_R^2 V_R$ (see text). The hatched part corresponds to a forbiden zone ($\mu < V_{\rm min}$).

$2L$ in the presence of a 1D random potential, $V(z)$, and trapped radially in a 2D harmonic trap with a trapping frequency $\omega_\perp$. We assume that the random potential is bounded below [$V_{\rm min} = \min(V)$] and we use the scaling form $V(z) = V_{\rm R} v(z/\sigma_{\rm R})$, where $v(u)$ is a random function with both typical amplitude and correlation length equal to unity [15]. This form is suitable for scaling arguments, which will turn out to be fruitful in the following.

For numerical purposes, we will consider a 1D speckle potential [10, 16] similar to that used in [8, 9, 10]. In brief, $v(u)$ is random, with an exponential probability distribution $\mathcal{P}(v) = \Theta(v+1)\exp[-(v+1)]$, where Θ is the Heaviside function. Thus, v is bounded below by $v_{\rm min} = -1$, and we have $\langle v\rangle = 0$ and $\langle v^2\rangle = 1$. In addition, for a square aperture, the correlation function reads $\langle v(u)v(u')\rangle = \mathrm{sinc}\left[\sqrt{3/2}\,(u-u')\right]^2$.

Depending on the competition between interactions and disorder, the Bose gas will enter various regimes that we discuss below.

BEC regime - For large interactions, the Bose gas enters a BEC regime [9, 18] (possibly quasi-BEC in 1D [17]) where the local density is given by the meanfield Gross-Pitaevskii equation (GPE),

$$\mu = \frac{-\hbar^2\nabla^2(\sqrt{n})}{2m\sqrt{n}} + \frac{m\omega_\perp^2\rho^2}{2} + V(z) + gn(\mathbf{r})\,, \qquad (1)$$

with ρ, the radial coordinate and μ, the chemical potential of the Bose gas. This regime has been investigated in details in the pure 1D case in Ref. [18]. Here, we focus on the case of a shallow radial trap ($\hbar\omega_\perp \ll \mu$) such that the radial profile of the Bose gas is a TF inverted parabola. Proceeding as in Ref [18], we find that the first-order perturbative solution of Eq. (1) is a generalized TF profile with a *screened random potential*:

$$\sqrt{n(\rho,z)} \simeq \sqrt{\mu(\rho)/g}\left[1 - \widetilde{V}(\rho,z)/2\mu(\rho)\right] \qquad (2)$$

where $\mu(\rho) = \mu\left[1-(\rho/R_\perp)^2\right]$ is the local chemical potential, and $R_\perp = \sqrt{2\mu/m\omega_\perp^2}$ is the radial TF half-size [19]. In Eq. (2), $\widetilde{V}(\rho,z) = \int \mathrm{d}z'\; G(\rho,z')V(z-z')$ is the *screened potential* [18] with $G(\rho,z) = \frac{1}{\sqrt{2}\xi(\rho)}\exp\left(-\frac{|z|}{\xi(\rho)/\sqrt{2}}\right)$, and $\xi(\rho) = \hbar/\sqrt{2m\mu(\rho)}$ being the local healing length. The solution (2) corresponds to a delocalized *disordered BEC*.

The validity of the perturbative approach requires the last term in Eq. (2) to be much smaller than unity, *i.e.* $\mu(\rho) \gg \Delta\widetilde{V}(\rho)$, where $\Delta\widetilde{V}(\rho)$ is the standard deviation of the screened random potential at the radial position ρ. From the above expression for the screened potential, $\widetilde{V}$, we can generally write $\Delta\widetilde{V}(\rho) = V_{\rm R}\sqrt{\Sigma^0(\sigma_{\rm R}/\xi(\rho))}$. In the case of the speckle potential, approximating the correlation function to a Gaussian, $V_{\rm R}^2\exp\left(-z^2/2\sigma_{\rm R}^2\right)$, we find [18]

$$\Sigma^0(\widetilde{\sigma}_{\rm R}) = \widetilde{\sigma}_{\rm R}^2 + (1-2\widetilde{\sigma}_{\rm R}^2)\widetilde{\sigma}_{\rm R}\,\mathrm{e}^{\widetilde{\sigma}_{\rm R}^2}\int_{\widetilde{\sigma}_{\rm R}}^{\infty}\mathrm{d}\theta\,\mathrm{e}^{-\theta^2}\,, \qquad (3)$$

