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ASYMPTOTIC ESTIMATE FOR PERTURBED SCALAR CURVATURE EQUATION.

SAMY SKANDER BAHOURA

ABSTRACT. We consider the equation $\Delta u_\epsilon = V_\epsilon u_\epsilon^{(n+2)/(n-2)} + \epsilon W_\epsilon u_\epsilon^\alpha$ with $\alpha \in]\frac{n}{n-2}, \frac{n+2}{n-2}[$ and we give some minimal conditions on ∇V and ∇W to have an uniform estimate for their solutions when $\epsilon \rightarrow 0$.

1. INTRODUCTION AND RESULTS.

We denote $\Delta = -\sum_i \partial_{ii}$ the geometric Laplacian on \mathbb{R}^n , $n \geq 3$.

Let us consider on open set Ω of \mathbb{R}^n , $n \geq 3$, the following equation:

$$\Delta u_\epsilon = V_\epsilon u_\epsilon^{(n+2)/(n-2)} + \epsilon W_\epsilon u_\epsilon^\alpha \quad (E_\epsilon)$$

where V_ϵ and W_ϵ are two regular functions and $\alpha \in]\frac{n}{n-2}, \frac{n+2}{n-2}[$.

We assume:

$$0 < a \leq V_\epsilon(x) \leq b, \quad \|\nabla V_\epsilon\|_{L^\infty} \leq A \quad (C_1)$$

$$0 < c \leq W_\epsilon(x) \leq d, \quad \|\nabla W_\epsilon\|_{L^\infty} \leq B \quad (C_2)$$

Problem: Can we have an $\sup \times \inf$ estimate with the minimal conditions (C_1) and (C_2) ?

Note that for $W \equiv 0$, the equation (E_ϵ) is the wellknown scalar curvature equation on open set of \mathbb{R}^n , $n \geq 3$. In this case, there is many results about this equation, see for example [B] and [C-L 1].

When $\Omega = \mathbb{S}_n$ YY. Li, give a flatness condition to have the boundedness of the energy and the existence of the simple blow-up points, see [L1] and [L2].

In [C-L 2], Chen and Lin gave a conterexample of solutions of the scalar curvature equation with unbounded energy. The conditions of Li are minimal in heigh dimension.

Note that, in [C-L 1] and [C-L 3], there is some results concerning Harnack inequalities of type $\sup \times \inf$ with the "Li-flatness" conditions for the following equation:

$$\Delta u = V u^{(n+2)/(n-2)} + g(u)$$

where g is a regular function (at least C^1) such that $g(t)/[t^{(n+2)/(n-2)}]$ is deacrising and tends to 0 when $t \rightarrow +\infty$. They extend Li result ([L1]) to any open set of the euclidian space.

We can find in [A], some existence results for the presribed scalar curvature equation.

In our work we have no assumption on the energy. We use the blow-up analysis and the moving-plane method, developped by Gidas-Ni-Nirenberg, see [G-N-N]. This method was used by different authors to have a priori estimates, look for example, [B], [B-L-S] (in dimension 2), [C-L 1], [C-L 3], [L 1] and [L 2].

We set $\delta = [(n+2) - \alpha(n-2)]/2$, $\delta \in]0, 1[$. We have:

Theorem 1. For all $a, b, c, d, A, B > 0$, for all $\alpha \in]\frac{n}{n-2}, \frac{n+2}{n-2}[$ and all compact set K of Ω , there is a positive constant $c = c(a, b, c, d, A, B, \alpha, K, \Omega, n)$ such that:

$$\epsilon^{(n-2)/2(1-\delta)} (\sup_K u_\epsilon)^{1/3} \times \inf_\Omega u_\epsilon \leq c$$

for all u_ϵ solution of (E_ϵ) with V_ϵ and W_ϵ satisfying the conditions (C_1) and (C_2) .

