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# Discussion on: “A Comparison of Sliding Mode and Unknown Input Observer for Fault Reconstruction”

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## 1 Introduction

The paper by C. Edwards and C.P. Tan [1] presents a comparative study between the sliding mode observers and the unknown input observers for faults reconstruction. This study is based on two works [2] and [3]. Let us give the situations considered by these works.

First the model considered in [2] is

$$\dot{x}(t) = Ax(t) + Bu(t) + Dv(t) \quad (1a)$$

$$y(t) = Cx(t) \quad (1b)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the known input,  $v(t) \in \mathbb{R}^q$  is the unknown input, which can be considered as the effect of the actuator failure, and  $y(t) \in \mathbb{R}^p$  is the measurement output. Without loss of generality, we have  $\text{rank } D = q$  and  $\text{rank } C = p$ .

The problem of the state estimation in this case is the well known unknown input one, see [4] and the references given in [2]. The necessary and sufficient conditions for the existence and the stability of the full order or reduced order observers are

**A.1.1**  $\text{rank } CD = \text{rank } D = q$ ,

**A.1.2** no invariant zeros of  $(C, A, D)$  are in  $\mathbb{C}_+$ .

*Remark 1.* No assumption is made on the nature of the unknown input  $v(t)$ . □

The model given in [3] is of the form

$$\dot{x}(t) = Ax(t) + Bu(t) + Ff_i(t, u) + M\xi(t, y, u) \quad (2a)$$

$$y(t) = Cx(t) \quad (2b)$$

where  $f_i : \mathbb{R}_+ \times \mathbb{R}^m \rightarrow \mathbb{R}^q$  models an unknown fault / disturbance and  $\xi : \mathbb{R}_+ \times \mathbb{R}^p \times \mathbb{R}^m \rightarrow \mathbb{R}^k$  describes the system uncertainty. Without loss of generality, we have  $\text{rank } F = q$ . The sliding mode observer developed for this system use the following assumptions

**A.2.1**  $\text{rank } CF = \text{rank } F = q$ ,

**A.2.2** no invariant zeros of  $(C, A, F)$  are in  $\mathbb{C}_+$ ,

**A.2.3**  $\|f_i(t, u)\| \leq \alpha(t, u)$ , where  $\alpha : \mathbb{R}_+ \times \mathbb{R}^m \rightarrow \mathbb{R}_+$  is known,

**A.2.4**  $\|\xi(t, y, u)\| < \beta$ , where  $\beta \in \mathbb{R}_+$  is known.

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First we can compare the assumptions used in [2] and [3]. Assumptions A.1.1 and A.1.2 are exactly A.2.1 and A.2.2, however boundedness conditions are added in [3]. The advantage of [2] is that  $v(t)$  is arbitrary and can be unbounded, in this case the observer of [3] can not be used. In addition, the unknown input observer developed in [2] is based on the results given in [5] and it is shown in [6] that these results are equivalent to those presented in [4, 7, 8].

## 2 Unknown input observer

In this section, we present an unified approach of the unknown input observer design for system (1) without using any transformation of the system as in [2]. Consider the following reduced order observer

$$\dot{z}(t) = Nz(t) + Ly(t) + Gu(t) \quad (3a)$$

$$\hat{x}(t) = Mz(t) + Ey(t) \quad (3b)$$

where  $z \in \mathbb{R}^{n-p}$  is the state of the observer.

It is easy to see that  $e(t) = \hat{x}(t) - x(t) \xrightarrow[t \rightarrow \infty]{} 0$  if there exists a full row rank matrix  $T \in \mathbb{R}^{(n-p) \times n}$  such that

- (i)  $N$  is a stability matrix,
- (ii)  $TA - NT - LC = 0$ ,
- (iii)  $MT + EC = I_n$ ,
- (iv)  $TD = 0$ ,
- (v)  $G = TB$ .

