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Comments on “State observation of a fixed-bed reactor based on reduced bilinear models”

H. RAFARALAHY, M. ZASADZINSKI, M. BOUTAYEB and M. DAROUACH[†]

Abstract

In this note, we point out that the proof of the convergence of the bilinear systems observer proposed by Hua (1993) is erroneous. Furthermore, the proposed existence conditions are not sufficient as it was claimed. To overcome these difficulties, we propose a straightforward proof of the observer's convergence based on Lyapunov stability and we deduce sufficient existence conditions.

1. Introduction

In a recent paper, X.-M. Hua (1993) presented a new bilinear observer design via pole-placement technique. This observer is used to estimate the temperature and the concentration profiles of a fixed-bed reactor approximated as bilinear system through orthogonal collocation. The proposed approach seems to be attractive since it can be applied to a large class of bilinear systems whereas the observers proposed by Hara and Furuta (1976), Funahashi (1979) or Derese *et al.* (1979), fail. Unfortunately, the proof of the convergence of this observer, based on Lyapunov stability, is erroneous. In addition, we demonstrate, via a counter-example, that the given existence conditions are not sufficient. Thus, we propose a simple proof and we deduce sufficient existence conditions of the observer which are stronger than those given by Hua.

2. Preliminaries

Consider the following continuous-time bilinear system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^p N_i x(t) u_i(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \mathfrak{R}^n$, $y(t) \in \mathfrak{R}^m$ and $u(t) = \begin{bmatrix} u_1(t) \dots u_p(t) \end{bmatrix}^T \in \mathfrak{R}^p$ are state, output and input vectors respectively. The matrices A , N_i ($i = 1 \dots p$), B and C are constant and of appropriate dimensions. Assume that the inputs of (1) are bounded, i.e. $u(t) \in \Omega$ where

$$\Omega = \left\{ u(t) / u_{i,\min} \leq u_i(t) \leq u_{i,\max} \text{ for all } t, \text{ and } i = 1 \dots p \right\}$$

The full-order state observer of (1) and (2) is of the following form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \sum_{i=1}^p N_i \hat{x}(t) u_i(t) + Bu(t) + G_0(y(t) - C\hat{x}(t)) + \sum_{i=1}^p G_i(y(t) - C\hat{x}(t)) u_i(t) \quad (3)$$

where $\hat{x}(t) \in \mathfrak{R}^n$ is the state estimate. The observation error is defined by

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$$e(t) = \hat{x}(t) - x(t) \quad (4)$$

which satisfies the following dynamical equation

$$\dot{e}(t) = \Gamma_0 e(t) + \sum_{i=1}^p \Gamma_i e(t) u_i(t) \quad e(0) = e_0 \quad (5)$$

where $\Gamma_0 = A - G_0 C$ and $\Gamma_i = N_i - G_i C, i = 1 \dots p$

The main objective is to find suitable observer's gain matrices G_0 and G_i such that the observation error approaches zero asymptotically regardless of the initial error.

The two-step design procedure of the bilinear state observer, proposed by Hua, is given by

Step 1 : choose G_i such that Γ_i are Hurwitz, $i = 1 \dots p$,

Step 2 : choose G_0 such that $\bar{\Gamma}_0$ is Hurwitz,

where $\bar{\Gamma}_0 = F - G_0 C$ and $F = A + \sum_{i=1}^p \Gamma_i u_{i,\min}$.

The sufficient existence conditions proposed by Hua are

- (ia) the pair (F, C) is observable,
- (iia) and the pairs (N_i, C) are observable, $i = 1 \dots p$

which correspond to the observers convergence conditions (ib) and (iib) given by Hua supposedly sufficient

- (ib) $\bar{\Gamma}_0$ is Hurwitz,
- (iib) and Γ_i are Hurwitz, $i = 1 \dots p$.

In the following section, we pointed out that the proof proposed by Hua is found to be incorrect and we demonstrate, via a counter-example, that the given existence conditions are not sufficient.

