Influence of Jc(B) on the full penetration current of superconducting tube

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Abstract

It is well known that the critical current density J_c of a superconducting material depends on the magnetic field B. If magnetic independent J_c is chosen for analytical calculation of current distribution, the critical current I_c corresponds to full penetration current I_p. I_c is a measured current with 1μV/cm criterion and I_p is a calculated current. The aim of this paper is to calculate the influence of the $J_c(B)$ variation on I_p of a superconducting tube. To calculate I_p, which is depending on the material itself, a linear function $J_c(B)$ is sufficient to obtain realistic values by analytic way. We need to have a linear $J_c(B)$ law that is close to the measured $J_c(B)$ characteristics presented in this paper. The linear $J_c(B)$ law chosen was used for the calculation of the distribution of both magnetic field B(r, t) and the current density J(r, t). These distributions allow the analytical calculation of I_p. The calculated results of magnetic field distribution and full penetration current with Bean model and linear model are compared. We also present the variation of critical current with the characteristic parameters of the material. The present results, allow to understand the relationship between the full penetration current variation of a sample and the variation of the $J_c(B)$ characteristics.

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Keywords: HTc superconductor, magnetic field dependence, critical current

1. Introduction

To relate the irreversible magnetization to the field and current profiles within the interior of superconducting sample, C.P. Bean [3] introduced the critical state concept and assumed that the critical current density at any point of the sample can only take one constant unique value, the critical current density J_c, among three different states J_c, $-J_c$ and zero. Thanks to this model, the calculation of the current distribution is possible in case of a superconducting cylinder [1] or tube [2] fed with transport current. With this model, when fed current i(t) increases into a superconducting sample, the current density penetrates up to the critical current I_c. For I_c current density is equals to J_c everywhere in superconducting material case and the critical current corresponds to the full penetration current I_p. This is the complete penetration state and the critical current corresponds to the full penetration current I_p. So the relation between J_c and I_c is very simple, $I_c = J_c.S$, with S the section of the sample. I_c is used to calculate the AC losses [8]. Experimentally I_c is defined though the critical electric field E_C. For high temperature superconductors this value is generally fixed to 1µV/cm. Unfortunately, for superconducting material, the real critical current density varies with magnetic flux density B [4][5]. In this case I_c doesn't correspond to I_p. I_c remains a measured current with the 1µV/cm criterion. I_p is the value of the applied current i(t) at which the current density arrives to the centre of the sample. It's a calculated current. The aim of this paper is to calculate the influence of $J_c(B)$ variation on the full penetration current of a superconducting tube.

Considering $J_c(B)$, for complete penetration, the current density is not constant in the superconducting material because the magnetic flux density is not constant. As consequence, the relation between I_p and $J_c(B)$ is not easy to calculate.

For the calculation of I_p, the first step is to set $J_c(B)$ law that must be closed to the measured $J_c(B)$ characteristics. In a second step, the analytical calculation of the current density, the electric field and the magnetic flux density distributions are presented. Finally the formula of I_p that takes into account the variation of $J_c(B)$ is derived.

2. Analytical calculation of the flux density distribution in a superconducting tube

2.1 Studied sample

The aim of this paper is to calculate the full penetration current I_p of a tube based on the $J_C(B)$ variation. To simplify the calculations, without reducing the generality of the problem, the tube applied current i(t) increases from 0 to I_{max} according to (Fig. 1), which is sufficient to determine the critical current. The internal radius R_{in}, the external radius R_e. and the length h of the tube are defined in Fig. 2.

The current i(t) circulates along [Oz]. The edge effects are neglected in our calculations; E, J and B don't depend on z.

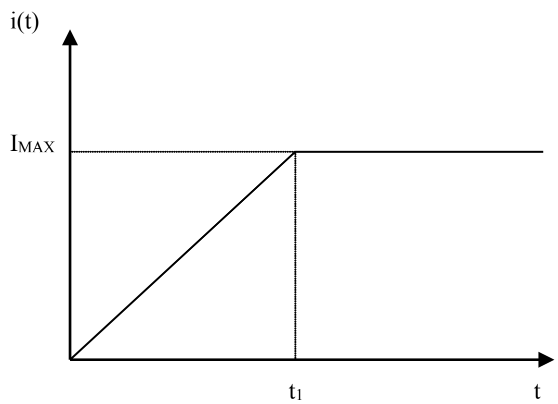

Fig.1: Current supply.