Now let  $R \in \mathbb{R}^{(n-p) \times n}$  be a full row rank matrix such that matrix  $[R^T \ C^T]$  is non singular (this is always possible since  $\text{rank } C = p$ ) and let

$$\begin{bmatrix} T \\ C \end{bmatrix} = \begin{bmatrix} I_{n-p} & -K \\ 0 & I_p \end{bmatrix} \begin{bmatrix} R \\ C \end{bmatrix}.$$

Then from conditions (ii), (iii) and (iv) we obtain

$$\underbrace{[N \ J \ K]}_{\Sigma} \underbrace{\begin{bmatrix} R & 0 \\ C & 0 \\ CA & CD \end{bmatrix}}_{\Theta} = \underbrace{[RA \ RD]}_{\Theta} \quad (4)$$

where  $J = L - NE$ . Equation (4) has a solution if and only if

$$\text{rank} \begin{bmatrix} \Sigma \\ \Theta \end{bmatrix} = \text{rank } \Sigma.$$

This is equivalent to assumption A.1.1. In this case we have

$$[N \ J \ K] = \Theta \Sigma^\dagger + Z \left( I_{n+p} - \Sigma \Sigma^\dagger \right) \quad (5)$$

which leads to

$$N = \Theta \Sigma^\dagger \begin{bmatrix} I_{n-p} \\ 0 \\ 0 \end{bmatrix} + Z \left( I_{n+p} - \Sigma \Sigma^\dagger \right) \begin{bmatrix} I_{n-p} \\ 0 \\ 0 \end{bmatrix}, \quad (6)$$

$$J = \Theta \Sigma^\dagger \begin{bmatrix} 0 \\ I_p \\ 0 \end{bmatrix} + Z \left( I_{n+p} - \Sigma \Sigma^\dagger \right) \begin{bmatrix} 0 \\ I_p \\ 0 \end{bmatrix}, \quad (7)$$

$$K = \Theta \Sigma^\dagger \begin{bmatrix} 0 \\ 0 \\ I_p \end{bmatrix} + Z \left( I_{n+p} - \Sigma \Sigma^\dagger \right) \begin{bmatrix} 0 \\ 0 \\ I_p \end{bmatrix}, \quad (8)$$

where  $Z$  is an arbitrary matrix of appropriate dimension and  $A^\dagger$  is a generalized inverse of matrix  $A$  satisfying  $A = AA^\dagger A$ .

It is easy to see that there exists  $Z$  such that  $N$  is a stability matrix if and only if assumption A.1.2 is satisfied.

Now we can see that

$$\Sigma^\dagger = \begin{bmatrix} \begin{bmatrix} R \\ C \end{bmatrix}^\dagger & 0 \\ -(CD)^\dagger CA \begin{bmatrix} R \\ C \end{bmatrix}^\dagger & (CD)^\dagger \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} R \\ C \end{bmatrix}^{-1} & 0 \\ -(CD)^\dagger CA \begin{bmatrix} R \\ C \end{bmatrix}^{-1} & (CD)^\dagger \end{bmatrix}$$

is a generalized inverse of  $\Sigma$  since  $\Sigma\Sigma^\dagger\Sigma = \Sigma$ . Let

$$\begin{bmatrix} H_1 & E_1 \end{bmatrix} = \begin{bmatrix} R \\ C \end{bmatrix}^{-1},$$

then we obtain

$$\Sigma^\dagger = \begin{bmatrix} H_1 & E_1 & 0 \\ -(CD)^\dagger CAH_1 & -(CD)^\dagger CAE_1 & (CD)^\dagger \end{bmatrix}.$$