3. Comments on the proof of the observers convergence

In order to investigate the stability of the error dynamics, Hua used the Lyapunov stability approach and the fact that, for an observable pair (F, C) , there exists a gain matrix G_0 such that $\bar{\Gamma}_0 = F - G_0 C$ is Hurwitz. Therefore, given a symmetric positive-definite matrix Q , there exists a symmetric positive-definite matrix P which satisfies the following equation

$$P(F - G_0 C) + (F - G_0 C)^T P = -Q < 0 \quad (6)$$

which is equivalent to

$$P\Gamma_0 + \Gamma_0^T P + \sum_{i=1}^p (P\Gamma_i + \Gamma_i^T P) u_{i,\min} = -Q < 0 \quad (7)$$

Let $V(e)$ be a Lyapunov function candidate

$$V(e) = e^T P e \quad (8)$$

where P is a symmetric positive-definite matrix. Differentiation of $V(e)$ yields

$$\dot{V}(e) = e^T \left(P\Gamma_0 + \Gamma_0^T P \right) e + \sum_{i=1}^p e^T (P\Gamma_i + \Gamma_i^T P) e u_i \quad (9)$$

In his paper, Hua concluded, from equation (7), that relations (10) and (11) hold

$$P\Gamma_i + \Gamma_i^T P < 0 \text{ for } i=1, \dots, p \quad (10a)$$

$$\text{and } P\Gamma_0 + \Gamma_0^T P < 0 \quad (11a)$$

This is equivalent to the existence of a unique solution P which satisfies simultaneously the following set of Lyapunov equations

$$P\Gamma_i + \Gamma_i^T P = -Q_i \text{ for } i=1, \dots, p \quad (10b)$$

$$\text{and } P\Gamma_0 + \Gamma_0^T P = -Q_0 \quad (11b)$$

where Q_0 and Q_i are symmetric positive-definite matrices. With the following counter-example, we demonstrate that if relation (7) holds, relations (10a-b) and (11a-b) are not necessarily satisfied. Indeed, consider the following stable matrices Γ_0 and Γ_1

$$\Gamma_0 = \begin{bmatrix} g_{011} & g_{012} \\ g_{021} & g_{022} \end{bmatrix}, \Gamma_1 = \begin{bmatrix} g_{111} & g_{112} \\ g_{121} & g_{122} \end{bmatrix}, \text{ with } g_{011} < 0, g_{012} > 0, g_{021} < 0, g_{022} = 0, g_{111} < 0, g_{112} < 0, g_{121} > 0 \text{ and } g_{122} = 0. \text{ Assume that } u_{\min} = 1, g_{012} < -g_{112} \text{ and } g_{021} > -g_{121}, \text{ then } (\Gamma_0 + u_{\min} \Gamma_1) \text{ is stable and there exists a symmetric positive-definite } P \text{ matrix such that (7)}$$

holds

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}, \text{ with } p_1 > 0 \text{ (} p_3 > 0 \text{)} \text{ and } p_1 p_3 > p_2^2.$$

Then the constraint $\left(-\frac{g_{011}}{g_{021}} p_1 < p_2 < 0 \right)$ is necessary to obtain $P\Gamma_0 + \Gamma_0^T P < 0$ while the

constraint $\left(0 < p_2 < -\frac{g_{111}}{g_{121}} p_1 \right)$ is necessary to achieve $P\Gamma_1 + \Gamma_1^T P < 0$. Unfortunately, these

two constraints cannot be satisfied simultaneously. Hence, the implication used by Hua in the observer convergence proof does not hold.

4. Comments on the existence conditions

Consider the numerical example worked out in Hua

$$A = \begin{bmatrix} -63.704 & 6.137 & -1.142 & 0.476 & -0.298 & 0.241 & -0.113 \\ 17.728 & -12.306 & -1.749 & 1.276 & -0.952 & 0.826 & -0.398 \\ -8.328 & 4.960 & -12.297 & -2.742 & 1.821 & -1.477 & 0.696 \\ 3.761 & -4.921 & 4.613 & -13.577 & -3.348 & 2.414 & -1.092 \\ -3.132 & 1.451 & -3.749 & 5.705 & -14.161 & -4.214 & 1.809 \\ 2.172 & -1.966 & 1.404 & -3.524 & 8.113 & -15.781 & -2.678 \\ -2.242 & 1.185 & -2.038 & 2.207 & -5.173 & 17.789 & -26.755 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -56.361 & 2.908 & -0.305 & 0.073 & -0.026 & 0.012 & -0.004 \\ 18.412 & -12.995 & -2.729 & 1.524 & -1.074 & 0.914 & -0.438 \\ -7.102 & 8.706 & -8.308 & -3.281 & 1.967 & -1.558 & 0.730 \\ 4.173 & -3.776 & 6.470 & -6.943 & -3.774 & 2.548 & -1.139 \\ -2.890 & 2.378 & -2.961 & 6.201 & -6.508 & -4.747 & 1.926 \\ 2.225 & -1.783 & 2.000 & -3.048 & 7.777 & -7.205 & -3.525 \\ -2.111 & 1.641 & -1.769 & 2.452 & -4.652 & 16.176 & -17.890 \end{bmatrix},$$

