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# Measuring flow resistivity of porous materials at low frequencies range via acoustic transmitted waves (L)

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An acoustic transmissivity method is proposed for measuring flow resistivity of porous materials having rigid frame. Flow resistivity of porous material is defined as the ratio between the pressure difference across a sample and the velocity of flow of air through that sample per unit cube. The proposed method is based on a temporal model of the direct and inverse scattering problem for the diffusion of transient low-frequency waves in a homogeneous isotropic slab of porous material having a rigid frame. The transmission scattering operator for a slab of porous material is derived from the response of the medium to an incident acoustic pulse. The flow resistivity is determined from the solution of the inverse problem. The minimization between experiment and theory is made in the time domain. Tests are performed using industrial plastic foams. Experimental and numerical results, and prospects are discussed.

## I. INTRODUCTION

The acoustic characterization of porous materials saturated by air<sup>1,2</sup> such as plastic foams, fibrous, or granular materials is of great interest for a wide range of industrial applications. These materials are frequently used in the automotive and aeronautics industries and in the building trade. One important parameter that appears in theories of sound propagation in porous materials at a low-frequency range<sup>3,4</sup> is the specific flow resistivity  $\sigma$ .<sup>5-11</sup> This parameter is defined as the ratio between the pressure difference across a sample and the velocity of flow of air through that sample per unit cube; the flows being considered are steady and nonpulsating. The permeability  $k_0$  is related to the specific flow resistivity  $\sigma$  by the relation  $k_0 = \eta / \sigma$ , where  $\eta$  is the fluid viscosity.

Among the various systems that have been developed for the measurement of flow resistance, a distinction can be made between direct and comparative methods. With direct methods,<sup>6,8,9,11</sup> the pressure drop across a sample and the rate of air flow through the porous sample are determined separately and the specific flow resistivity is computed as the ratio of the two quantities. With comparative methods,<sup>10,12</sup> a calibrated flow resistivity is placed in series with the porous sample. The ratio of pressure drops across each element is

the same as the ratio of the values of flow resistivity, since the volumetric flow of air in the line is the constant.

In this work, we present a simple acoustical method of measuring specific flow resistivity by measuring a diffusive wave transmitted by a slab of air-saturated porous material in a guide (pipe).

## II. MODEL

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory.<sup>13</sup> In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid,<sup>4</sup> which is a particular case of the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium  $\alpha(\omega)$  given by Johnson *et al.*<sup>14</sup> and the dynamic compressibility of the fluid included in the porous material  $\beta(\omega)$  given by Allard<sup>1</sup> ( $\omega$  is the pulsation frequency). In the frequency domain, these factors multiply the density of the fluid and its compressibility, respectively, and represent the deviation from the behavior of the fluid in free space as the frequency changes.

Consider a homogeneous porous material that occupies the region  $0 \leq x \leq L$ . A sound pulse impinges normally on the medium. It generates an acoustic pressure field  $p(x, t)$  and an acoustic velocity field  $v(x, t)$  within the material. In the low-frequency range, the acoustic fields satisfy the Euler equation and the constitutive equation (along the  $x$  axis):

$$\sigma \phi v(x, t) = - \frac{\partial p(x, t)}{\partial x},$$

$$\frac{\gamma}{K_a} \frac{\partial p(x, t)}{\partial t} = - \frac{\partial v(x, t)}{\partial x}.$$

In these equations,  $\phi$  is the porosity,  $K_a$  is the bulk modulus of the fluid, and  $\gamma$  is the adiabatic constant. The Euler equation is reduced to the Darcy's law, which expresses the balance between the driving force of the wave and the drag forces  $\sigma \phi v$ , due to the flow resistance of the material. The fields that are varying in time, the pressure, the acoustic velocity, etc., follow a diffusion equation,<sup>4</sup>

$$\frac{\partial^2 p(x, t)}{\partial x^2} - D \frac{\partial p(x, t)}{\partial t} = 0, \quad (1)$$

where  $D = \sigma \phi \gamma / K_a$  is the diffusion constant. The solution of this diffusive equation gives the Green's function of the porous material:

$$G(x, t) = \frac{x \sqrt{D}}{2 \sqrt{\pi}} \frac{1}{t^{3/2}} \exp\left(-\frac{x^2 D}{4t}\right). \quad (2)$$