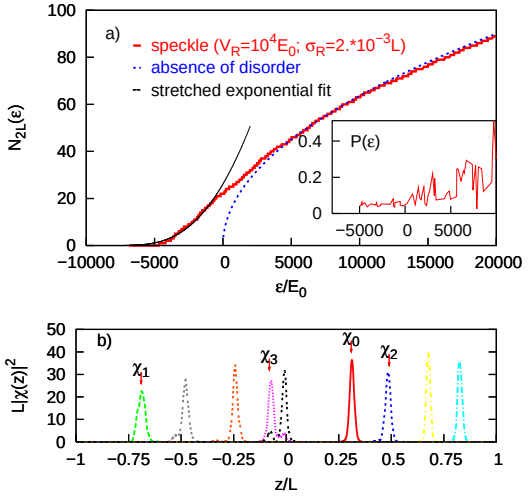

Figure 2: (color online) a) Cumulative density of states of single-particles in a speckle random potential. Inset: Participation ratio. b) Low-energy Lifshits eigenstates. Here $E_0 = \hbar^2/2mL^2$.

with $\widetilde{\sigma}_{\rm R} = \sigma_{\rm R}/\xi(\rho)$. In the center, $\rho = 0$, (or strictly in 1D), the validity condition of the BEC regime thus reduces to

$$\mu \gg V_{\rm R}\sqrt{\Sigma^0(\sigma_{\rm R}/\xi)}\,, \qquad (4)$$

with $\xi = \xi(0)$. In the limit $\xi \ll \sigma_{\rm R}$, *i.e.* for

$$\mu \gg \hbar^2/2m\sigma_{\rm R}^2\,, \qquad (5)$$

the random potential is hardly screened: $\widetilde{V}(z) \simeq V(z)$.

If condition (4) is not fulfilled, the BEC will fragment at the positions of some local maxima of the random potential and thus form an insulator. In each fragment, the Bose gas will be in the BEC regime and will thus be compressible. Therefore, the fragmented BEC can be identified with a *Bose glass* [13].

Non-interacting regime - Consider now the non interacting case. Here, we are left with the eigenproblem of the single-particle Hamiltonian, $\widehat{h} = -\hbar^2\partial_z^2/2m + V(z)$. In the presence of disorder, the eigenstates are *usually* all localized [2]. Usual characteristics of localized states are (i) a finite localization length, (ii) a dense pure point density of states, $\mathcal{D}_{2L}$, and (iii) a large inverse participation ratio, $P_\nu^{-1} = \int \mathrm{d}z\;|\chi_\nu(z)|^4$. If the random potential is bounded below, so is the spectrum ($\epsilon_\nu > V_{\rm min}$) and the low-lying energy states belong to the so-called *Lifshits tail* of the spectrum which is characterized by a stretched exponential cumulative density of states (cDOS), $\mathcal{N}_{2L}(\epsilon) = \int^\epsilon \mathrm{d}\epsilon'\;\mathcal{D}_{2L}(\epsilon') \sim \exp\left(-c\sqrt{\frac{V_{\rm R}}{\epsilon - V_{\rm min}}}\right)$, in 1D [20].

Typical numerical results for the case of the speckle potential are shown in Fig. 2. As expected, the cDOS follows the stretched exponential form, the lowest LSs are well spatially localized, and the participation ratio increases with energy, indicating a decreasing localization. Note, however, that $P(\epsilon)$ is almost constant at low energy. It is also worth noting that, if the size of the system ($2L$) is much larger than the extension

of the LSs, they hardly overlap. These properties will turn out to be crucial in the following.