Let  $[Z_1 \ Z_2 \ Z_3]$  be the partition of  $Z$  according to  $\Sigma^\dagger$ , then we obtain

$$N = R \left( I_n - D(CD)^\dagger C \right) AH_1 + Z_3 \left( I_p - (CD)(CD)^\dagger \right) CAH_1, \quad (9)$$

$$J = R \left( I_n - D(CD)^\dagger C \right) AE_1 + Z_3 \left( I_p - (CD)(CD)^\dagger \right) CAE_1, \quad (10)$$

$$K = RD(CD)^\dagger + Z_3 \left( I_p - (CD)(CD)^\dagger \right). \quad (11)$$

The observer design can be done as follows : under assumptions A.1.1 and A.1.2, choose  $Z_3$  such that  $N$ , given by (9), is a stability matrix and deduce  $J$  and  $K$  from (10) and (11), then calculate  $L = J - NE$  and  $T = R - KC$ . We deduce  $G = TB$  and we obtain  $M$  and  $E$  from

$$\begin{bmatrix} M & E \end{bmatrix} = \begin{bmatrix} T \\ C \end{bmatrix}^{-1}.$$

One can see that from the above results, by taking  $C = [0 \ I_p]$ , we obtain directly the results of [2, 5] (see [6]. On the other hand, and since matrix  $\begin{bmatrix} T^T & C^T \end{bmatrix}$  is regular, model (1) is structurally equivalent to

$$\begin{aligned} \begin{bmatrix} T \\ C \end{bmatrix} \dot{x}(t) &= \begin{bmatrix} T \\ C \end{bmatrix} Ax(t) + \begin{bmatrix} T \\ C \end{bmatrix} Bu(t) + \begin{bmatrix} T \\ C \end{bmatrix} Dv(t) \\ y(t) &= Cx(t) \end{aligned}$$

or equivalently

$$T\dot{\hat{x}}(t) = TAx(t) + TBu(t) \quad (12a)$$

$$C\dot{\hat{x}}(t) = \dot{y}(t) = CAx(t) + CBu(t) + CDv(t) \quad (12b)$$

$$y(t) = Cx(t) \quad (12c)$$

which gives

$$\hat{v}(t) = (CD)^\dagger (\dot{y}(t) - CA\hat{x}(t) - CBu(t)). \quad (13)$$

We can also see that (3) is an observer for the singular system (12a) and (12c). We can also obtain the unknown input estimation from (1a) as follows

$$\hat{v}(t) = D^\dagger \left( \dot{\hat{x}}(t) - A\hat{x}(t) - Bu(t) \right) = (D^T D)^{-1} D^T \left( \dot{\hat{x}}(t) - A\hat{x}(t) - Bu(t) \right) \quad (14)$$

where  $D^\dagger D = I_q$  since  $\text{rank } D = q$ .

*Remark 2.* The full order case can be obtained directly from the above results by putting  $R = I_n$ , then

$$\begin{bmatrix} R \\ C \end{bmatrix}^\dagger = \begin{bmatrix} I_n \\ C \end{bmatrix}^\dagger = [I_n \ 0]$$

which leads to

$$\Sigma^\dagger = \begin{bmatrix} I_n & 0 & 0 \\ -(CD)^\dagger CA & 0 & (CD)^\dagger \end{bmatrix}.$$

Let  $[Z_1 \ Z_2 \ Z_3]$  be the partition of  $Z$  according to  $\Sigma^\dagger$ , then we obtain

$$J = Z_2, \quad (15)$$

$$E = -D(CD)^\dagger - Z_3 \left( I_p - (CD)(CD)^\dagger \right), \quad (16)$$

$$N = A + D(CD)^\dagger CA - KC - Z_3 \left( I_p - (CD)(CD)^\dagger \right) CA. \quad (17)$$

and the observer (3) becomes [4]

$$\dot{z}(t) = Nz(t) + Ly(t) + Gu(t) \quad (18a)$$

$$\hat{x}(t) = z(t) - Ey(t) \quad (18b)$$

where  $J = L + NE$ ,  $\Theta = [A \ -D]$  and  $T = I_n + EC$ .

