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CHARACTERIZATION OF POISSON INTEGRALS FOR NON-TUBE BOUNDED SYMMETRIC DOMAINS

ABDELHAMID BOUSSEJRA AND KHALID KOUFANY

ABSTRACT. We characterize the L^p -range, $1 < p < +\infty$, of the Poisson transform on the Shilov boundary for non-tube bounded symmetric domains. We prove that this range is a Hua-Hardy type space for harmonic functions satisfying a Hua system.

1. INTRODUCTION

Let $\Omega = G/K$ be a Riemannian symmetric space of non-compact type. To each boundary G/P one can define a Poisson transform, which is an integral operator from hyperfunctions on G/P into the space of eigenfunctions on Ω of the algebra $\mathcal{D}(\Omega)^G$ of invariant differential operators. For the maximal boundary, G/P_{\min} , the most important result is the Helgason conjecture, proved by Kashiwara *et al.* [8] which states that a function is eigenfunction of all invariant differential operators on Ω if and only if it is Poisson integral

$$\mathcal{P}_\lambda f(gK) = \int_K f(k) e^{-\langle \lambda + \rho, H(g^{-1}k) \rangle} dk.$$

of a hyperfunction on the maximal boundary, for a generic $\lambda \in \mathfrak{a}_\mathbb{C}^*$. For other function spaces such as $L^p(G/P_{\min})$ the characterization is in connection with Fatou's theorems. We mention here the work of Helgason [5] and Michelson [14] for $p = \infty$, and Sjörgen [17] for $1 \leq p < \infty$ using weak L^p -spaces. Another characterization for $1 \leq p \leq \infty$, using Hardy-type spaces, was done by Stoll [18] in the harmonic case and by Ben Saïd *et al.* [1] in the general case.

If Ω is a bounded symmetric domain, one is interested in functions whose boundary values are supported on the Shilov boundary (minimal boundary) $S := G/P_{\max}$ rather than the maximal boundary G/P_{\min} .

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For the Shilov boundary the Poisson transform is defined by

$$\mathcal{P}_s f(gK) = \int_K f(k) e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle} dk, \quad s \in \mathbb{C}.$$

In this case, Hua [6] had proved that the algebra of invariant differential operators is not necessarily the most appropriate for characterizing harmonic functions (i.e., annihilated by the algebra $\mathcal{D}(\Omega)^G$). Johnson and Korányi [7], generalizing the earlier work of Hua, Korányi and Stein [10], and Korányi and Malliavin [11], introduced an invariant second order ($\mathfrak{k}_{\mathbb{C}}$ -valued) operator \mathcal{H} , called since, second-order Hua operator (or Hua system). They showed, in the tube case, that a function is annihilated by the Hua operator if and only if it is the Poisson integral $\mathcal{P}_{s_0} f$ ($s_0 = n/r$) of a hyperfunction on the Shilov boundary. Thus, in the tube case, The Hua operator plays the same role with respect to the Shilov boundary as the algebra $\mathcal{D}(\Omega)^G$ does with respect to the maximal boundary. In his paper [13], Lassalle showed the existence of a smaller system (a projection of the Hua operator) with the same properties.

Later Shimeno [16] generalize the result of Johnson and Korányi; namely he proved that a function is eigenfunction of \mathcal{H} if and only if it is a Poisson transform $\mathcal{P}_s f$ of a hyperfunction on the Shilov boundary for generic $s \in \mathbb{C}$.

In [2], the first author gave a characterization of the Poisson transform \mathcal{P}_s on $L^p(S)$, which closes the tube type symmetric domains case characterization.

It thus arises the question of characterizing the range of the Poisson transform \mathcal{P}_s on $L^p(S)$, $1 < p < +\infty$, for non-tube bounded symmetric domains on $L^p(S)$. The purpose of this paper is to answer this question.

For general bounded symmetric domains the Poisson integrals are not eigenfunctions of the second-order Hua operator \mathcal{H} , see for instance [3] or [12]. However for type $\mathbf{I}_{r,r+b}$ domains of non-tube type, (see [3] and [12]) there is a variant of the second-order Hua operator, $\mathcal{H}^{(1)}$, by taking the first component of \mathcal{H} , since in this case $\mathfrak{k}_{\mathbb{C}}$ is a sum of two irreducible ideals $\mathfrak{k}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}}^{(1)} \oplus \mathfrak{k}_{\mathbb{C}}^{(2)}$. It is proved, in [12] (and in [3] for the harmonic case, $s = (2r + b)/r$) that a smooth function f on $\mathbf{I}_{r,r+b}$ is a solution of the Hua system, $\mathcal{H}^{(1)} f = \frac{1}{4}(s^2 - (r + b)^2) f I_r$ if and only if it is the Poisson transform \mathcal{P}_s of a hyperfunction on the Shilov boundary.

For general non-tube domains, and for the harmonic case (i.e., for $s = n/r$ in our parametrization) the characterization of the image of the Poisson transform $\mathcal{P}_{\frac{n}{r}}$ on hyperfunctions over the Shilov boundary was done by Berline and Vergne [3] where certain third-order Hua operator was introduced. Recently, the second author and Zhang [12] generalize the result of Berline and Vergne to any (generic) s . They introduce two third-order Hua operators \mathcal{U} and \mathcal{W} (different from the Berline and Vergne operator) and prove that an eigenfunction f of $\mathcal{D}(\Omega)^G$ is a solution the Hua system (6) if and only if it is a Poisson transform of a hyperfunction on the Shilov boundary.