Now, we suppose that V_ϵ satisfies:

$$0 < a \leq V_\epsilon(x) \leq b \text{ and } \|\nabla V_\epsilon\|_{L^\infty(\Omega)} \leq k\epsilon \quad (C_3)$$

We have:

Theorem 2. For all $a, b, c, d, k, B > 0$, for all $\alpha \in]\frac{n}{n-2}, \frac{n+2}{n-2}[$ and all compact set K of Ω , there is a positive constant $c = c(a, b, c, d, k, B, \alpha, K, \Omega, n)$ such that:

$$\sup_K u_\epsilon \times \inf_\Omega u_\epsilon \leq c$$

for all u_ϵ solution of (E_ϵ) with V_ϵ and W_ϵ satisfying the conditions (C_3) and (C_2) .

Note that in [B], we have some results as the previous but for prescribed scalar curvature equation with subcritical exponent tending to the critical. Here, we have a $\sup \times \inf$ inequality for the scalar curvature equation, with critical exponent, perturbed by a nonlinear term. We can see the influence of this non-linear term.

2. PROOFS OF THE THEOREMS.

Proof of the theorem 1.

Without loss of generality, we suppose $\Omega = B_1$ the unit ball of \mathbb{R}^n . We want to prove an a priori estimate around 0. We can also suppose $\epsilon \rightarrow 0$, the case $\epsilon \not\rightarrow 0$ is solved in [B].

Let (u_i) and (V_i) be a sequences of functions on Ω such that:

$$\Delta u_i = V_i u_i^{(n+2)/(n-2)} + \epsilon_i W_i u_i^\alpha, \quad u_i > 0,$$

with $0 < a \leq V_i(x) \leq b, 0 < a \leq W_i(x) \leq d, \|V_i\|_{L^\infty} \leq A$ and $\|W_i\|_{L^\infty} \leq B$.

We argue by contradiction and we suppose that the $\sup \times \inf$ is not bounded.

We have:

$\forall c, R > 0 \exists u_{c,R}$ solution of (E_1) such that:

$$\epsilon^{(n-2)/2(1-\delta)} R^{n-2} (\sup_{B(0,R)} u_{\epsilon,c,R})^{1/3} \times \inf_\Omega u_{\epsilon,c,R} \geq c, \quad (H)$$

Proposition : (blow-up analysis)

There is a sequence of points $(y_i)_i, y_i \rightarrow 0$ and two sequences of positive real numbers $(l_i)_i, (L_i)_i, l_i \rightarrow 0, L_i \rightarrow +\infty$, such that if we set $v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}$, we have:

$$0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \rightarrow 1.$$

$$v_i(y) \rightarrow \left(\frac{1}{1 + |y|^2} \right)^{(n-2)/2}, \text{ uniformly on all compact set of } \mathbb{R}^n.$$

$$l_i^{(n-2)/2} \epsilon_i^{(n-2)/2(1-\delta)} [u_i(y_i)]^{1/3} \times \inf_{B_1} u_i \rightarrow +\infty,$$

Proof of the proposition:

We use the hypothesis (H) , we take two sequences $R_i > 0, R_i \rightarrow 0$ and $c_i \rightarrow +\infty$, such that,

$$\epsilon_i^{(n-2)/2(1-\delta)} R_i^{(n-2)} \left(\sup_{B(0, R_i)} u_i \right)^{1/3} \times \inf_{B_1} u_i \geq c_i \rightarrow +\infty,$$

Let $x_i \in B(x_0, R_i)$ be a point such that $\sup_{B(0, R_i)} u_i = u_i(x_i)$ and $s_i(x) = (R_i - |x - x_i|)^{(n-2)/2} u_i(x)$, $x \in B(x_i, R_i)$. Then, $x_i \rightarrow 0$.

We have:

$$\max_{B(x_i, R_i)} s_i(x) = s_i(y_i) \geq s_i(x_i) = R_i^{(n-2)/2} u_i(x_i) \geq \sqrt{c_i} \rightarrow +\infty.$$

We set:

$$l_i = R_i - |y_i - x_i|, \quad \bar{u}_i(y) = u_i(y_i + y), \quad v_i(z) = \frac{u_i[y_i + (z/[u_i(y_i)]^{2/(n-2)})]}{u_i(y_i)}.$$