The observer design can be done as follows : under assumptions A.1.1 and A.1.2, determine  $J$  and  $Z_3$  such that  $N$ , given by (17), is a stability matrix and deduce  $E$  from (16), then calculate  $L = J - NE$  and  $G = TB = (I_n + EC)B$ . The above results summarize the results given in [4].

Let  $T = I_n + EC$  with  $TD = 0$ , then model (1) is structurally equivalent to

$$T\dot{x}(t) = TAx(t) + TBu(t) \quad (19a)$$

$$D^\dagger \dot{x}(t) = D^\dagger Ax(t) + D^\dagger Bu(t) + v(t) \quad (19b)$$

$$y(t) = Cx(t). \quad (19c)$$

We can also see that (18) is an observer for the singular system (19a) and (19c).  $\square$

The drawback of equation (13) or (14) is the use of the derivative of  $y(t)$  or  $\hat{x}(t)$ .

In reference [2], the estimation of the unknown input for  $B = 0$  is given by

$$\hat{v}(k\tau) = (D^T D)^{-1} D^T A (e^{A\tau} - I_n)^{-1} (\hat{x}((k+1)\tau) - e^{A\tau} \hat{x}(k\tau))$$

which is a discretization of (14) with sampling time  $\tau$ , used to avoid the derivative. This approach was used in [2] to estimate the unknown input affecting the output  $y(t)$  by adding the model of the bias resulting from sensor failures.

In reference [2], the parameter uncertainty in the system is modeled as

$$\dot{x}(t) = (A_0 + \Delta_A)x(t) + (B_0 + \Delta_B)u(t) + Dv(t) \quad (20a)$$

$$y(t) = Cx(t) + Hh(t) \quad (20b)$$

where  $h(t)$  is an unmeasurable vector describing the additive sensor failure. Model (20) is then reduced to standard unknown input system

$$\dot{x}(t) = A_0x(t) + B_0u(t) + D^*v^*(t) \quad (21a)$$

$$y(t) = Cx(t) + Hh(t) \quad (21b)$$

where  $D^* = [D \ I_n \ I_n]$  and  $v^*(t) = [v^T(t) \ (\Delta_A x(t))^T \ (\Delta_B u(t))^T]^T$ .

*Remark 3.* The sensor failure  $h(t)$  in (20) et (21) is assumed to be satisfy equation (33) in [2] or relation (16) in [1].  $\square$

As can be seen from [1], the method used is simple and can give good results in failures detection and identification. However, it was not used to estimate the failures. The problem induced by this approach is the choice of the decision rules (see [9] and references therein). In addition, the failure estimation uses the derivative of the state or the output, this explain why this method is very sensitive to the uncertainty and the noises.

### 3 Sliding mode observer

In [1, 10, 11], the proposed observer for the uncertain system (2) is in the form

$$\dot{z}(t) = Az(t) + Bu(t) - G_\ell e_y(t) + G_\eta \nu(t) \quad (22)$$

with

$$\nu(t) = -\rho(t, y, u) \frac{P_o e_y(t)}{\|P_o e_y(t)\|} \quad (23)$$

where  $e_y(t) = Cz(t) - y(t)$  is the output estimation error and  $P_o$  is a symmetric positive definite matrix. The function  $\rho(\cdot)$  is a design parameter which depends on the magnitude of the fault and the uncertainty.

The necessary and sufficient conditions for the existence of the observer (22), which must be insensitive to the unknown input or fault are assumptions A.2.1 and A.2.2. Contrary to observers (3) and (18) which are linear ones, the sliding mode observer (22) is nonlinear and discontinuous.

The advantage of the approach presented in the section 2 of [1] is that it permits in addition to estimate, under assumptions A.2.3 et A.2.4, the fault or the unknown input without using the derivative of the output (see equations (10)-(13) and (16)-(17) of [1]).