Let $\mathcal{E}_s(\Omega)$ be the space of harmonic functions on Ω that are solutions of the Hua system (for type $\mathbf{I}_{r,r+b}$ domains, an eigenfunction of $\mathcal{H}^{(1)}$ is indeed harmonic). Then the image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of $\mathcal{E}_s(\Omega)$. For $1 < p < +\infty$, we introduce the Hua-Hardy type space, $\mathcal{E}_s^p(\Omega)$ of functions $f \in \mathcal{E}_s(\Omega)$ such that

$$\|f\|_{s,p} = \sup_{t>0} e^{-t(\Re(s)r-n)} \left(\int_K |f(ka_t)|^p dk \right)^{1/p} < +\infty.$$

Our main result (see Theorem 4.10) says that *if $s \in \mathbb{C}$ is such that $\Re(s) > \frac{a}{2}(r-1)$, a smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^p(S)$ if and only if $f \in \mathcal{E}_s^p(\Omega)$* . Our method of proving this characterization uses an L^2 version of this theorem (see Theorem 4.8) and an inversion formula for the Poisson transform (see Proposition 4.9) which needs Fatou-type theorems (see Theorem 4.3 and Theorem 4.5).

2. PRELIMINARIES

Let Ω be an irreducible bounded symmetric domain in a complex n -dimensional space V . Let G be the identity component of the group of biholomorphic automorphisms of Ω , and K be the isotropy subgroup of G at the point $0 \in \Omega$. Then K is a maximal compact subgroup of G and as a Hermitian symmetric space, $\Omega = G/K$. Let \mathfrak{g} be the Lie algebra of G , and

$$\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$$

be its Cartan decomposition. The Lie algebra \mathfrak{k} of K has one dimensional center \mathfrak{z} . Then there exists an element $Z_0 \in \mathfrak{z}$ such that $\text{ad}Z_0$ defines the complex structure of \mathfrak{p} . Let

$$\mathfrak{g}_{\mathbb{C}} = \mathfrak{p}^+ \oplus \mathfrak{k}_{\mathbb{C}} \oplus \mathfrak{p}^-$$

be the corresponding eigenspaces decomposition of $\mathfrak{g}_{\mathbb{C}}$, the complexification of \mathfrak{g} . Let $G_{\mathbb{C}}$ be a connected Lie group with Lie algebra $\mathfrak{g}_{\mathbb{C}}$ and P^+ , $K_{\mathbb{C}}$, P^- be the analytic subgroups of $G_{\mathbb{C}}$ corresponding to \mathfrak{p}^+ , $\mathfrak{k}_{\mathbb{C}}$, \mathfrak{p}^- . Denote by σ the conjugaison of $G_{\mathbb{C}}$ with respect to G . Then we have, $\sigma(P^{\pm}) \subset P^{\mp}$ and $\sigma(K_{\mathbb{C}}) \subset K_{\mathbb{C}}$.

Let \mathfrak{h} be a maximal Abelian subalgebra of \mathfrak{k} , and let $\Delta(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$ be the corresponding set of roots. As Z_0 belongs to \mathfrak{h} , the space \mathfrak{p}^+ is stable by $\text{ad}\mathfrak{h}$. The roots $\gamma \in \Delta(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$ such that $\mathfrak{g}^{\gamma} \subset \mathfrak{p}^+$ are said to be positive non-compact, and we denote by Φ the set of such roots. Let $\gamma \in \Phi$, then one may choose elements $H_{\gamma} \in i\mathfrak{h}$, $E_{\gamma} \in \mathfrak{g}^{\gamma}$, $E_{-\gamma} \in \mathfrak{g}^{-\gamma}$ such that $[E_{\gamma}, E_{-\gamma}] = H_{\gamma}$ and $\sigma(E_{\gamma}) = -E_{-\gamma}$. Let $X_{\gamma} = E_{\gamma} + E_{-\gamma}$ and $Y_{\gamma} = i(E_{\gamma} - E_{-\gamma})$. Then, by a classical Harish-Chandra construction, there exists a maximal set $\Gamma = \{\gamma_1, \dots, \gamma_r\}$ of strongly orthogonal roots in Φ . For simplicity, let us set for, $1 \leq j \leq r$,

$$E_j = E_{\gamma_j}, \quad X_j = X_{\gamma_j}, \quad Y_j = Y_{\gamma_j}.$$

Then,

$$\mathfrak{a} = \sum_{j=1}^r \mathbb{R}X_j,$$

is a Cartan subspace of the pair $(\mathfrak{g}, \mathfrak{k})$. Let \mathfrak{a}^* denote the dual of \mathfrak{a} and let $\{\beta_1, \beta_2, \dots, \beta_r\}$ be a basis of \mathfrak{a}^* determined by

$$\beta_j(X_k) = 2\delta_{j,k}, \quad 1 \leq j, k \leq r.$$

The restricted root system $\Sigma = \Sigma(\mathfrak{g}, \mathfrak{a})$ of \mathfrak{g} relative to \mathfrak{a} is (of type C_r or BC_r) given by

$$\pm\beta_j \quad (1 \leq j \leq r) \quad \text{each with multiplicity } 1,$$

$$\pm\frac{1}{2}(\beta_j \pm \beta_k) \quad (1 \leq j \neq k \leq r) \quad \text{each with multiplicity } a,$$

and possibly

$$\pm\frac{1}{2}\beta_j \quad (1 \leq j \leq r) \quad \text{each with multiplicity } 2b.$$

Let $\Sigma^+ = \{\beta_j, \frac{1}{2}\beta_j, \frac{1}{2}(\beta_{\ell} \pm \beta_k); \quad 1 \leq j \leq r, \quad 1 \leq \ell \neq k \leq r\}$ the set of positive restricted roots. Then the set $\Lambda = \{\alpha_1, \dots, \alpha_{r-1}, \alpha_r\}$ of simple roots in Σ^+ is such that

$$\alpha_j = \frac{1}{2}(\beta_{r-j+1} - \beta_{r-j}), \quad 1 \leq j \leq r-1$$

and

$$\alpha_r = \begin{cases} \beta_1 & \text{for tube case} \\ \frac{1}{2}\beta_1 & \text{for non-tube case.} \end{cases}$$