To derive the transmission scattering operator, it is assumed that the pressure field and flow velocity are continuous at the material boundary:  $p(0^+, t) = p(0^-, t)$ ,  $p(L^-, t) = p(L^+, t)$ ,  $v(0^-, t) = \phi v(0^+, t)$ ,  $v(L^+, t) = \phi v(L^-, t)$ , where  $\phi$  is the porosity of the medium and the  $\mp$  superscript denotes the limit from left and right, respectively. Assumed initial conditions are  $p(x, t)|_{t=0} = 0$  and  $\partial p / \partial t|_{t=0} = 0$ , which means that the medium is idle for  $t=0$ . The incident  $p^i(t)$  and transmitted  $p^t(t)$  fields are related in the time domain by the transmission scattering operator:

$$p^t(x, t) = \int_0^t \tilde{T}(\tau) p^i\left(t - \tau - \frac{(x-L)}{c_0}\right) d\tau. \quad (3)$$

The transmission operator is independent of the incident signal and depends only on the properties of the porous material. Using the relations (1)–(3), we obtain the transmission scattering operator  $\tilde{T}(t) = D(t) * G(L, t)$ , where  $*$  denotes the time convolution operation. The operator  $D(t)$  is given by

$$D(t) = - \frac{8}{B^2} \left(1 + \frac{t}{B^2}\right) \exp\left(\frac{t}{B^2}\right) \operatorname{Erfc}\left(\frac{\sqrt{t}}{B}\right) + \frac{4}{B \sqrt{\pi t}} \left(2 \frac{t}{B^2} + 1\right), \quad B = \frac{1}{\eta} \sqrt{\phi^3 \gamma \sigma^3 \rho},$$

where  $\operatorname{erfc}$  is the error function and  $\rho_0$  is the fluid density.

### III. INVERSE PROBLEM

The inverse problem is to find value for parameter  $\sigma$  that minimizes the function  $U(\sigma) = \int_0^t [p'_{\text{exp}}(x, t) - p^t(x, t)]^2 dt$ ,

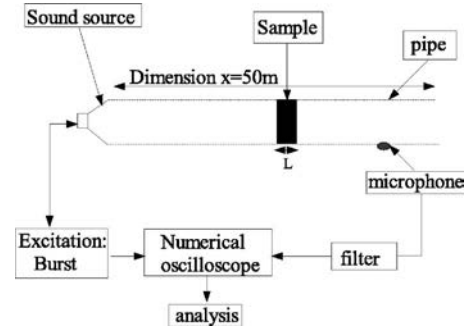


FIG. 1. Experimental setup of acoustic measurements.

where  $p'_{\text{exp}}(x, t)$  is the experimentally determined transmitted signal and  $p^t(x, t)$  is the transmitted wave predicted from Eq. (3). The inverse problem is not solved for the porosity  $\phi$  because the transmission is much more sensitive to flow resistivity than to porosity; the effect of the porosity in transmitted mode is negligible, as it has been observed in the asymptotic domain<sup>15</sup> (high-frequency range). The analytical method of solving the inverse problem using the conventional least-square method is tedious. In our case, a numerical solution of the least-square method can be found that minimizes  $U(\sigma)$  defined by  $U(\sigma) = \sum_{i=1}^N [p'_{\text{exp}}(x, t_i) - p^t(x, t_i)]^2$ , where  $p'_{\text{exp}}(x, t_i)_{i=1,2,\dots,N}$  represents the discrete set of values of the experimental transmitted signal and  $p^t(x, t_i)_{i=1,2,\dots,N}$  is the discrete set of values of the simulated transmitted signal. The inverse problem is solved numerically by the least-square method.