$B^T = [-9.722 \quad -3.057 \quad 6.983 \quad 5.236 \quad 3.455 \quad 1.866 \quad 1.694], C = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$
and $u_{\min} = -0.3, u_{\max} = 0.25$. N_1 being stable, G_1 can be chosen as $G_1 = 0$ i.e. $\Gamma_1 = N_1$.

The pair $(A + u_{\min} \Gamma_1, C)$ being observable, we can choose a gain matrix G such that $(A + u_{\min} \Gamma_1 - GC)$ is stable. Selecting the eigenvalues of $(A + u_{\min} \Gamma_1 - GC)$ as $\{-10, -5, -2, -3 \pm i, -1 \pm i\}$, one obtains, by pole-placement techniques, the gain matrix $G = [-1296.7 \quad 386.1 \quad -218.2 \quad 135.3 \quad -101.2 \quad 82.4 \quad -98.7]^T$. Unfortunately, for this choice of gain matrix G , the observation error is not stable for all values of $u \in [-0.29; -0.12] \subset [u_{\min}; u_{\max}]$. We can conclude that even if conditions (ia-b) and (iia-b) in Hua are satisfied, i.e. matrices $\Gamma_0 - GC$ and Γ_1 are stable, the observer convergence is not necessarily guaranteed. This means that these conditions are not sufficient.

5. Convergence analysis and existence conditions

Before stating the convergence of the observer, let introduce the following notation

$$\text{for all } u_i(t) \in \Omega, u_i(t) = u_{i,\min} + \varepsilon_i(t) \quad (12)$$

where $\varepsilon_i(t) \geq 0$ for all t . The observation error can be rewritten as follows

$$\dot{e}(t) = \left(\Gamma_0 + \sum_{i=1}^p \Gamma_i u_{i,\min} \right) e(t) + \sum_{i=1}^p \Gamma_i e(t) \varepsilon_i(t) \quad (13)$$

or equivalently

$$\dot{e}(t) = \left(\bar{\Gamma}_0 + \sum_{i=1}^p \Gamma_i \varepsilon_i(t) \right) e(t) \quad (14)$$

where $\bar{\Gamma}_0$ and Γ_i are Hurwitz and defined above.

In this form, the problem is reduced to the stability analysis of linear system with time-varying structured uncertainties. A sufficient condition guaranteeing the asymptotic stability of (14) was given in Mansour (1988) as follows.

The observation error (14) is asymptotically stable if

$$(iii) \quad \mu(\bar{\Gamma}_0) < 0 \text{ and } \mu(\Gamma_i) < 0 \text{ for all } \Gamma_i, \text{ where } \mu(\cdot) \text{ denotes the matrix measure of } (\cdot)$$

(Desoer 1975).

Note that the numerical example worked out in Hua verifies this condition.

A less conservative version of this condition can be stated as follows

$$(iv) \quad \mu(\bar{\Gamma}_0) < 0 \text{ and } \mu(\bar{\Gamma}_0) + \sum_{i=1}^p \mu(\Gamma_i) \tilde{\varepsilon}_i < 0 \text{ where } \tilde{\varepsilon}_i = u_{i,\max} - u_{i,\min} \text{ if } \mu(\Gamma_i) > 0 \\ \text{otherwise } \tilde{\varepsilon}_i = 0.$$

The proof of this condition is obvious by choosing $V(e) = e^T e$ as Lyapunov function candidate. Notice that the class of matrix pairs (A, C) for which there exists a gain matrix G such that $\mu(A - GC) < 0$ neither includes nor is included by the set of observable pairs (Mori *et al.* 1983) but is only a subset of the detectable pairs. Then the observer design cannot be reduced to unconstrained pole-placements as proposed by Hua, the convergence conditions (iii) or (iv) are stronger than (ib) and (iib).

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