Another alternative of the sliding mode observers which can be used to estimate the faults or the unknown inputs is the step by step sliding mode observer [12, 13, 14]. The latter will be presented in the next section.

### 4 Comparison based on a crane system

The system considered for the comparison is described by a nonlinear model of the form(45)-(46) of [1], which can be written as

$$(I + m_p l^2) \ddot{\theta} + c\dot{\theta} + m_p g l \sin \theta + m_p l \ddot{d} \cos \theta = 0 \quad (24a)$$

$$(m_t + m_p) \ddot{d} + b\dot{d} + m_p l \ddot{\theta} \cos \theta - m_p l \dot{\theta}^2 \sin \theta = u. \quad (24b)$$

Choosing  $[x_1 \ x_2 \ x_3 \ x_4]^T = [\theta \ \dot{\theta} \ d \ \dot{d}]^T$  as the state vector and using standard small approximations, the crane system (24) is equivalent to

$$\dot{x}_1 = x_2 \quad (25a)$$

$$\dot{x}_2 = \frac{-\beta\gamma g}{\beta\alpha - \gamma^2} x_1 - \frac{c\beta}{\beta\alpha - \gamma^2} x_2 + \frac{b\gamma}{\beta\alpha - \gamma^2} x_4 - \frac{\gamma}{\beta\alpha - \gamma^2} u \quad (25b)$$

$$\dot{x}_3 = x_4 \quad (25c)$$

$$\dot{x}_4 = \frac{\gamma^2 g}{\beta\alpha - \gamma^2} x_1 + \frac{c\gamma}{\beta\alpha - \gamma^2} x_2 - \frac{b\alpha}{\beta\alpha - \gamma^2} x_4 + \frac{\alpha}{\beta\alpha - \gamma^2} u \quad (25d)$$

with

$$\alpha = I + m_p l^2, \quad \beta = m_t + m_p \text{ and } \gamma = m_p l$$

and corresponds to the linear model (1) with matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-\beta\gamma g}{\beta\alpha - \gamma^2} & \frac{-c\beta}{\beta\alpha - \gamma^2} & 0 & \frac{b\gamma}{\beta\alpha - \gamma^2} \\ 0 & 0 & 0 & 1 \\ \frac{\gamma^2 g}{\beta\alpha - \gamma^2} & \frac{c\gamma}{\beta\alpha - \gamma^2} & 0 & \frac{-b\alpha}{\beta\alpha - \gamma^2} \end{bmatrix}, C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = D = \begin{bmatrix} 0 \\ \frac{-\gamma}{\beta\alpha - \gamma^2} \\ 0 \\ \frac{\alpha}{\beta\alpha - \gamma^2} \end{bmatrix}.$$

The observer matching condition is verified (see([12, 14]), therefore it is possible to design the following step by step sliding mode observer [12, 13, 14]

$$\dot{\hat{x}}_1 = \hat{x}_2 + \lambda_1 \text{sign}(x_1 - \hat{x}_1) \quad (26a)$$

$$\dot{\hat{x}}_2 = \frac{-\beta\gamma g}{\beta\alpha - \gamma^2}x_1 - \frac{c\beta}{\beta\alpha - \gamma^2}\tilde{x}_2 + \frac{b\gamma}{\beta\alpha - \gamma^2}x_4 + E_1\lambda_2 \text{sign}(\tilde{x}_2 - \hat{x}_2) \quad (26b)$$

$$\dot{\hat{x}}_3 = x_4 + \lambda_3 \text{sign}(x_3 - \hat{x}_3) \quad (26c)$$

$$\dot{\hat{x}}_4 = \frac{\gamma^2 g}{\beta\alpha - \gamma^2}x_1 + \frac{c\gamma}{\beta\alpha - \gamma^2}\tilde{x}_2 - \frac{b\alpha}{\beta\alpha - \gamma^2}x_4 + \lambda_4 \text{sign}(x_4 - \hat{x}_4) \quad (26d)$$