Lifshits regime - We turn now to the regime of weak interactions, where the chemical potential, μ, lies in the Lifshits tail of the spectrum. In this regime, owing to the fact that the lowest single particle LSs hardly overlap, it is useful to work in the basis of the LSs, $\{\chi_\nu, \nu \in \mathbb{N}\}$. These can be regarded as trapping micro-sites populated with N_ν Bosons in the quantum state $\phi_\nu(\rho)\chi_\nu(z)$, where the longitudinal motion is frozen to χ_ν, and ϕ_ν accounts for the radial extension in the micro-site ν. Therefore, the many body wavefunction reads:

$$\Psi(\mathbf{r}_1, ..., \mathbf{r}_N) \;=\; \mathcal{S} \prod_{\nu=0}^{\mathcal{N}-1} \left[\prod_{l=M_\nu}^{M_\nu+N_\nu-1} \phi_\nu(\rho_l)\chi_\nu(z_l) \right] \quad (6)$$

where l labels the particles, $M_\nu = \sum_{0\le l<\nu} N_l$, and $\mathcal{S}$ is the bosonic symmetrization operator, accounting for permutations of couples (ρ_l, z_l). The meanfield energy functional of the interacting Bose gas then reads

$$\begin{aligned} E[\Psi] \;&=\; \sum_\nu N_\nu \int \mathrm{d}\rho\; \phi_\nu^* \left(\frac{-\hbar^2\nabla_\perp^2}{2m} + \frac{m\omega_\perp^2\rho^2}{2} + \epsilon_\nu \right) \phi_\nu \\ &+\; \sum_\nu \frac{N_\nu^2}{2} \int \mathrm{d}\rho\; U_\nu |\phi_\nu|^4, \end{aligned} \quad (7)$$

where $U_\nu = g\int \mathrm{d}z\; |\chi_\nu(z)|^4$ is the local interaction energy in the Lifshits state (LS), χ_ν. Interestingly, the interaction energy is directly proportional to the participation ratio in the ν-th LS: $U_\nu = gP_\nu^{-1}$. Minimizing $E[\Psi]$ for a fixed number of particles ($E[\Psi] - \mu\sum_\nu N_\nu \to$ min), we end up with the equation

$$(\mu - \epsilon_\nu)\phi_\nu = \left[\frac{-\hbar^2\nabla_\perp^2}{2m} + \frac{m\omega_\perp^2\rho^2}{2} + N_\nu U_\nu |\phi_\nu|^2 \right] \phi_\nu \; . \quad (8)$$

Solving Eq. (8) for each micro-site ν, one can unambiguously determine the atom numbers, N_ν, and the corresponding wavefunctions, ϕ_ν. Since Eq. (8) is a 2D GPE, $|\phi_\nu|^2$ will continuously turn from a Gaussian (for $\hbar\omega_\perp \gg \mu$) to an inverted parabola (for $\hbar\omega_\perp \ll \mu$) as μ increases (see below).

Let us determine the validity condition of the Lifshits regime. We call $\nu^{\max}$, the index of the highest LS such that all lower LSs hardly overlap. The above Lifshits description is correct if the chemical potential, μ, is small enough so that the number of populated LSs is smaller than $\nu^{\max}$, *i.e.* if

$$\mathcal{N}_{2L}(\mu) \le \nu^{\max} \; . \quad (9)$$

Obviously, both $\mathcal{N}_{2L}$ and $\nu^{\max}$ depend on the model of disorder, v, and on both parameters V_{R} and σ_{R}, and are difficult to compute in a general fashion. General properties can nevertheless be obtained using scaling arguments. To do so, we rewrite the single-particle eigenproblem as

$$\frac{\epsilon_\nu}{V_{\mathrm{R}}}\varphi_\nu(u) = -\alpha_{\mathrm{R}}\partial_u^2\varphi_\nu(u) + v(u)\varphi_\nu(u) \; , \quad (10)$$

where $u = z/\sigma_{\mathrm{R}}$, $\varphi_\nu(u) = \sqrt{\sigma_{\mathrm{R}}}\chi_\nu(z)$ and $\alpha_{\mathrm{R}} = \hbar^2/2m\sigma_{\mathrm{R}}^2 V_{\mathrm{R}}$. It is then clear that, upon renormalization of energies and lengths, all characteristics of the spectrum depend only on the parameter α_{R} (and on the model of disorder, v).