Let $\Lambda_1 = \{\alpha_1, \dots, \alpha_{r-1}\}$ and write $\Sigma_1 = \Sigma \cap \mathbb{Z} \cdot \Lambda_1$. Define

$$\mathfrak{m}_{1,1} = \mathfrak{m} + \mathfrak{a} + \sum_{\gamma \in \Sigma_1} \mathfrak{g}^\gamma, \quad \mathfrak{n}_1^+ = \sum_{\gamma \in \Sigma^+ \setminus \Sigma_1} \mathfrak{g}^\gamma.$$

Let

$$\mathfrak{a}_1 = \{H \in \mathfrak{a} : \gamma(H) = 0 \forall \gamma \in \Lambda_1\},$$

then $\mathfrak{m}_{1,1}$ is the centralizer of \mathfrak{a}_1 in \mathfrak{g} and $\mathfrak{p}_1 = \mathfrak{m}_{1,1} + \mathfrak{n}_1^+$ is a standard parabolic subalgebra of \mathfrak{g} with Langlands decomposition $\mathfrak{m}_1 + \mathfrak{a}_1 + \mathfrak{n}_1^+$, where \mathfrak{m}_1 is the orthocomplement of \mathfrak{a}_1 in $\mathfrak{m}_{1,1}$ with respect to the Killing form. Note that $\theta(\mathfrak{n}_1^+) = \sum_{\gamma \in \Sigma^+ \setminus \Sigma_1} \mathfrak{g}^{-\gamma}$. Let P_1 be the corresponding parabolic subgroup and $P_1 = M_1 A_1 N_1^+$ its Langlands decomposition. Obviously, P_1 is a maximal parabolic subgroup of G , thus the Shilov boundary S can be viewed as $S = G/P_1 = K/K_1$, where $K_1 = M_1 \cap K$

If we define the element $X_0 = \sum_{j=1}^r X_j$, Then $\mathfrak{a}_1 = \mathbb{R}X_0$. Let

$$\mathfrak{a}(1) = \sum_{j=1}^{r-1} \mathbb{R}(X_j - X_{j+1})$$

be the orthocomplement of \mathfrak{a}_1 in \mathfrak{a} with respect to the Killing form,

$$(1) \quad \mathfrak{a} = \mathfrak{a}_1 \oplus^\perp \mathfrak{a}(1) = \mathbb{R}X_0 \oplus^\perp \sum_{j=1}^{r-1} \mathbb{R}(X_j - X_{j+1}).$$

We denote ρ_0 the linear form on \mathfrak{a}_1 such that, $\rho_0(X_0) = r$. We extend ρ_0 to \mathfrak{a} via the orthogonal projection (1). If ρ_1 is the restriction of ρ to \mathfrak{a}_1 , then it is clear that

$$\rho_1(X_0) = rb + r + a \frac{r(r-1)}{2} = n.$$

Again, we extend ρ_1 to \mathfrak{a} via the orthogonal projection (1). Then

$$\rho_1 = (b + 1 + a \frac{(r-1)}{2}) \rho_0 = \frac{n}{r} \rho_0.$$

For $g \in G$, define $H(g) \in \mathfrak{a}$ as the unique element such that

$$g \in K \exp(H(g))N \subset KAN = G.$$

We also denote by $\kappa(g) \in K$ and $H_1(g) \in \mathfrak{a}_1$ the unique elements such that

$$g \in \kappa(g)M_1 \exp(H_1(g))N_1 \subset KM_1A_1N_1 = G.$$

The following lemma will be useful for the sequel.

Lemma 2.1 ([15, Lemma 6.1.6]). (i) Let $x, y \in G$, $\bar{n} \in \bar{N}_1$ and $a \in A_1$. Then

$$(2) \quad H_1(x\kappa(y)) = H_1(xy) - H_1(y)$$

$$(3) \quad H_1(\bar{n}a^{-1}) = H_1(\bar{n}) - H_1(a)$$

(ii) Let $t > 0$ and $\bar{n} \in \bar{N}_1$. Then

$$(4) \quad \rho_0(H_1(a_t\bar{n}a_{-t})) \leq \rho_0(H_1(\bar{n})).$$

3. THE POISSON TRANSFORM AND THE HUA OPERATORS

For any real analytic manifold X , we denote by $\mathcal{B}(X)$ the space of all hyperfunctions on X . We will view a function on the Shilov boundary $S = G/P_1$ as a P_1 -invariant function on G . For $s \in \mathbb{C}$, we denote by $\mathcal{B}(G/P_1; s)$ the space of hyperfunctions f on G satisfying

$$f(gman) = e^{(s\rho_0 - \rho_1)\log a} f(g), \quad \forall g \in G, m \in M_1, a \in A_1, n \in N_1^+,$$

The Poisson transform of a function $f \in \mathcal{B}(G/P_1; s)$, is defined by

$$\mathcal{P}_s f(gK) = \int_K e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle} f(k) dk.$$

Since $G = KP_1$, the restriction from G to K defines a G -isomorphism from $\mathcal{B}(G/P_1, s)$ onto the space $\mathcal{B}(K/K_1)$ of all hyperfunctions f on K such that $f(kh) = f(k)$ for all $h \in K_1$.

We review the construction of Hua operators of the second order (see [7]) and the third order (see [3], [12]).