with the following condition : if  $\hat{x}_1 = x_1$  then  $E_1 = 1$  else  $E_1 = 0$ . And by using the equivalent vector (see [15, 10])

$$\tilde{x}_2 = \hat{x}_2 + E_1\lambda_1 \text{sign}(x_1 - \hat{x}_1).$$

After recovering the states  $x_1, x_2, x_3$  and  $x_4$  (i.e.  $\hat{x}_1 = x_1, \tilde{x}_2 = \hat{x}_2 = x_2, \hat{x}_3 = x_3$  and  $\hat{x}_4 = x_4$ ), we can estimate the unknown input  $u$  by calculating the observation error dynamic

$$\begin{aligned} \dot{e}_1 &= e_2 - \lambda_1 \text{sign}(x_1 - \hat{x}_1) = 0 \\ \dot{e}_2 &= \frac{-\gamma}{\beta\alpha - \gamma^2}u - E_1\lambda_2 \text{sign}(\tilde{x}_2 - \hat{x}_2) = 0 \\ \dot{e}_3 &= -\lambda_3 \text{sign}(\tilde{x}_3 - \hat{x}_3) = 0 \\ \dot{e}_4 &= \frac{\alpha}{\beta\alpha - \gamma^2}u - \lambda_4 \text{sign}(x_4 - \hat{x}_4) = 0 \end{aligned}$$

which gives

$$\begin{aligned} \hat{u} &= \frac{-\beta\alpha - \gamma^2}{\gamma}E_1\lambda_2 \text{sign}(\tilde{x}_2 - \hat{x}_2) \\ \hat{u} &= \frac{\beta\alpha - \gamma^2}{\alpha}\lambda_4 \text{sign}(x_4 - \hat{x}_4). \end{aligned}$$

To explore the effect of parametric uncertainty the mass of the pendulum has been changed by 5% to be  $m_p = m'_p = 0.525$ . However, the observation error dynamics ( $e_i = x_i - \hat{x}_i$ ) are

$$\begin{aligned} \dot{e}_1 &= e_2 - \lambda_1 \text{sign}(x_1 - \hat{x}_1) \\ \dot{e}_2 &= \left( \frac{\beta\gamma g}{\beta\alpha - \gamma^2} - \frac{\beta'\gamma'g}{\beta'\alpha' - \gamma'^2} \right) x_1 + \left( \frac{c\beta}{\beta\alpha - \gamma^2} - \frac{c'\beta'}{\beta'\alpha' - \gamma'^2} \right) x_2 \\ &\quad + \left( \frac{b\gamma'}{\beta'\alpha' - \gamma'^2} - \frac{b\gamma}{\beta\alpha - \gamma^2} \right) x_4 - \frac{\gamma'}{\beta'\alpha' - \gamma'^2}u - E_1\lambda_2 \text{sign}(\tilde{x}_2 - \hat{x}_2) \\ \dot{e}_3 &= -\lambda_3 \text{sign}(\tilde{x}_3 - \hat{x}_3) \\ \dot{e}_4 &= \left( \frac{\gamma'^2 g}{\beta'\alpha' - \gamma'^2} - \frac{\gamma^2 g}{\beta\alpha - \gamma^2} \right) x_1 + \left( \frac{c\gamma'}{\beta'\alpha' - \gamma'^2} - \frac{c\gamma}{\beta\alpha - \gamma^2} \right) x_2 \\ &\quad + \left( \frac{b\alpha}{\beta\alpha - \gamma^2} - \frac{b\alpha'}{\beta'\alpha' - \gamma'^2} \right) x_4 + \frac{\alpha'}{\beta'\alpha' - \gamma'^2}u - \lambda_4 \text{sign}(x_4 - \hat{x}_4) \end{aligned}$$

with

$$\alpha' = I + m'_p l^2, \beta' = m_t + m'_p \text{ and } \gamma' = m'_p l.$$

When we estimated  $x_1, x_2, x_3$  and  $x_4$  (i.e.  $\hat{x}_1 = x_1, \tilde{x}_2 = \hat{x}_2 = x_2, \hat{x}_3 = x_3$  and  $\hat{x}_4 = x_4$ ), we have  $\dot{e} = 0$ .