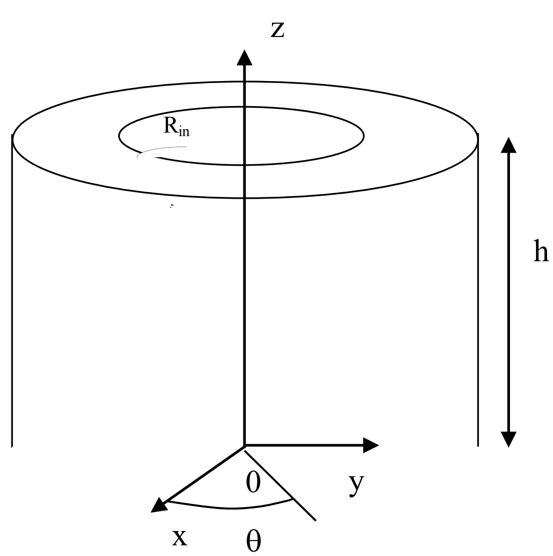

Fig. 2 : Studied superconductor tube.

2.2 Modeling of the problem

The electromagnetic behavior of superconductor is governed by the Maxwell equations:

$$\vec{\mathrm{rot}}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad , \qquad (1)$$

$$\vec{\mathrm{rot}}\,\vec{B} = \mu_0.\vec{J} \ , \qquad (2)$$

For the superconducting material, we consider:

$$B = \mu_0.H \ . \qquad (3)$$

E(J) is given by the critical state model that defines the relation between electric field E and current density J :

$$J = \pm J_c \text{ and } E > 0 \qquad (4)$$

or

$$E = 0 \text{ and } -J_c < J < +J_c$$

When the current rises, the current density penetrates from the external radius R_e toward the internal radius R_{in} and its direction is [Oz].

Because of symmetries, the current density being oriented along the [Oz] axis, because of symmetries, the flux density B(r, t) has only one component:

$$\vec{B}(r,t) = B(r,t).\vec{u}_\theta$$

First we are going to prove that an analytical expression can be found for b(r, t) with a linear model for $J_c(B)$. Considering the relation (2) and (4):

$$\frac{1}{r}\frac{\partial\left(r.B(r,t)\right)}{\partial r} = \mu_0.J_c(B) = \mu_0 J_{c0}\left(1 - \frac{B(r,t)}{B_{j0}}\right) \qquad (5)$$

$|B| = B$ because $B > 0$

2.3 Flux density distribution B(r, t)

The analytical solution of expression (5) is:

$$B(r,t) = B_{j0} - \frac{B_{j0}^2}{\mu_0.J_{c0}.r} + K\frac{\exp\left(-\frac{\mu_0 J_{c0}.r}{B_{j0}}\right)}{r} \qquad (6)$$

K is a constant which can be calculated thanks to the boundary condition at $r = R_e$. The Ampere law enables to write:

$$B(r = R_e, t) = b_e(t) = \frac{\mu_0.i(t)}{2\pi R_e} \qquad (7)$$

Considering (6) and (7), we can deduce:

$$B(r,t) = \frac{B_{j0}(\mu_0.J_{c0}.r - B_{j0}) + \exp\left(\frac{\mu_0.J_{c0}.(R_e - r)}{B_{j0}}\right)\left(B_{j0}^2 + \mu_0.J_{c0}.R_e\left(b_e(t) - B_{j0}\right)\right)}{\mu_0.J_{c0}.r} \qquad (8)$$

This expression is valid for $B(r, t) > 0$. So we have to determine the point where $B(r,t) = 0$.

To do that, one can remark that it is equivalent to solve $B(r = d(t), t) = 0$ with $d(t) = R_e - pd(t)$, where pd(t) is the penetration depth. Using the relation (8) and mathematical software, we obtain:

$$\frac{B_{j0}(\mu_0.J_{c0}.d(t) - B_{j0}) + \exp\left(\frac{\mu_0.J_{c0}.(R_e - d(t))}{B_{j0}}\right)\left(B_{j0}^2 + \mu_0.J_{c0}.R_e\left(b_e(t) - B_{j0}\right)\right)}{\mu_0.J_{c0}.d(t)} = 0$$

$$d(t) = \frac{B_{j0}}{\mu_0.J_{c0}}\left(1 + W(X_1)\right) \qquad (9)$$

$X_1(t) = -\frac{\left(B_{j0}^2 + \mu_0 J_{c0} R_e\left(b_e(t) - B_{j0}\right)\right)}{B_{j0}^2}\exp\left(\frac{\mu_0 J_{c0} R_e}{B_{j0}} - 1\right)$, where $X_1 > 0$. $W(X_1)$ is Lambert's W-function and is well-known in many mathematical libraries.