Let $\{v_j\}$ be a basis of \mathfrak{p}^+ and $\{v_j^*\}$ be the dual basis of \mathfrak{p}^- with respect to the Killing form. Let $\mathcal{U}(\mathfrak{g}_{\mathbb{C}})$ denote the universal enveloping algebra of $\mathfrak{g}_{\mathbb{C}}$. The second-order Hua operator, is the element of $\mathcal{U}(\mathfrak{g}_{\mathbb{C}}) \otimes \mathfrak{k}_{\mathbb{C}}$ defined by

$$\mathcal{H} = \sum_{i,j} v_i v_j^* \otimes [v_j, v_i^*]$$

It is known that the Hua operator does not depend on basis, therefore, for computations one can choose the root vectors basis $\{E_j\}_{j=1}^r$.

For tube domains the Hua operator \mathcal{H} maps the Poisson kernels

$$P_s(gk) = e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}) \rangle}$$

into the center of $\mathfrak{k}_{\mathbb{C}}$, namely the Poisson kernels are its eigenfunctions up to an element in the center, but it is not true for non-tube domains, see [12, Theorem 5.3]. However for non-tube type **I** domains, $\mathbf{I}_{r,r+b} \simeq SU(r, r+b)/S(U(r) \times U(r+b))$, the situation is not quite different

from the tube case. In fact, $\mathfrak{k}_{\mathbb{C}}$ is a sum of two irreducible ideals, $\mathfrak{k}_{\mathbb{C}} = \mathfrak{k}_{\mathbb{C}}^{(1)} + \mathfrak{k}_{\mathbb{C}}^{(2)}$ where

$$\begin{aligned}\mathfrak{k}_{\mathbb{C}}^{(1)} &= \left\{ \begin{pmatrix} A & 0 \\ 0 & \frac{\text{tr}(A)}{r+b} I_{r+b} \end{pmatrix}, A \in \mathfrak{gl}(r+b, \mathbb{C}) \right\}, \\ \mathfrak{k}_{\mathbb{C}}^{(2)} &= \left\{ \begin{pmatrix} 0 & 0 \\ 0 & D \end{pmatrix}, D \in \mathfrak{sl}(r+b, \mathbb{C}) \right\}.\end{aligned}$$

There is a variant of the Hua operator, $\mathcal{H}^{(1)}$ see [3], [12], by taking the projection of \mathcal{H} onto $\mathfrak{k}_{\mathbb{C}}^{(1)}$. In [12], the second author and Zhang showed that the operator $\mathcal{H}^{(1)}$ has the Poisson kernels as its eigenfunctions and they found the eigenvalues. They proved further that the eigenfunctions of the Hua operator $\mathcal{H}^{(1)}$ are harmonic functions (i.e., eigenfunctions of all invariant differential operators on Ω), and gave the following characterization of the range of the Poisson transform for $\mathbf{I}_{r,r+b}$.

Theorem 3.1 ([12, Theorem 6.1]). *Suppose $s \in \mathbb{C}$ satisfies the following condition*

$$-4[b+1+j+\frac{1}{2}(s-r-b)] \notin \{1, 2, 3, \dots\}, \text{ for } j = 0 \text{ and } 1.$$

Then the Poisson transform \mathcal{P}_s is a G -isomorphism of $\mathcal{B}(S)$ onto the space of smooth functions f on Ω that satisfy

$$(5) \quad \mathcal{H}^{(1)} f = \frac{1}{4}(s^2 - (r+b)^2) f I_r.$$

For the characterization of range of the Poisson transform for general non-tube domains the second author and Zhang [12] introduced new third-order Hua operators \mathcal{U} and \mathcal{W} :

$$\mathcal{U} = \sum_{i,j,k} v_i^* v_j^* v_k \otimes [v_i, [v_j, v_k^*]],$$

$$\mathcal{W} = \sum_{i,j,k} v_k v_i^* v_j \otimes [[v_k^*, v_i], v_j],$$

Similarly to \mathcal{H} , the operators \mathcal{U} and \mathcal{W} do not depend on the basis.

Denote

$$c = 2(n+1) + \frac{1}{n}(a^2 - 4) \dim(\mathcal{P}^{(1,1)}),$$

where $\mathcal{P}^{(1,1)}$ is the dimension of the irreducible subspaces of holomorphic polynomials on \mathfrak{p}^+ with lowest weight $-\gamma_1 - \gamma_2$. For any $s \in \mathbb{C}$, put $\sigma = \frac{1}{2}(s + \frac{n}{r})$. For general non-tube domains we have the following

Theorem 3.2 ([12, Theorem 7.2]). *Let Ω be a bounded symmetric non-tube domain of rank r in \mathbb{C}^n . Suppose $s \in \mathbb{C}$ satisfies*

$$-4[b + 1 + j\frac{a}{2} + \frac{1}{2}(s - \frac{r}{n})] \notin \{1, 2, 3, \dots\}, \text{ for } j = 0 \text{ and } 1.$$

Then the Poisson transform \mathcal{P}_s is a G -isomorphism of $\mathcal{B}(S)$ onto the space of harmonic functions f on Ω that satisfy

$$(6) \quad \left(\mathcal{U} - \frac{-2\sigma^2 + 2p\sigma + c}{\sigma(2\sigma - p - b)} \mathcal{W} \right) f = 0,$$

4. THE L^p -RANGE OF THE POISSON TRANSFORM

For $1 < p < +\infty$, we will consider the space $L^p(S) = L^p(K/K_1)$ as the space of all complex valued measurable (classes) functions f on K that are K_1 -invariant and satisfying

$$\|f\|_p = \left(\int_K |f(k)|^p dk \right)^{1/p} < +\infty,$$

where dk is the Haar measure of K . Let $d\bar{n}$ be the invariant measure on $\bar{N}_1 = \theta(N_1)$ with the normalization

$$(7) \quad \int_{\bar{N}_1} e^{\langle -2\rho_1, H_1(\bar{n}) \rangle} d\bar{n} = 1.$$

Then for a continuous function f on S we have

$$(8) \quad \int_K f(k) dk = \int_{\bar{N}_1} f(\kappa(\bar{n})) e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} d\bar{n}.$$

The space $L^p(S)$ can be viewed as a subspace of $\mathcal{B}(S)$, thus its image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of $\mathcal{E}_s(\Omega)$. We will now, for specific s , characterize this image. For this we need some information on the integral \mathbf{c}_s in following proposition.