Then the estimation of unknown input is given by

$$\begin{aligned}\hat{u} &= \frac{\gamma'}{\beta'\alpha' - \gamma'^2} \left( \left( \frac{\beta\gamma g}{\beta\alpha - \gamma^2} - \frac{\beta'\gamma'g}{\beta'\alpha' - \gamma'^2} \right) x_1 + \left( \frac{c\beta}{\beta\alpha - \gamma^2} - \frac{c'\beta'}{\beta'\alpha' - \gamma'^2} \right) x_2 \right. \\ &\quad \left. + \left( \frac{b\gamma'}{\beta'\alpha' - \gamma'^2} - \frac{b\gamma}{\beta\alpha - \gamma^2} \right) x_4 - E_1\lambda_2 \text{sign}(\tilde{x}_2 - \hat{x}_2) \right) \\ \hat{u} &= \frac{-\alpha'}{\beta'\alpha' - \gamma'^2} \left( \left( \frac{\gamma'^2 g}{\beta'\alpha' - \gamma'^2} - \frac{\gamma^2 g}{\beta\alpha - \gamma^2} \right) x_1 + \left( \frac{c\gamma'}{\beta'\alpha' - \gamma'^2} - \frac{c\gamma}{\beta\alpha - \gamma^2} \right) x_2 \right. \\ &\quad \left. + \left( \frac{b\alpha}{\beta\alpha - \gamma^2} - \frac{b\alpha'}{\beta'\alpha' - \gamma'^2} \right) x_4 - \lambda_4 \text{sign}(x_4 - \hat{x}_4) \right).\end{aligned}$$

The simulation results are shown in figures 1 and 2.