3. Experimental $J_c(B)$ of the sample

The critical current density J_c varies with magnetic flux density B. Several variation laws $J_c(B)$ are presented in Table 1. The experimental tests are made with a cylindrical current lead of BiSCCO. The dimensions of this sample are: R_{in} = 3.8 mm, R_e = 5 mm, tube section S = 33mm^2 and h = 11.7 cm. Without external magnetic field, with a 1µV/cm criterion, the measured critical current I_{mc0} = 96 A. To obtain the experimental curve $J_c(B)$ of this tube, it was fed with direct current I and plunged in an external magnetic flux density B_{ext} parallel to the axis [Oz] (Fig. 3). The sample voltage drop U versus I is measured for different external magnetic flux densities B_{ext} (Fig. 4).

Table 1 : Jc(B) laws, J_c J_{K0}, B_K, J_{c0}, and B_{j0} are constants and depend on the superconducting material.

$J_c(B) = J_{cB}$ (10)	Bean Model [3]
$J_c(B) = \dfrac{J_{K0}}{1 + \dfrac{\lvert B \rvert}{B_K}}$ (11)	Kim Model [7]
$J_c(B) = \dfrac{J_{c0}\left(B_{j0} - \lvert B \rvert\right)}{B_{j0}}$ (12)	Linear Model [5]

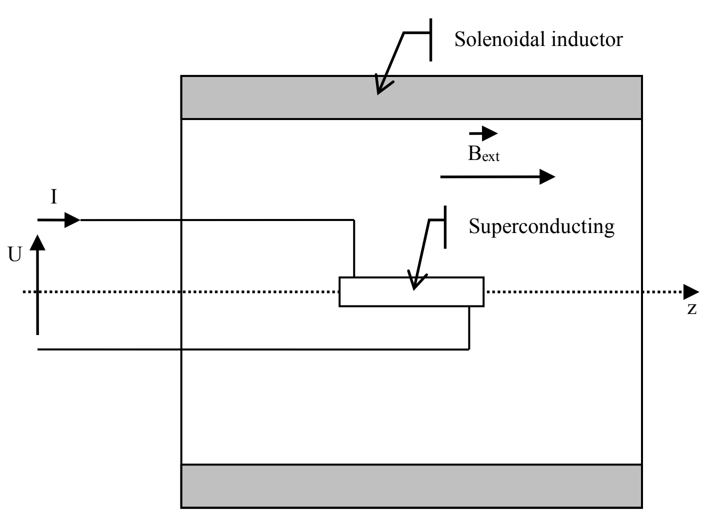

Fig. 3: $J_c(B)$ measurement device.

From the measured critical current $I_{mc}(B_{ext})$ corresponding to voltage drop equal to 1µV/cm*h (Fig. 4) we can deduce the $J_{mc}(B_{ext}) = \frac{I_{mc}(B_{ext})}{S}$. Fig. 5 shows the curve related to this function.