Clearly we have, $y_i \rightarrow x_0$. We also obtain:

$$L_i = \frac{l_i}{(c_i)^{1/2(n-2)}} [u_i(y_i)]^{2/(n-2)} = \frac{[s_i(y_i)]^{2/(n-2)}}{c_i^{1/2(n-2)}} \geq \frac{c_i^{1/(n-2)}}{c_i^{1/2(n-2)}} = c_i^{1/2(n-2)} \rightarrow +\infty.$$

If $|z| \leq L_i$, then $y = [y_i + z/[u_i(y_i)]^{2/(n-2)}] \in B(y_i, \delta_i l_i)$ with $\delta_i = \frac{1}{(c_i)^{1/2(n-2)}}$ and $|y - y_i| < R_i - |y_i - x_i|$, thus, $|y - x_i| < R_i$ and, $s_i(y) \leq s_i(y_i)$. We can write:

$$u_i(y) (R_i - |y - y_i|)^{(n-2)/2} \leq u_i(y_i) (l_i)^{(n-2)/2}.$$

But, $|y - y_i| \leq \delta_i l_i$, $R_i > l_i$ and $R_i - |y - y_i| \geq R_i - \delta_i l_i > l_i - \delta_i l_i = l_i(1 - \delta_i)$. We obtain,

$$0 < v_i(z) = \frac{u_i(y)}{u_i(y_i)} \leq \left[\frac{l_i}{l_i(1 - \delta_i)} \right]^{(n-2)/2} \leq 2^{(n-2)/2}.$$

We set, $\beta_i = \left(\frac{1}{1 - \delta_i} \right)^{(n-2)/2}$, clearly, we have, $\beta_i \rightarrow 1$.

The function v_i satisfies:

$$\Delta v_i = \tilde{V}_i v_i^{(n+2)/(n-2)} + \epsilon_i \tilde{W}_i \frac{v_i^{n/(n-2)}}{[u_i(y_i)]^{[(n+2)/(n-2)] - \alpha}}$$

where, $\tilde{V}_i(y) = V_i [y + y/[u_i(y_i)]^{2/(n-2)}]$ and $\tilde{W}_i(y) = W_i [y + y/[u_i(y_i)]^{2/(n-2)}]$.

Without loss of generality, we can suppose that $\tilde{V}_i \rightarrow V(0) = n(n-2)$.

We use the elliptic estimates, Ascoli and Ladyzenskaya theorems to have the uniform convergence of (v_i) to v on compact set of \mathbb{R}^n . The function v satisfies:

$$\Delta v = n(n-2)v^{N-1}, \quad v(0) = 1, \quad 0 \leq v \leq 1 \leq 2^{(n-2)/2},$$

By the maximum principle, we have $v > 0$ on \mathbb{R}^n . If we use Caffarelli-Gidas-Spruck result, (see [C-G-S]), we obtain, $v(y) = \left(\frac{1}{1 + |y|^2} \right)^{(n-2)/2}$. We have the same properties that in [B].

Polar Coordinates (Moving-Plane method)

Now, we must use the same method than in the Theorem 1 of [B]. We will use the moving-plane method.

We must prove the lemma 2 of [B].

We set $t \in]-\infty, -\log 2]$ and $\theta \in \mathbb{S}_{n-1}$:

$$w_i(t, \theta) = e^{(n-2)t/2} u_i(y_i + e^t \theta), \quad \bar{V}_i(t, \theta) = V_i(y_i + e^t \theta) \text{ and } \bar{W}_i(t, \theta) = W_i(y_i + e^t \theta).$$

We consider the following operator $L = \partial_{tt} - \Delta_\sigma - \frac{(n-2)^2}{4}$, with Δ_σ the Laplace-Baltrami operator on \mathbb{S}_{n-1} .

The function w_i is solution of:

$$-Lw_i = \bar{V}_i w_i^{N-1} + \epsilon_i e^{[(n+2)-(n-2)\alpha]t/2} \bar{W}_i w_i^\alpha.$$

For $\lambda \leq 0$ we set :

$$t^\lambda = 2\lambda - t \quad w_i^\lambda(t, \theta) = w_i(t^\lambda, \theta), \quad \bar{V}_i^\lambda(t, \theta) = \bar{V}_i(t^\lambda, \theta) \text{ et } \bar{W}_i^\lambda(t, \theta) = \bar{W}_i(t^\lambda, \theta).$$

Remark: Here we work on $[\lambda, t_i] \times \mathbb{S}_{n-1}$, with $\lambda \leq -\frac{2}{n-2} \log u_i(y_i) + 2$ and $t_i \leq \log \sqrt{t_i}$, where t_i is chooses as in the proposition.