Proposition 4.1. *For $s \in \mathbb{C}$ such that $\Re(s) > \frac{\alpha}{2}(r - 1)$, the integral*

$$\mathbf{c}_s = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle} d\bar{n}$$

converges absolutely to a constant $\mathbf{c}_s \neq 0$.

Proof. For $s \in \mathbb{C}$ let $\lambda_s \in \mathfrak{a}_{\mathbb{C}}^*$ be the linear form defined by

$$\lambda_s(H) = (s\rho_0 - \rho_1)(H_1) + \rho(H), \quad H \in \mathfrak{a}$$

where H_1 is the projection of H onto \mathfrak{a}_1 . Then the condition $\Re(s) > \frac{\alpha}{2}(r - 1)$ is equivalent to

$$(9) \quad \Re(\langle \lambda_s, \alpha \rangle) > 0 \quad \forall \alpha \in \Sigma^+ \setminus \Sigma_1.$$

Moreover, we can choose (see for example [15, Lemma 6.1.4]) ω in the Weyl group W of Σ such that

$$\begin{aligned} (i) \quad & \omega \cdot H = H, \quad \forall H \in \mathfrak{a}_1, \\ (ii) \quad & \omega(\Sigma^+ \cap \Sigma_1) = -\Sigma^+ \cap \Sigma_1, \\ (iii) \quad & \omega(\Sigma^+ \setminus \Sigma_1) = \Sigma^+ \setminus \Sigma_1. \end{aligned}$$

Since $\langle \omega \lambda_s, \alpha \rangle = \langle \lambda_s, \omega^{-1} \alpha \rangle$, the condition (9) is equivalent to

$$\Re(\langle \omega \lambda_s, \alpha \rangle) > 0, \quad \forall \alpha \in \Sigma^+$$

Furthermore

$$\langle s\rho_0 + \rho_1, H_1(g) \rangle = \langle \omega \lambda_s + \rho, H(g) \rangle$$

so that

$$\int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle} d\bar{n} = \int_{\bar{N}_1} e^{-\langle \omega \lambda_s + \rho, H(\bar{n}) \rangle} d\bar{n}$$

and the right hand side is the Harish-Chandra c function, $c(\omega \lambda_s)$ associated with the maximal parabolic subgroup, which converges absolutely, see [15]. □

Let $s \in \mathbb{C}$. Let $\mathcal{E}_s(\Omega)$ be the space of harmonic functions on Ω that satisfy (5) in type I domains or (6) in general domains. It is clear that the image $\mathcal{P}_s(L^p(S))$ is a proper closed subspace of the eigenspace $\mathcal{E}_s(\Omega)$. Hence, it is natural to look for a characterization of those $F \in \mathcal{E}_s(\Omega)$ that are Poisson transform of some $f \in L^p(S)$.

For any $1 < p < \infty$, let $\mathcal{E}_s^p(\Omega)$ denote the Hua-Hardy type space of functions $f \in \mathcal{E}_s(\Omega)$ such that

$$\|f\|_{s,p} = \sup_{a \in A_1} e^{-\langle \Re(s)\rho_0 - \rho_1, \log a \rangle} \left(\int_K |f(ka)|^p dk \right)^{1/p} < +\infty.$$

Since $\mathfrak{a}_1 = \mathbb{R}X_0$, the above integral becomes

$$\|f\|_{s,p} = \sup_{t > 0} e^{-t\langle \Re(s)r - n \rangle} \left(\int_K |f(ka_t)|^p dk \right)^{1/p},$$

where $a_t = \exp(tX_0)$.

4.1. Fatou type theorems. As a preparation to Fatou-type theorems we prove the following

Proposition 4.2. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Let Ψ_t be the function defined on \bar{N}_1 by*

$$\Psi_t(\bar{n}) = e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle + \langle s\rho_0 - \rho_1, H_1(a_t \bar{n} a_{-t}) \rangle}.$$

Then there exists a non-negative function $\Phi \in L^1(\bar{N}_1)$ such that $\Psi_t \leq \Phi$ for each t .

Proof. It follows from (4), that for any $t > 0$ and for any $\bar{n} \in \bar{N}_1$,

$$0 \leq \rho_0(H_1(a_t \bar{n} a_{-t})) \leq \rho_0(H_1(\bar{n})).$$

Therefore,

$$|\Psi_t(\bar{n})| \leq \begin{cases} e^{-\langle \Re(s)\rho_0 + \rho_1, H_1(\bar{n}) \rangle} & \text{if } \frac{a}{2}(r-1) < \Re(s) \leq \frac{a}{2}(r-1) + b + 1 \\ e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} & \text{if } \Re(s) > \frac{a}{2}(r-1) + b + 1 \end{cases}$$

and the second hand is an integrable function on \bar{N}_1 by (7) and Proposition 4.1 . □

Let $\mathcal{C}(S)$ be the space of complex-valued continuous functions on S with the topology of uniform convergence.

Theorem 4.3. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then*

$$f(k) = \mathbf{c}_s^{-1} \lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t)$$

uniformly, for $f \in \mathcal{C}(S)$.