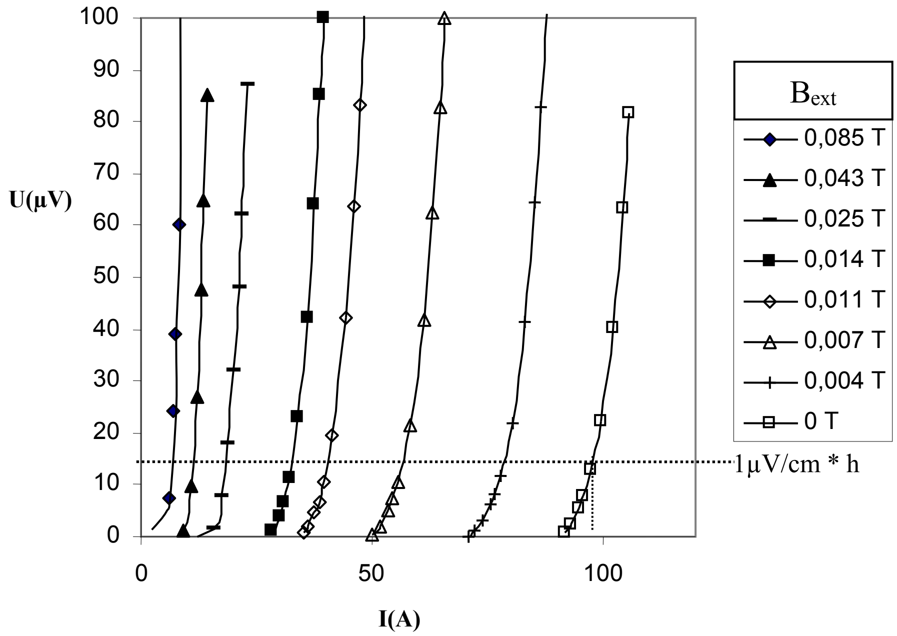

Fig. 4 : Measure of sample voltage drop versus direct current for different external magnetic flux density.

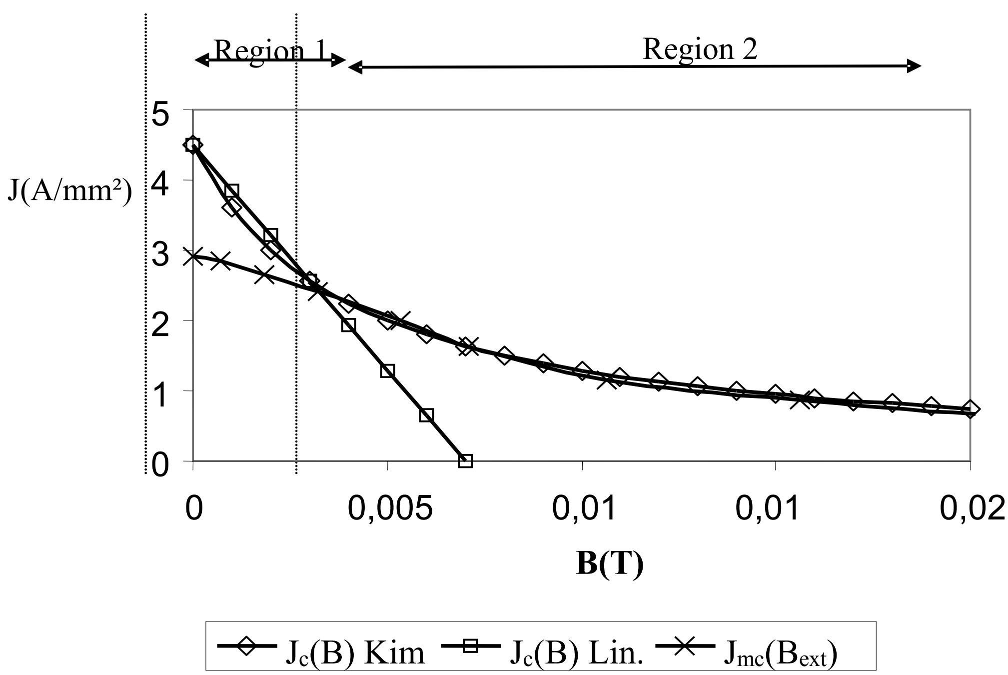

Fig. 5 : Measured $J_c(B_{ext})$, $J_c(B)$ with Kim model and linear model.

$J_{mc}(B_{ext})$ and $J_c(B)$ are different because :

- $J_c(B)$ is a locally law and $J_{MC}(B_{ext})$ is a macroscopic law
- B is not equal to B_{ext} because of self magnetic field B_{SF} : $\vec{B} = \vec{B}_{SF} + \vec{B}_{ext}$

When the value of B_{ext} is sufficiently large, B_{SF} becomes negligible and the values of J_{mc} are very close to $J_C(B)$. Among the laws represented in Table 1, Kim model is the most suitable to extrapolate $J_{mc}(B)$. With the appropriate values of J_{K0} and B_K, we propose the following expression of the Kim law:

$$J_c(B) = \frac{4.6.10^6}{1 + \frac{|B|}{0.004}} \qquad (13)$$

As shown in Fig. 5, values of $J_c(B)$ provided by the Kim model are close to measurements for high values of B. An important discussion has to be set before going on. We have to find I_C in the region where the magnetic field is low, so we have to eliminate the self field effect. To do that we allow that the Kim law continues to be true near B = 0. To develop an analytical study in the previous region, we approximate the function by a linear one, with $J_{c0} = 4.6A/mm^2$ and $B_{j0} = 7mT$.

$$J_c(B) = \frac{4.6.10^6(0.007 - |B|)}{0.007} \qquad (14)$$

Fig. 5 represents the different results of the previous approaches. The following part deals with the study of the flux density in the region 1.