Experiments are performed in a guide (pipe), having a diameter of 5 cm and of length 50 m. This length has been chosen for the propagation of transient signals at low frequency. It is not important to keep the pipe straight; it can be rolled in order to save space without perturbations on experimental signals (the cutoff frequency of the tube  $f_c \sim 4$  kHz). A sound source Driver unit "Brand" constituted by loudspeaker Realistic 40-9000 is used. Bursts are provided by synthesized function generator Stanford Research Systems model DS345-30 MHz. The signals are amplified and filtered using model SR 650-Dual channel filter, Standford Research Systems. The signals (incident and transmitted) are measured using the same microphone (Bruel&Kjaer, 4190) in the same position in the tube. The incident signal is measured without a porous sample, however, the transmitted signal is measured with the porous sample. The experimental setup is shown in Fig. 1. Consider a cylindrical sample of plastic foam M of diameter 5 cm, porosity  $\phi=0.9$ , and thickness 2.5 cm. Sample M was characterized using classical methods<sup>6</sup> given  $\sigma = 38\,000 \pm 6000 \text{ N m}^{-4} \text{ s}$ . Figure 2 shows the experimental incident signal (solid line) generated by the loudspeaker in the frequency bandwidth (85–115) Hz, and the experimental transmitted signal (dashed line). After solving the inverse problem numerically for the flow resistivity, we find the following optimized value:  $\sigma = 39\,500 \pm 2000 \text{ N m}^{-4} \text{ s}$ . We present in Fig. 3 the variation of the minimization function  $U$  with the flow resistivity  $\sigma$ . In Fig. 4, we show a comparison between an experimental transmitted signal and simulated transmitted signal for the optimized value of the flow resistivity. The difference between the two curves is slight, which

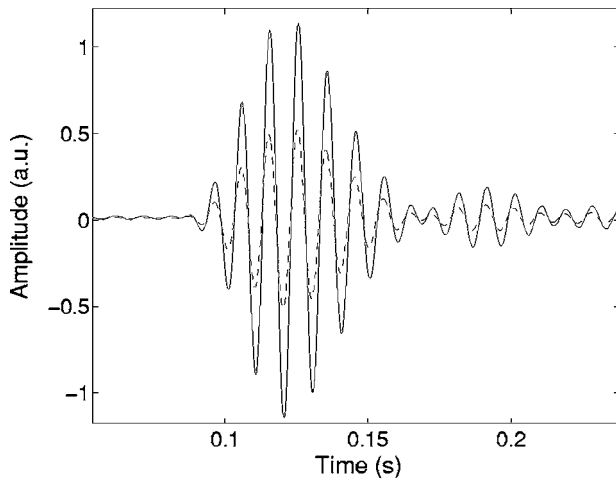


FIG. 2. Experimental incident signal (solid line) and experimental transmitted signal (dashed line).

leads us to conclude that the optimized value of the flow resistivity is correct. This study has been carried on, in the frequency bandwidths (25–60) Hz and (130–190) Hz and has also given good results.

The classical method<sup>1</sup> using continuous sound can be used only for the intermediary frequencies (it means between 1 and 3 or 4 KHz, depending of the radius of the tube). However, in this domain of frequency, we cannot use our approximation of very low frequency in which only the flow resistivity (or permeability) and porosity intervene in the propagation. The advantage of the proposed method is that the measurement can be done at very low frequency (less than 100 Hz by taking a long tube of 50 m), and thus the flow resistivity can be obtained easily. In the domain of frequency corresponding to measurement in the Kundt Tube (continuous sound), all physical parameters (porosity, tortuosity, viscous, and thermal characteristics lengths, viscous and thermal permeability, etc) intervene in the model of propagation. In this case, the inverse problem becomes very difficult to solve. In this proposed method, the use of tran-

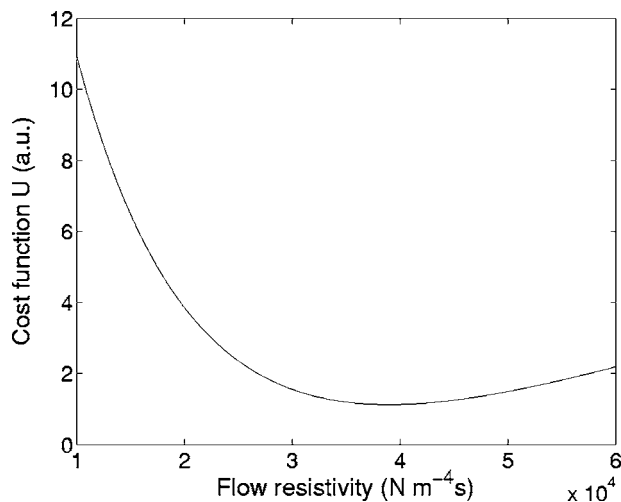


FIG. 3. Variation of the cost function  $U$  with flow resistivity  $\sigma$ .