Proof. Let $f \in \mathcal{C}(S)$, then

$$\mathcal{P}_s f(ka_t) = \int_K e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}h) \rangle} f(kh) dh.$$

We transform this integral using the formula (8) to an integral over \bar{N}_1 ,

$$\mathcal{P}_s f(ka_t) = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}\kappa(\bar{n})) \rangle} f(k\kappa(\bar{n})) e^{-2\langle \rho_1, H_1(\bar{n}) \rangle} d\bar{n},$$

and by (2) we get

$$\mathcal{P}_s f(ka_t) = \int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}\bar{n}) \rangle} e^{\langle s\rho_0 - \rho_1, H_1(\bar{n}) \rangle} f(k\kappa(\bar{n})) d\bar{n}.$$

which by the substitution $\bar{n} \mapsto a_{-t}\bar{n}a_t$ and (3), becomes

$$\begin{aligned} \mathcal{P}_s f(ka_t) &= e^{\langle s\rho_0 - \rho_1, H_1(a_t) \rangle} \times \\ &\int_{\bar{N}_1} e^{-\langle s\rho_0 + \rho_1, H_1(\bar{n}) \rangle + \langle s\rho_0 - \rho_1, H_1(a_t \bar{n} a_{-t}) \rangle} f(k\kappa(a_t \bar{n} a_{-t})) d\bar{n}. \end{aligned}$$

But $\rho_1 = \frac{n}{r}\rho_0$, and $a_t \bar{n} a_{-t} \rightarrow e$ when $t \rightarrow +\infty$, thus, by Proposition 4.2,

$$\lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t) = \mathbf{c}_s f(k).$$

□

Let

$$\varphi_s(a_t) := \int_K e^{-(s\rho_0 + \rho_1, H(a_{-t}k))} dk.$$

then, it follows from the above theorem that

$$(10) \quad \lim_{t \rightarrow \infty} e^{-(rs-n)t} \varphi_s(a_t) = \mathbf{c}_s, \quad \text{if } \Re(s) > \frac{a}{2}(r-1).$$

As a consequence we can prove the following

Proposition 4.4. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then there exists a positive constant γ_s such that, for $1 < p < \infty$ and $f \in L^p(S)$, we have*

$$\left(\int_K |\mathcal{P}_s f(ka_t)|^p dk \right)^{1/p} \leq \gamma_s e^{(rs-n)t} \|f\|_p.$$

Proof. For $t > 0$, we define the function p_s^t by

$$p_s^t(k) = e^{-(s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}))}, \quad k \in K.$$

Then the Poisson transform can be written as the convolution

$$\mathcal{P}_s f(ka_t) = (f * p_s^t)(k).$$

Hence, to prove the proposition we use the Hausdorff-Young inequality,

$$\left(\int_K |\mathcal{P}_s f(ka_t)|^p dk \right)^{1/p} \leq \|p_s^t\|_1 \|f\|_p \quad (p > 1),$$

and (10). □

Let, as usual, \hat{K} , be the set of equivalence classes of finite dimensional irreducible representations of K . For $\delta \in \hat{K}$, let $\mathcal{C}(S)_\delta$ be the linear span of all K -finite vectors on S of type δ . It is well known that the space $\mathcal{C}^K(S) := \bigoplus_{\delta \in \hat{K}} \mathcal{C}(S)_\delta$ is dense in $\mathcal{C}(S)$. Recall also, that the space $\mathcal{C}(S)$ is dense in $L^p(S)$ for $1 < p < \infty$.

Theorem 4.5. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Then*

$$f(k) = \mathbf{c}_s^{-1} \lim_{t \rightarrow +\infty} e^{-(rs-n)t} \mathcal{P}_s f(ka_t)$$

in $L^p(S)$, for $1 < p < \infty$.

Proof. Let $f \in L^p(S)$. By the above density arguments, for any $\epsilon > 0$, there exists $\varphi \in \mathcal{C}^K(S)$ such that $\|f - \varphi\|_p < \epsilon$. Then we have

$$\begin{aligned} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t f - f\|_p &\leq \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t (f - \varphi)\|_p + \\ &+ \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t \varphi - \varphi\|_p + \|\varphi - f\|_p \end{aligned}$$

where the function $P_s^t f$ is defined by

$$(11) \quad P_s^t f(k) = \mathcal{P}_s f(ka_t).$$

By Proposition 4.4,

$$\|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t (f - \varphi)\|_p \leq \gamma_s |\mathbf{c}_s^{-1}| \|f - \varphi\|_p,$$

and by Theorem 4.3

$$\lim_{t \rightarrow +\infty} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t \varphi - \varphi\|_p = 0.$$

Thus, $\lim_{t \rightarrow +\infty} \|\mathbf{c}_s^{-1} e^{-(rs-n)t} P_s^t f - f\|_p \leq \epsilon(\gamma_s + 1)$ and this proves the theorem. \square

We can now prove the following estimates

Proposition 4.6. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{\alpha}{2}(r-1)$. Then there exists a positive constant γ_s such that for $1 < p < +\infty$ and $f \in L^p(S)$,*

$$(12) \quad |\mathbf{c}_s| \|f\|_p \leq \|\mathcal{P}_s f\|_{s,p} \leq \gamma_s \|f\|_p.$$

Proof. In fact, the right hand side of the estimate (12) follows from Proposition 4.4. On the other hand, by Theorem 4.5 we have

$$\lim_{t \rightarrow \infty} e^{(n-rs)t} \mathcal{P}_s f(ka_t) = \mathbf{c}_s f(k)$$

in $L^p(S)$. Hence, there exists a sequence $(t_j)_j$, with $t_j \rightarrow +\infty$ when $j \rightarrow +\infty$ such that $\lim_{j \rightarrow +\infty} e^{(n-rs)t_j} \mathcal{P}_s f(ka_{t_j}) = \mathbf{c}_s f(k)$, almost every where in K . Therefore, by the classical Fatou lemma,

$$|\mathbf{c}_s| \|f\|_p \leq \sup_j e^{(n-r\Re(s))t_j} \|P_s^{t_j} f\|_p$$

and this is how we prove the left hand side of (12). \square

4.2. The L^2 -Poisson transform range. Recall that

$$L^2(S) = \bigoplus_{\delta \in \hat{K}} V_\delta$$

where V_δ is the finite linear span of $\{\varphi_\delta \circ k, k \in K\}$, where φ_δ is the zonal spherical function corresponding to δ .