First, like in [B], we have the following lemma:

Lemma 1:

Let A_λ be the following property:

$$A_\lambda = \{ \lambda \leq 0, \exists (t_\lambda, \theta_\lambda) \in]\lambda, t_i] \times \mathbb{S}_{n-1}, \bar{w}_i^\lambda(t_\lambda, \theta_\lambda) - \bar{w}_i(t_\lambda, \theta_\lambda) \geq 0 \}.$$

Then, there is $\nu \leq 0$, such that for $\lambda \leq \nu$, A_λ is not true.

Like in the proof of the Theorem 1 of [B], we want to prove the following lemma:

Lemma 2:

For $\lambda \leq 0$ we have :

$$w_i^\lambda - w_i \leq 0 \Rightarrow -L(w_i^\lambda - w_i) \leq 0,$$

on $]\lambda, t_i] \times \mathbb{S}_{n-1}$.

Like in [B], we have:

A useful point:

$$\xi_i = \sup\{ \lambda \leq \bar{\lambda}_i = 2 + \log \eta_i, w_i^\lambda - w_i < 0, \text{ on }]\lambda, t_i] \times \mathbb{S}_{n-1} \}. \text{ The real } \xi_i \text{ exists.}$$

First, we have:

$$w_i(2\xi_i - t, \theta) = w_i[(\xi_i - t + \xi_i - \log \eta_i - 2) + (\log \eta_i + 2)],$$

the definition of w_i and the fact that, $\xi_i \leq t$, we obtain:

$$w_i(2\xi_i - t, \theta) = e^{[(n-2)(\xi_i - t + \xi_i - \log \eta_i - 2)]/2} e^{n-2} v_i [\theta e^2 e^{(\xi_i - t) + (\xi_i - \log \eta_i - 2)}] \leq 2^{(n-2)/2} e^{n-2} = \bar{c}.$$

Proof of the Lemma 2:

We know that:

$$-L(w_i^{\xi_i} - w_i) = [\bar{V}_i^{\xi_i} (w_i^{\xi_i})^{N-1} - \bar{V}_i w_i^{N-1}] + \epsilon_i [e^{\delta t \xi_i} \bar{W}_i^{\xi_i} (w_i^{\xi_i})^\alpha - e^{\delta t} \bar{W}_i w_i^\alpha],$$

with $\delta = [(n+2) - (n-2)\alpha]/2$.

We denote by Z_1 and Z_2 the following terms:

$$Z_1 = (\bar{V}_i^{\xi_i} - \bar{V}_i)(w_i^{\xi_i})^{N-1} + \bar{V}_i [(w_i^{\xi_i})^{N-1} - w_i^{N-1}],$$

and

$$Z_2 = \epsilon_i(\bar{W}_i^{\xi_i} - \bar{W}_i)(w_i^{\xi_i})^\alpha e^{\delta t^{\xi_i}} + \epsilon_i e^{\delta t^{\xi_i}} \bar{W}_i [(w_i^{\xi_i})^\alpha - w_i^\alpha] + \epsilon_i \bar{W}_i w_i^\alpha (e^{\delta t^{\xi_i}} - e^{\delta t}).$$

But, using the same method as in the proof of the theorem 1 of [B], we have:

$$w_i^{\xi_i} \leq w_i \text{ et } w_i^{\xi_i}(t, \theta) \leq \bar{c} \text{ pour tout } (t, \theta) \in [\xi_i, \log 2] \times \mathbb{S}_{n-1},$$

where \bar{c} is a positive constant not depending on i for $\xi_i \leq \log \eta_i + 2$;

$$|\bar{V}_i^{\xi_i} - \bar{V}_i| \leq A(e^t - e^{t^{\xi_i}}) \text{ et } |\bar{W}_i^{\xi_i} - \bar{W}_i| \leq B(e^t - e^{t^{\xi_i}}),$$