4. Calculation of the magnetic flux density and current density penetration

In this part we present the calculations of B(r, t) and J(r, t) distributions. Results provided by both the Bean model and the linear model are compared. For B(r,t) distribution with linear model, relation (8) is used:

$$B(r,t) = \frac{B_{j0}(\mu_0.J_{c0}.r - B_{j0}) + \exp\left(\frac{\mu_0.J_{c0}.(R_e - r)}{B_{J0}}\right)\left(B_{j0}^2 + \mu_0.J_{c0}.R_e(b_e(t) - B_{j0})\right)}{\mu_0.J_{c0}.r}$$

with $J_c(B) = \frac{4.6.10^6(0.007 - B)}{0.007}$, J_{c0} = 4.6 A/mm², B_{j0} = 7 mT.

From now, the Bean model full penetration current is named I_{pB} and the linear model one remains named I_p.

For a B(r,t) distribution with the Bean model [9], on which case $J_c = J_{cB}$ and $I_{pB} = I_c = J_{cB}.S$, there is :

$$B(r,t) = \frac{\mu_0 . J_{cB}}{2} . \left[r - \frac{R_e^2}{r} \left(1 - \left(1 - \frac{R_e^2}{R_{in}^2} \right) \frac{i(t)}{I_{pB}} \right) \right]$$

To compare the linear model and the Bean model for the B(r, t) distributions, the critical current density J_{cB} has to be equal to J_{c0} and so $I_{pB} = I_c = J_{c0} .S = 156$ A.

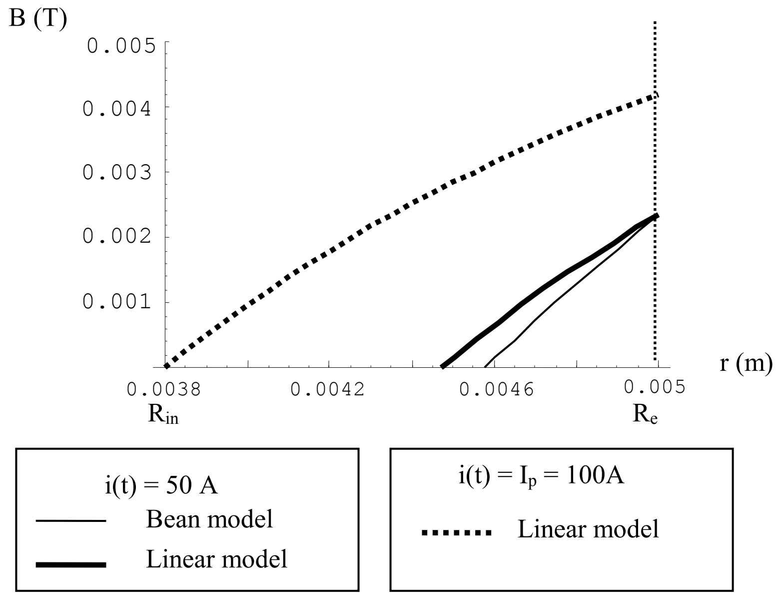

Fig. 6 : B(r) distributions with Bean model and linear model.

Fig. 6 represents B(r) for two values of current. For i(t) = 50A the penetration is incomplete for both, the Bean model and the linear model. For i(t) = 100A with the linear model there is complete penetration, so i(t) = I_p = 100A. There is complete penetration with the Bean model for i(t) = $I_p = I_c$ = 156A. It follows that the magnetic flux density penetrates faster into material with the linear model than with the Bean model. From B(r,t), the maximum penetration depth (R_e - r_S) and $B_{max}(r)$ are deduced, where B(r,t) for i(t) = I_{max} :

$$r_S = \frac{B_{j0}}{\mu_0 . J_{c0}} \left(1 + W\left(X_1(t_1) \right) \right)$$

$$r_S < r < R_e : B_{max}(r) = \frac{B_{j0}(\mu_0.J_{c0}.r - B_{j0}) + (B_{j0}^2 + \mu_0.J_{c0}.R_e(B_{emax} - B_{j0}))\exp\left(\frac{\mu_0.J_{c0}.(R_e - r)}{B_{j0}}\right)}{\mu_0.J_{c0}.r}, \qquad (15)$$

where $B_{emax} = B_{max}(r = R_e)$.