For $s \in \mathbb{C}$ and $\delta \in \hat{K}$, define the *generalized spherical function* $\Phi_{s,\delta}$ on A_1 by

$$\Phi_{s,\delta}(a_t) = (\mathcal{P}_s \varphi_\delta)(a_t).$$

Proposition 4.7. *Let $s \in \mathbb{C}$, $\delta \in \hat{K}$ and $f \in V_\delta$. Then for any $k \in K$ and $a_t \in A_1$,*

$$(\mathcal{P}_s f)(ka_t) = \Phi_{s,\delta}(a_t) f(k).$$

Proof. Since M_1 centralize A_1 , we can view the operator (11) as a bounded operator on $L^2(S)$. Moreover, P_s^t commutes with the left regular representation of K in $L^2(S)$. Hence, by Schur's lemma, $P_s^t = \Phi_{s,\delta}(a_t) \cdot I$ on each V_δ and the proposition follows. \square

The first main theorem of this section can now be stated as follows :

Theorem 4.8. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. A smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^2(S)$ if and only if $F \in \mathcal{E}_s^2(\Omega)$.*

Proof. The necessary condition follows from Proposition 4.6 and [12, Theorem 6.1 and Theorem 7.2]. On the other hand, let $F \in \mathcal{E}_s^2(\Omega)$. We apply again [12, Theorem 6.1 and Theorem 7.2]. Then, there exists a hyperfunction $f \in \mathcal{B}(S)$ such that $F = \mathcal{P}_s f$. Let $f = \sum_{\delta \in \hat{K}} f_\delta$ be its K -type decomposition. By Proposition 4.7 we can write

$$F(ka_t) = \sum_{\delta \in \hat{K}} \Phi_{s,\delta}(a_t) f_\delta(k)$$

in $\mathcal{C}^\infty(K \times [0, +\infty[)$.

Now observe that

$$\|F\|_{s,2}^2 = \sup_{t>0} e^{2(n-r\Re(s))t} \sum_{\delta \in \hat{K}} |\Phi_{s,\delta}(a_t)|^2 \|f_\delta\|_2^2 < \infty.$$

Then, if Λ is an arbitrary finite subset of \hat{K} , we get

$$e^{2(n-r\Re(s))t} \sum_{\delta \in \Lambda} |\Phi_{s,\delta}(a_t)|^2 \|f_\delta\|_2^2 \leq \|F\|_{s,2}^2$$

for every $t > 0$ and hence from Theorem 4.3 it follows immediately that

$$|\mathbf{c}_s|^2 \sum_{\delta \in \Lambda} \|f_\delta\|_2^2 \leq \|F\|_{s,2}^2$$

which implies that $f = \sum_{\delta \in \hat{K}} f_\delta$ in $L^2(S)$ and that

$$|\mathbf{c}_s|^2 \|f\|_2 \leq \|F\|_{s,2}.$$

This ends the proof of the theorem. \square

In the following proposition we show how to recover a function $f \in L^2(S)$ from its Poisson transform $\mathcal{P}_s f$.

Proposition 4.9. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{a}{2}(r-1)$. Let $F \in \mathcal{E}_s^2(\Omega)$ and $f \in L^2(S)$ its boundary value. Then the following inversion formula*

$$(13) \quad f(k) = |\mathbf{c}_s|^{-2} \lim_{t \rightarrow \infty} e^{2(n-r\Re(s))t} \int_K \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h) \rangle}} F(ha_t) dh$$

holds in $L^2(S)$.

Proof. Let $F \in \mathcal{E}_s^2(\Omega)$, then it follows from Theorem 4.8 that there exists a unique $f \in L^2(S)$ such that $F = \mathcal{P}_s f$. Let $f = \sum_{\delta \in \hat{K}} f_\delta$ be its K -type expansion, then similarly to the preceding proof, we get

$$(14) \quad F(ka_t) = \sum_{\delta \in \hat{K}} \Phi_{s,\delta}(a_t) f_\delta(k).$$

For any $t > 0$, define the complex-valued function on K by

$$g_t(ka_t) = |\mathbf{c}_s|^{-2} e^{2(n-rs)t} \int_K \overline{e^{-\langle s\rho_0 + \rho_1, H_1(a_t k^{-1}h) \rangle}} F(ha_t) dh.$$

Next, using the above series expansion we can write according to Theorem 4.5,

$$g_t(h) = |\mathbf{c}_s|^{-2} e^{2(n-rs)t} \sum_{\delta \in \hat{K}} |\Phi_{s,\delta}(a_t)|^2 f_\delta(h).$$

Thus,

$$\|g_t - f\|_2^2 = \sum_{\delta \in \hat{K}} \left| |\mathbf{c}_s|^{-2} e^{2(n-rs)t} |\Phi_{s,\delta}(a_t)|^2 - 1 \right|^2 \|f_\delta\|_2^2,$$

which shows that $\|g_t - f\|_2 \rightarrow 0$, since $\lim_{t \rightarrow \infty} e^{(n-rs)t} \Phi_{s,\delta}(a_t) = \mathbf{c}_s$. \square