Then,

$$Z_1 \leq A(w_i^{\xi_i})^{N-1} (e^t - e^{t^{\xi_i}}) \text{ et } Z_2 \leq \epsilon_i B ((w_i^{\xi_i})^\alpha (e^t - e^{t^{\xi_i}}) + \epsilon_i c (w_i^{\xi_i})^\alpha \times (e^{\delta t^{\xi_i}} - e^{\delta t})).$$

and,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [(A w_i^{\xi_i}{}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})].$$

But, $w_i^{\xi_i} \leq \bar{c}$, we obtain:

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [(A \bar{c}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})]. \quad (1)$$

We must see the sign of:

$$\bar{Z} = [(A \bar{c}^{N-1-\alpha} + \epsilon_i B) (e^t - e^{t^{\xi_i}}) + \epsilon_i c (e^{\delta t^{\xi_i}} - e^{\delta t})].$$

$$\text{But } \alpha \in]\frac{n}{n-2}, \frac{n+2}{n-2}[, \delta = \frac{n+2-(n-2)\alpha}{2} \in]0, 1[.$$

For $t \leq t_i < 0$, we have:

$$e^t \leq e^{(1-\delta)t_i} e^{\delta t}, \text{ for all } t \leq t_i.$$

and the fact that $t^{\xi_i} \leq t$ ($\xi_i \leq t$), by integration of the previous two members, we obtain:

$$e^t - e^{t^{\xi_i}} \leq \frac{e^{(1-\delta)t_i}}{\delta} (e^{\delta t} - e^{\delta t^{\xi_i}}), \text{ for all } t \leq t_i,$$

We can write:

$$(e^{\delta t^{\xi_i}} - e^{\delta t}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{t^{\xi_i}} - e^t).$$

Then,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha [-\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} + A \bar{c}^{N-1-\alpha} + \epsilon_i B] (e^t - e^{t^{\xi_i}}).$$

The term $\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} - A \bar{c}^{N-1-\alpha} - \epsilon_i B$ is positive if:

$$\epsilon_i e^{-(1-\delta)t_i} \rightarrow +\infty,$$

then,

$$\epsilon_i^{(n-2)/2(1-\delta)} e^{-(n-2)/2t_i} \rightarrow +\infty.$$

If we take, $t_i = -\frac{2}{3(n-2)} \log u_i(y_i)$, we have:

$$\epsilon_i^{(n-2)/2(1-\delta)} [u_i(y_i)]^{1/3} \rightarrow +\infty.$$

It is given by our Hypothesis in the proposition.

But the Hopf Maximum principle, gives:

$$\min_{\theta \in \mathbb{S}_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in \mathbb{S}_{n-1}} w_i(2\xi_i - t_i, \theta),$$

then,

$$e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

and,

$$[u_i(y_i)]^{1/3} \min_{B_2(0)} u_i \leq c,$$

Contradiction.

Proof of the Theorem 2.

The proof is similar than the proof of the theorem 1. Only the end of the proof is different.

Step 1: The blow-up analysis give:

There is a sequence of points $(y_i)_i$, $y_i \rightarrow 0$ and two sequences of positive real numbers $(l_i)_i$, $(L_i)_i$, $l_i \rightarrow 0$, $L_i \rightarrow +\infty$, such that if we set $v_i(y) = \frac{u_i(y + y_i)}{u_i(y_i)}$, we have:

$$0 < v_i(y) \leq \beta_i \leq 2^{(n-2)/2}, \quad \beta_i \rightarrow 1.$$

$$v_i(y) \rightarrow \left(\frac{1}{1 + |y|^2} \right)^{(n-2)/2}, \quad \text{uniformly on all compact set of } \mathbb{R}^n.$$

$$l_i^{(n-2)/2} u_i(y_i) \times \inf_{B_1} u_i \rightarrow +\infty,$$

Step 2: Application of the Hopf maximum principle.