Scaling arguments show that, in the Lifshits tail,

$$\mathcal{N}_{2L}(\epsilon) = \frac{L}{\sigma_{\mathrm{R}}}\zeta\left(\alpha_{\mathrm{R}}, \frac{\epsilon}{V_{\mathrm{R}}}\right) \quad \text{and} \quad \nu^{\max} = \frac{L}{\sigma_{\mathrm{R}}}\eta(\alpha_{\mathrm{R}}) \; , \quad (11)$$

where ζ and η are v-dependent functions. Finally, inserting these expressions into Eq. (9) and solving formally, we obtain the validity condition of the Lifshits regime:

$$\mu \le V_{\mathrm{R}} F(\alpha_{\mathrm{R}}), \quad (12)$$

where F is the solution of $\zeta\big(\alpha_{\mathrm{R}}, F(\alpha_{\mathrm{R}})\big) = \eta(\alpha_{\mathrm{R}})$, which can be numerically computed for any model of disorder v.

We are now in the position to draw the schematic phase diagram of the zero temperature Bose gas versus μ and V_{R} (see Fig. 1). From the discussion above, it appears useful to fix the parameter $\alpha_{\mathrm{R}} = \hbar^2/2m\sigma_{\mathrm{R}}^2 V_{\mathrm{R}}$, while varying V_{R}. The boundaries between the various regimes (Lifshits, fragmented, BEC and screened BEC) result from the competition between the interactions and the disorder and are given by Eqs. (4-5-12). We stress that these are smooth crossovers rather than phase transitions. Interestingly enough, it can be shown easily that all these boundaries are straight lines with slopes depending on the parameter α_{R}. This is clear from Eq. (12) for the boundary between the Lifshits and the fragmented regimes. In addition, since $V_{\mathrm{R}} = (\hbar^2/2m\sigma_{\mathrm{R}}^2)/\alpha_{\mathrm{R}}$, the non-screening condition (5) reduces to $\mu \gg \alpha_{\mathrm{R}} V_{\mathrm{R}}$. Finally, since $\mu = \hbar^2/2m\xi^2$, and thus $\sigma_{\mathrm{R}}/\xi = \frac{1}{\sqrt{\alpha_{\mathrm{R}}}}\sqrt{\mu/V_{\mathrm{R}}}$, the non-fragmented BEC condition (4) also corresponds to a straight line in the phase diagram of Fig. 1 with a slope depending on α_{R}.

Concretely, the chemical potential, μ, and thus the transitions between the various regimes, will be determined by the mean atomic density, $\overline{n}$, and the coupling constant, g. Both can be accurately controlled in experiments with ultracold atoms. It is thus useful to compute the equation of state, $\mu = \mu(\overline{n}, g)$, of the Bose gas in all the identified regimes.

Tight radial confinement - Let us first assume a tight radial confinement: $\mu' - \epsilon_\nu \ll \hbar\omega_\perp$ where $\mu' = \mu - \hbar\omega_\perp$. The radial wavefunctions are then frozen to zero point oscillations, $\phi_\nu(\rho) = \exp(-\rho^2/2l_\perp^2)/\sqrt{\pi}l_\perp$ where $l_\perp = \sqrt{\hbar/m\omega_\perp}$ is the width of the ground state of the 2D-radial harmonic oscillator.

In the BEC regime, $\mu' \gg \Delta\widetilde{V}$, it follows from Eq. (2) that

$$\mu' = \overline{n}g \quad (13)$$

up to first order of the perturbation series.

In the Lifshits regime, we find

$$\begin{aligned} N_\nu \;&=\; [\mu' - \epsilon_\nu]/U_\nu \quad \text{if } \mu' > \epsilon_\nu \\ N_\nu \;&=\; 0 \quad \text{otherwise} \; , \end{aligned} \quad (14)$$

by inserting the above expression for $\phi_\nu(\rho)$ into Eq. (8). Turning to a continuous formulation and using the normalization

condition $N = \int \mathrm{d}\epsilon\ \mathcal{D}_{2L}(\epsilon) N(\epsilon)$, we deduce the equation of state of the Bose gas in the Lifshits regime:

$$Ng = \int_{-\infty}^{\mu'} \mathrm{d}\epsilon\ \mathcal{D}_{2L}(\epsilon)\,(\mu' - \epsilon)\,P(\epsilon)\ . \qquad (15)$$