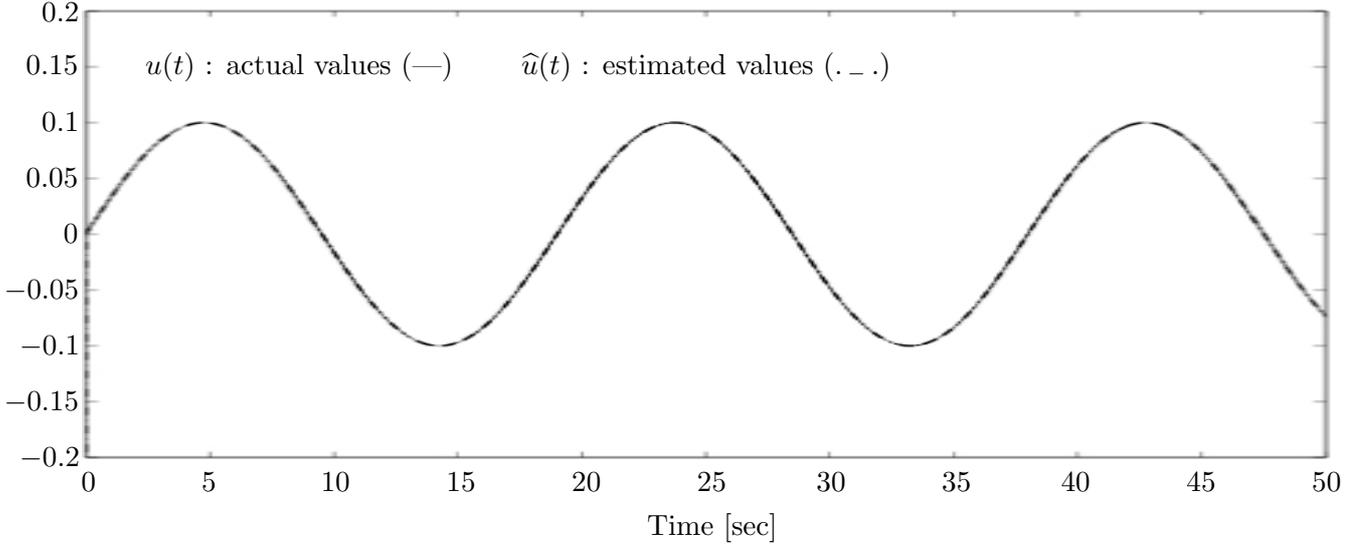


Figure 1: Actuator reconstruction without parametric uncertainties.

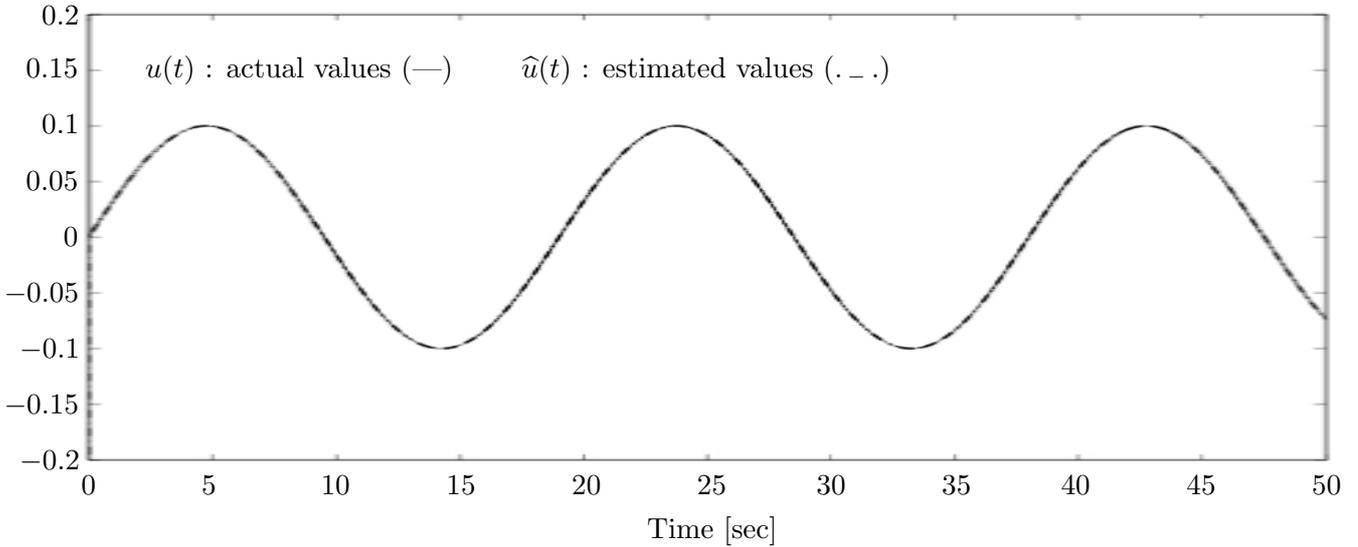


Figure 2: Actuator reconstruction in presence of parametric uncertainties.

Figures 1 and 2 show the obtained results of the actuator reconstruction by using the step by step sliding mode observer (26) when system (25) has no parametric uncertainties and in presence of parametric uncertainties in system (25), respectively.

Comparing the unknown input and classical sliding mode observers given in [1] and the step by step sliding mode observer in presence of parametric uncertainties, we notice that more significant errors appear for the unknown input and classical sliding mode observers (see figure 7 in [1]), however a good estimation is obtained by the step by step sliding mode observer (26) (see figure 2).

The step by step sliding mode observer is very useful and was developed to achieve robustness under parameter uncertainties, if a specific condition (dual of the matching condition [12, 14]) is verified.

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