We have the same notation that in the proof of the theorem 1. First, we take $t_i = \sqrt{l_i}$ as in the Step 1 and we look to the end of the proof of the theorem 1. We replace A by $k\epsilon_i$. We want to proof that:

$$w_i^\lambda - w_i \leq 0 \Rightarrow -L(w_i^\lambda - w_i) \leq 0,$$

on $]\xi_i, t_i] \times \mathbb{S}_{n-1}$. We have the same defintion for ξ_i (as in the proof of the theorem 1).

For $t \leq t_i < 0$, we have:

$$e^t \leq e^{(1-\delta)t_i} e^{\delta t}, \quad \text{for all } t \leq t_i.$$

and the fact that $t^{\xi_i} \leq t$ ($\xi_i \leq t$), by integration of the previous two members, we obtain:

$$e^t - e^{t^{\xi_i}} \leq \frac{e^{(1-\delta)t_i}}{\delta} (e^{\delta t} - e^{\delta t^{\xi_i}}), \quad \text{for all } t \leq t_i,$$

We can write:

$$(e^{\delta t^{\xi_i}} - e^{\delta t}) \leq \frac{\delta}{e^{(1-\delta)t_i}} (e^{t^{\xi_i}} - e^t).$$

Then,

$$-L(w_i^{\xi_i} - w_i) \leq (w_i^{\xi_i})^\alpha \left[-\frac{\epsilon_i \delta c}{e^{(1-\delta)t_i}} + k\epsilon_i \bar{c}^{N-1-\alpha} + \epsilon_i B \right] (e^t - e^{t^{\xi_i}}).$$

The term $\frac{\delta c}{e^{(1-\delta)t_i}} - k \bar{c}^{N-1-\alpha} - B$ is positive because $t_i \rightarrow -\infty$ and $\delta \in]0, 1[$.

But the Hopf Maximum principle, gives:

$$\min_{\theta \in \mathbb{S}_{n-1}} w_i(t_i, \theta) \leq \max_{\theta \in \mathbb{S}_{n-1}} w_i(2\xi_i - t_i, \theta),$$

then,

$$e^{(n-2)t_i} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

and,

$$l_i^{(n-2)/2} u_i(y_i) \min_{B_2(0)} u_i \leq c,$$

Contradiction with the step 1.

Références:

- [A] T. Aubin. *Nonlinear Problems in Riemannian Geometry*. Springer-Verlag 1998.
- [B] S.S Bahoura. Majorations du type $\sup u \times \inf u \leq c$ pour l'équation de la courbure scalaire prescrite sur un ouvert de \mathbb{R}^n , $n \geq 3$. *J.Math.Pures Appl.*(9) 83 (2004), no.9, 1109-1150.
- [B-L-S] H. Brezis, Yy. Li Y-Y, I. Shafrir. A sup+inf inequality for some nonlinear elliptic equations involving exponential nonlinearities. *J.Funct.Anal.* 115 (1993) 344-358.
- [C-G-S] Caffarelli L, Gidas B., Spruck J. Asymptotic symmetry and local behavior of semi-linear elliptic equations with critical Sobolev growth. *Commun. Pure Appl. Math.* 37 (1984) 369-402.
- [C-L 1] Chen C-C, Lin C-S. Estimates of the conformal scalar curvature equation via the method of moving planes. *Comm. Pure Appl. Math.* L(1997) 0971-1017.
- [C-L 2] Chen C-C. and Lin C-S. Blowing up with infinite energy of conformal metrics on \mathbb{S}_n . *Comm. Partial Differ Equations.* 24 (5,6) (1999) 785-799.
- [C-L 3] Chen C-C, Lin C-S. Prescribing scalar curvature on \mathbb{S}_n . I. A priori estimates. *J. Differential Geom.* 57 (2001), no. 1, 67-171.
- [G-N-N] B. Gidas, W. Ni, L. Nirenberg, Symmetry and Related Properties via the Maximum Principle, *Comm. Math. Phys.*, vol 68, 1979, pp. 209-243.
- [L1] Y.Y Li. Prescribing Scalar Curvature on \mathbb{S}_n and related Problems. I. *J. Differential Equations* 120 (1995), no. 2, 319-410.
- [L2] Y.Y Li. Prescribing Scalar Curvature on \mathbb{S}_n and related Problems. II. *Comm. Pure. Appl. Math.* 49(1996), no.6, 541-597.

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