Upon computing the integral in Eq. (15), one can unambiguously determine a relation between the chemical potential, μ', and the interaction parameter, g. This relation is in general non-universal (*i.e.* it depends on the model of disorder, v). In the case of a speckle random potential, we have found that $\mathcal{N}_{2L}(\epsilon) = A(\alpha_{\rm R}) \left(\frac{L}{\sigma_{\rm R}}\right) \exp\left(-c(\alpha_{\rm R})/\sqrt{\epsilon/V_{\rm R}+1}\right)$, and assuming a participation ratio, $P_\nu = \sigma_{\rm R} p_\nu(\alpha_{\rm R})$, independent of the energy in the Lifshits tail (see the Inset of Fig. 2a), we find

$$\overline{n}g = A(\alpha_{\rm R}) c^2(\alpha_{\rm R}) p_0(\alpha_{\rm R}) V_{\rm R} \Gamma\left(-2, \frac{c(\alpha_{\rm R})}{\sqrt{\mu'/V_{\rm R}+1}}\right)\ , \qquad (16)$$

with Γ being the incomplete gamma function, and with $A(\alpha_{\rm R})$, $c(\alpha_{\rm R})$ and $p_0(\alpha_{\rm R})$ determined numerically.

Using a numerical minimization of the energy functional (7) in the Gross-Pitaevskii formulation [11], we compute the chemical potential of the Bose gas in a wide range of interactions. The result presented in Fig. 3 shows a clear *crossover* from the Lifshits to the BEC regimes, while the interaction strength increases. The numerically obtained chemical potential, μ, perfectly agrees with our analytical formulae in both Lifshits and BEC regimes.

Shallow radial confinement - The equations of state can be obtained easily also in the case of a shallow radial confinement ($\mu - \epsilon_\nu \gg \hbar\omega_\perp$). In the BEC regime, $\mu \gg V_{\rm R}$, and for $\xi \ll \sigma_{\rm R}$, we easily find $\mu \simeq \sqrt{\frac{\overline{n}gm\omega_\perp^2}{\pi}} - V_{\rm R}^2$. In the Lifshits regime, the 2D wavefunctions, $\phi_\nu(\rho)$, are in the TF regime:

$$|\phi_\nu(\rho)|^2 = \frac{\mu - \epsilon_\nu}{N_\nu U_\nu}\left(1 - \rho^2/R_\nu^2\right), \qquad (17)$$

where $R_\nu = \sqrt{2(\mu-\epsilon_\nu)/m\omega_\perp^2}$ is the 2D-TF radius and $N_\nu = \pi(\mu-\epsilon_\nu)^2/m\omega_\perp^2 U_\nu$ for $\mu > \epsilon_\nu$, and 0 otherwise.

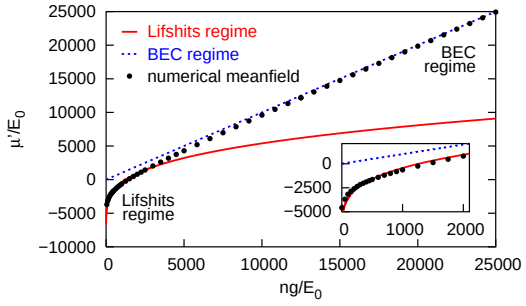

Figure 3: (color online) Chemical potential of a Bose gas in a speckle random potential with the same parameters as in Fig. 2 in the case of a tight radial confinement ($\mu' - \epsilon_\nu \ll \hbar\omega_\perp$). The points correspond to numerical calculations and the lines to the analytic formulae in both Lifshits [Eq. (16)] and BEC regimes [Eq. (13)].

Proceeding as in the 1D case, we finally find

$$Ng = \frac{\pi}{m\omega_\perp^2} \int_{-\infty}^{\mu} \mathrm{d}\epsilon\ \mathcal{D}_{2L}(\epsilon)\,(\mu - \epsilon)^2\,P(\epsilon)\ . \qquad (18)$$

Applying this formula to the considered model of disorder allows us to compute the populations, N_ν, of the various LSs, χ_ν, and the corresponding radial extensions, ϕ_ν.

