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# ON THE CODIMENSION-ONE FOLIATION THEOREM OF W. THURSTON

FRANÇOIS LAUDENBACH

ABSTRACT. This article has been withdrawn due to a mistake which is explained in version 2.

We consider a 3-simplex  $\sigma$  in an affine space  $E$ . Let  $x_1, x_2, x_3, x_4$  be its vertices; the edges are oriented by the ordering of the vertices. Let  $F_i$  be the 2-face opposite to  $x_i$ . We are looking at germs of codimension-one foliation along  $\sigma$  (or along a subcomplex of  $\sigma$ ) which are transversal to  $\sigma$  and to all its faces of positive dimension.

If such a foliation  $\mathcal{H}$  is given along the three 2-faces  $F_2, F_3, F_4$  through  $x_1$  and if  $\mathcal{H}$  does not trace spiralling leaves on  $F_2 \cup F_3 \cup F_4$ , then  $\mathcal{H}$  extends to  $\sigma$  transversally to  $F_1$ . If  $\mathcal{H}$  is only given along  $F_2 \cup F_4$  (resp.  $F_3 \cup F_4$ ), then  $\mathcal{H}$  extends to  $F_3$  (resp.  $F_2$ ) with no spiralling on  $F_2 \cup F_3 \cup F_4$ , and hence to  $\sigma$ .

But, on contrary of what is claimed on version 1 of this paper, it is in general not true when  $\mathcal{H}$  is given along  $F_2 \cup F_3$ . It is only true when an extra condition is fulfilled: *The separatrices of  $x_2$  in  $F_3$  and of  $x_3$  in  $F_2$  cross  $F_2 \cap F_3 = [x_1, x_4]$  respectively at points  $y_2$  and  $y_3$  which lie in the order  $y_2 < y_3$ .*

The first place where this extension argument is misused is corollary 4.5. Moreover the statement of this corollary is wrong. Let us explain why.

Let  $\sigma^{pl} \subset E$  be a so-called *pleated* 3-simplex associated to  $\sigma$  and  $\mathcal{H}$  be a germ of codimension-one foliation transversal to its simplices. We recall that  $\sigma^{pl}$  and  $\sigma$  have the same boundary and we assume that  $\mathcal{H}$  traces spiralling leaves on  $\partial\sigma$ , making the pleating necessary according to the Reeb stability theorem. Let  $x * \sigma^{pl}$  be the (abstract) cone on  $\sigma^{pl}$ . If  $\dim E$  is large enough, it embeds into  $E$ . Certainly  $\mathcal{H}$  does not extend to  $x * \sigma^{pl}$ , contradicting the statement of corollary 4.5. Indeed, if it does, then we get a foliation of  $x * \partial\sigma^{pl} = x * \partial\sigma$  transversal to all faces. Proposition 4.4 states that, if all 3-faces through  $x$  in the 4-simplex  $x * \sigma$  are foliated, then the foliation extends to the face opposite to  $x$ , which is  $\sigma$  itself. But this is impossible due to the spiralling leaves on  $\partial\sigma$ .

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