4.3. The L^p -Poisson transform range, $p \neq 2$. We shall now prove the second main result of this paper, more precisely, we shall characterize the L^p -range of the Poisson transform. We will need the following notation. For each function f on Ω , define the function f^t , $t > 0$, on K by

$$f^t(k) = f(ka_t).$$

Theorem 4.10. *Let $s \in \mathbb{C}$ be such that $\Re(s) > \frac{\alpha}{2}(r-1)$. A smooth function F on Ω is the Poisson transform $F = \mathcal{P}_s f$ of a function $f \in L^p(S)$ if and only if $F \in \mathcal{E}_s^p(\Omega)$.*

Proof. We will follow the technique used by Korányi [9]. Let $(\chi_n)_n$ be an approximation of the identity in $\mathcal{C}(K)$. That is $\chi_n \geq 0$, $\int_K \chi_n(k) dk = 1$ and $\lim_{n \rightarrow +\infty} \int_{K \setminus U} \chi_n(k) dk = 0$ for every neighborhood U of e in K . Let $F \in \mathcal{E}_s^p(\Omega)$. For each n , define the function F_n on Ω by

$$F_n(gK) = \int_K \chi_n(k) F(k^{-1}g) dk.$$

Then $(F_n)_n$ converges point-wise to F , and since the set $\mathcal{E}_s(\Omega)$ of harmonic functions satisfying the Hua system is G -invariant, $F_n \in \mathcal{E}_s(\Omega)$, for each n . Furthermore,

$$F_n^t(ka_tK) = (\chi_n * F^t)(k)$$

and this shows

$$(15) \quad \|F_n^t\|_2 \leq \|\chi_n\|_2 \|F^t\|_p,$$

and

$$(16) \quad \|F_n^t\|_p \leq \|F^t\|_p.$$

It follows from (15)

$$\sup_{t>0} e^{(n-rs)t} \left(\int_K |F_n(ka_t)|^2 dk \right)^{1/2} \leq \|\chi_n\|_2 \|F\|_{s,p}.$$

Thus $F_n \in \mathcal{E}_{s,2}(\Omega)$ and by Theorem 4.8, there exists $f_n \in L^2(S)$ such that $F_n = \mathcal{P}_s f_n$. Now, our goal is to prove that f_n belongs to $L^p(S)$. Using the inversion formula (13) we can write in $L^2(S)$,

$$f_n(k) = \lim_{t \rightarrow +\infty} g_n^t(k)$$

where

$$g_n^t(h) = g_n(ha_t) = |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \int_K \overline{e^{-(s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h))}} F_n(ka_t) dk.$$

Let $\varphi \in \mathcal{C}(S)$ be a continuous function on S , then

$$\int_K f_n(h) \varphi(h) dh = \lim_{t \rightarrow \infty} \int_K g_n^t(h) \varphi(h) dh.$$

Moreover,

$$\begin{aligned} \int_K g_n^t(h) \varphi(h) dh &= |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \times \\ &\times \int_K \int_K F_n(ka_t) \varphi(h) \overline{e^{-(s\rho_0 + \rho_1, H_1(a_{-t}k^{-1}h))}} dk dh \\ &= |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \int_K \overline{\mathcal{P}_s \varphi(ka_t)} F_n(ka_t) dk. \end{aligned}$$

By the Holder inequality, if q is such that $1/p + 1/q = 1$, we get

$$\begin{aligned} \left| \int_K g_n^t(h) \varphi(h) dh \right| &\leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F_n^t\|_p, \\ &\leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F^t\|_p, \end{aligned}$$

where the second inequality follows from (16). But $F \in \mathcal{E}_{s,p}(\omega)$, then

$$\left| \int_K g_n^t(h) \varphi(h) dh \right| \leq |\mathbf{c}_s|^{-2} e^{2(n-r\Re(s))t} \|\mathcal{P}_s \varphi\|_q \|F\|_{s,p}.$$

Therefore, by Theorem 4.3,

$$\left| \int_K f_n(h)\varphi(h)dh \right| \leq |\mathbf{c}_s|^{-1} \|\varphi\|_q \|F\|_{s,p},$$

and by taking the supremum over $\varphi \in \mathcal{C}(S)$ with $\|\varphi\|_q = 1$, we get

$$\|f_n\|_p \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}.$$

Now, for each $\varphi \in L^q(S)$, define the functional

$$T_n(\varphi) = \int f_n(h)\varphi(h)dk.$$

Then it is obvious by (4.3) that

$$|(T_n)\varphi| \leq |\mathbf{c}_s|^{-1} \|\varphi\|_q \|F\|_{s,p}$$

hence, T_n is uniformly bounded operator in $L^q(S)$ with $\sup_n \|T_n\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}$. Thanks to Banach-Alaouglu-Bourbaki's theorem, there exists a subsequence of bounded operators $(T_{n_j})_j$ which converges as $n_j \rightarrow +\infty$ to a bounded operator T in $L^q(S)$, under the $*$ -weak topology, with $\|T\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}$. Then, by the Riesz representation theorem, there exists a unique function $f \in L^p(S)$ such that

$$T(\varphi) = \int_K f(h)\varphi(h)dh, \quad \forall \varphi \in L^q(S)$$

with

$$(17) \quad \|f\|_p \leq \|T_n\| \leq |\mathbf{c}_s|^{-1} \|F\|_{s,p}.$$

Now, observe that

$$F_{n_j}(g) = T_{n_j}(e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle}),$$

thus, by taking the limit as $n \rightarrow +\infty$ we get

$$F(g) = T(e^{-\langle s\rho_0 + \rho_1, H_1(g^{-1}k) \rangle}) = \mathcal{P}_s f(g)$$

with $|\mathbf{c}_s| \|f\|_p \leq \|F\|_{s,p}$, by (17), and this finishes the proof of the theorem. \square

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