In summary, we have presented a complete picture of the quantum phases of a Bose gas in a 1D random potential, including the novel description of the weakly interacting *Lifshits glass* phase [21]. We have provided analytical formulae for the frontiers between the various phases and we have shown that the corresponding crossovers are controlled by the interaction parameter. Since this can be controlled in cold gases, future experiments should be able to probe the whole diagram.

We thank G. Shlyapnikov, L. Santos, D. Gangardt and C. Mora for fruitful discussions. This work was supported by the DGA, the MENRT, the ANR, the European Union, BEC2000+ and QUDEDIS. M.L. acknowleges support from the Spanish MEC Programs, DUQUAG, QDIT and EU IP SCALA. LCFIO is a member of IFRAF.

* URL: `http://www.atomoptic.fr`

[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[2] N.F. Mott and W.D. Towes, Adv. Phys. **10**, 107 (1961).
[3] E. Abrahams *et al.*, Phys. Rev. Lett. **42**, 673 (1979).
[4] B. van Tiggelen, in *Wave Diffusion in Complex Media*, lectures notes at Les Houches 1998, edited by J. P. Fouque, NATO Science (Kluwer, Dordrecht, 1999).
[5] G. Grynberg, P. Horak, and C. Mennerat-Robilliard, Europhys. Lett. **49**, 424 (2000); L. Guidoni, C. Triché, P. Verkerk, and G. Grynberg, Phys. Rev. Lett **79**, 3363 (1997).
[6] B. Damski *et al.*, Phys. Rev. Lett. **91**, 080403 (2003); R. Roth and K. Burnett, Phys. Rev. A **68**, 023604 (2003); U. Gavish and Y. Castin, Phys. Rev. Lett. **95**, 020401 (2005); L. Sanchez-Palencia and L. Santos, Phys. Rev. A **72**, 053607 (2005); R.C. Kuhn *et al.*, Phys. Rev. Lett. **95**, 250403 (2005); T. Paul *et al.*, Phys. Rev. A **72**, 063621 (2005).
[7] V. Ahufinger *et al.*, Phys. Rev. A **72**, 063616 (2005).
[8] J.E. Lye *et al.*, Phys. Rev. Lett. **95**, 070401 (2005); C. Fort *et al.*, Phys. Rev. Lett. **95**, 170410 (2005).
[9] D. Clément *et al.*, Phys. Rev. Lett. **95**, 170409 (2005).
[10] D. Clément *et al.*, New J. Phys. **8**, 165 (2006).
[11] T. Schulte *et al.*, Phys. Rev. Lett. **95**, 170411 (2005).
[12] L. Fallani *et al.*, cond-mat/0603655.
[13] M.P.A. Fisher, P.B. Weichman, G. Grinstein, and D.S. Fisher, Phys. Rev. B **40**, 546 (1989); R.T. Scalettar, G.G. Batrouni, and G.T. Zimanyi, Phys. Rev. Lett. **66**, 3144 (1991).
[14] I.M. Lifshits, S.A. Gredeskul, and L.A. Pastur, *Introduction to the Theory of Disordered Systems*, (Wiley, New York, 1988).
[15] Therefore, the random potential $V(z)$ is characterized by a typical amplitude $V_{\rm R}$ and a typical correlation length $\sigma_{\rm R}$.
[16] J.W. Goodman, *Statistical Properties of Laser Speckle Patterns* in *Laser Speckle and Related Phenomena*, J.-C. Dainty ed. (Springer-Verlag, Berlin, 1975).
[17] D.S. Petrov, G.V. Shlyapnikov, and J.T.M. Walraven, Phys. Rev. Lett. **85**, 3745 (2000).

[18] L. Sanchez-Palencia, cond-mat/0609036.
[19] Equation (2) is valid also in the pure 1D case where $\rho = 0$ [18].
[20] I.M. Lifshits, Adv. Phys. **13**, 483 (1964); T.P. Eggarter, Phys. Rev. B **5**, 3863 (1972); J.M. Luttinger and H.K. Sy, Phys. Rev. A **7**, 701 (1973).
[21] The Lifshits glass can be identified with what is termed the 'Anderson glass' in [13]. We stress however that its nature is that of a Fock state of *Lifshits states* as shown in this work.