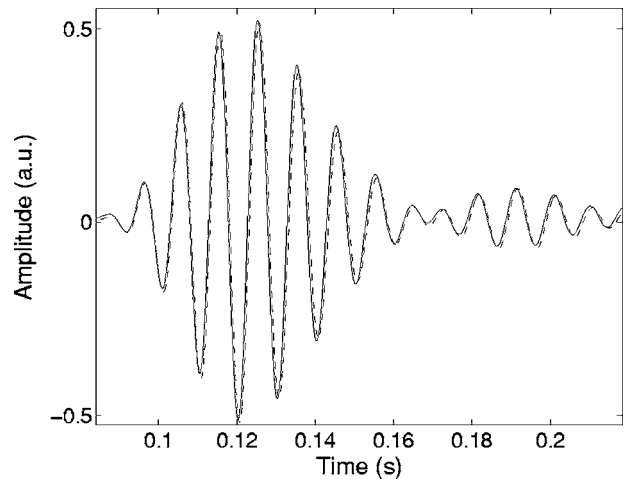


FIG. 4. A comparison between the experimental transmitted signal (dashed line) and the simulated transmitted signal (solid line) for the sample M.

sient signals (having a large spectrum) is possible, this is not the case for the classic Kundt tube (continuous frequency). For transient signals, the temporal approach are more appropriate, we do not need to use the Fourier transform. In our case the inverse problem is solved directly in time domain using the waveform without any transformation in Fourier domain.

<sup>1</sup>J. F. Allard, *Propagation of Sound in Porous Media: Modeling Sound Absorbing Materials* (Chapman and Hall, London, 1993).

<sup>2</sup>K. Attenborough, "Models for the acoustical properties of air-saturated granular media," *Acta. Acust. Acust.* **1**, 213 (1993).

<sup>3</sup>N. Chandler, "Transient streaming potential measurements on fluid-saturated porous structures: An experimental verification of Biot's slow wave in the quasi-static limit," *J. Acoust. Soc. Am.* **70**, 116–121 (1981).

<sup>4</sup>Z. E. A. Fellah and C. Depollier, "Transient acoustic wave propagation in rigid porous media: a time-domain approach," *J. Acoust. Soc. Am.* **107**, 683–688 (2000).

<sup>5</sup>M. E. Delany and E. N. Bazley, "Acoustical properties of fibrous materials," *Appl. Acoust.* **3**, 105–116 (1970).

<sup>6</sup>D. A. Bies and C. H. Hansen, "Flow resistance information for acoustical design," *Appl. Acoust.* **13**, 357–391 (1980).

<sup>7</sup>T. F. W. Embleton, J. E. Piercy, and G. A. Daigle, "Effective flow resistivity of ground surfaces determined by acoustical measurements," *J. Acoust. Soc. Am.* **74**, 1239–1244 (1983).

<sup>8</sup>R. L. Brown and R. H. Bolt, "The measurement of flow resistance of porous acoustic materials," *J. Acoust. Soc. Am.* **13**, 337–344 (1942).

<sup>9</sup>R. W. Leonard, "Simplified flow resistance measurements," *J. Acoust. Soc. Am.* **17**, 240–241 (1946).

<sup>10</sup>M. R. Stinson and G. A. Daigle, "Electronic system for the measurement of flow resistance," *J. Acoust. Soc. Am.* **83**, 2422–2428 (1988).

<sup>11</sup>P. M. Morse, R. H. Bolt, and R. L. Brown, "Relation between acoustic impedance and low resistance of porous acoustic materials," *J. Acoust. Soc. Am.* **12**, 475–476 (1941) (abstract).

<sup>12</sup>A. Gemant, "Frictional phenomena IV," *J. Appl. Phys.* **12**, 725–734 (1941).

<sup>13</sup>M. A. Biot, "The theory of propagation of elastic waves in fluid-saturated porous solid. I. Low frequency range," *J. Acoust. Soc. Am.* **28**, 168–178 (1956).

<sup>14</sup>D. L. Johnson, J. Koplik, and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media," *J. Fluid Mech.* **176**, 379–402 (1987).

<sup>15</sup>Z. E. A. Fellah, M. Fellah, W. Lauriks, and C. Depollier, "Direct and inverse scattering of transient acoustic waves by a slab of rigid porous material," *J. Acoust. Soc. Am.* **113**, 61–73 (2003).