From $J_c(B)$ and $B(r,t)$, $J(r,t)$ can be deduced:

$$J(r,t) = J_c(B(r,t)) = \frac{4.6.10^6(0.007 - B(r,t))}{0.007}$$

We represent J(r) for the two models at the same instant for i(t) = 50A (Fig. 7). We understand that J(r) is not the same for the models because for linear model J(r) is weaker than for Bean model except where B(r)=0. It follows that the current penetrates deeper in the case of linear model.

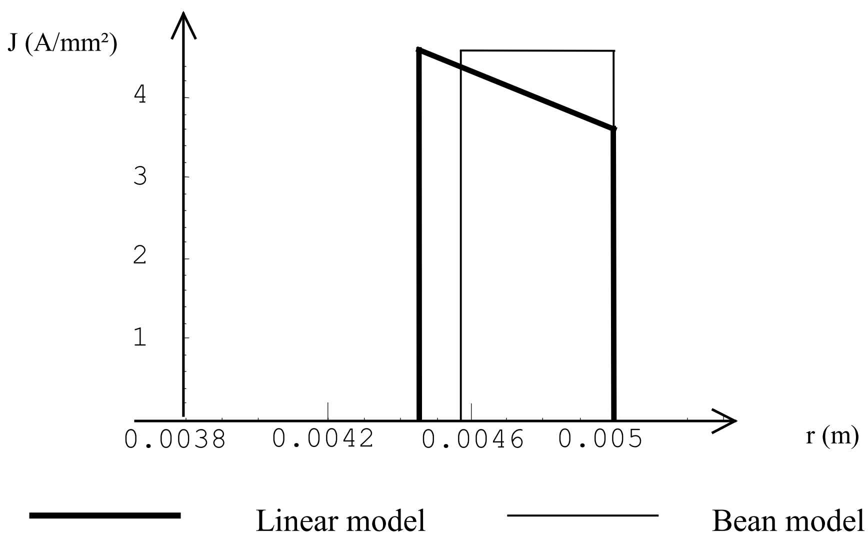

Fig. 7 : Distribution of J(r) with the Bean model and the linear model for i(t) = 50A.

5. Influence of flux density on the full penetration current I_p

5.1 Analytical calculation of I_p

This part shows the most important difference between the linear model and the Bean model. With the Bean model, the current corresponding to complete penetration of current density is I_{pB} with $I_{pB} = I_c = J_{cB}.S$.

With the linear model, the current density penetrates faster into the tube, as presented above. The full penetration current I_p can be calculated with the linear model with:

$$B(r = R_{in}, i(t) = I_p) = 0 =$$

$$\frac{B_{j0}\left(\mu_0.J_{c0}.R_{in} - B_{j0}\right) + \exp\left(\frac{\mu_0.J_{c0}.(R_e - R_{in})}{B_{j0}}\right)\left(B_{j0}^2 + \mu_0.J_{c0}.R_e\left(\frac{\mu_0.I_p}{2\pi R_e} - B_{c0}\right)\right)}{\mu_0.J_{c0}.R_{in}}$$

So :

$$I_p = \frac{2\pi.B_{j0}}{\mu_0^2.J_{c0}}.\left(\mu_0.J_{c0}.R_e - B_{j0} + \left(B_{j0} - \mu_0.J_{c0}.R_{in}\right)\exp\left(\frac{\mu_0.J_{c0}\left(R_{in} - R_e\right)}{B_{j0}}\right)\right), \quad (16)$$

I_p is smaller than I_{c0} because the current penetrates deeper for the linear model than for the Bean model (for the same current).

5.2 I_p variation with B_{j0}

Fig. 8 represents the ratio I_p / I_{c0} according to B_{j0} for different internal radii for the same section. It shows that I_p is closer to I_{c0} for large internal radii because the magnetic flux density is weaker and the influence of $J_c(B)$ is weaker. It also shows that I_p is close to I_{c0} for large value of B_{j0}. On the other hand, I_p is much smaller than I_{c0} for small values of B_{j0}. For this small values of B_{j0}, the self field creates by the tube (around 5mT) is close to B_{j0}. Now, as shown in part 3, one has to use only the linear model for B much smaller than B_{J0}. So there is no sense to use this I_p formula (16) for very small values of B_{J0}, where $B > B_{J0}$.

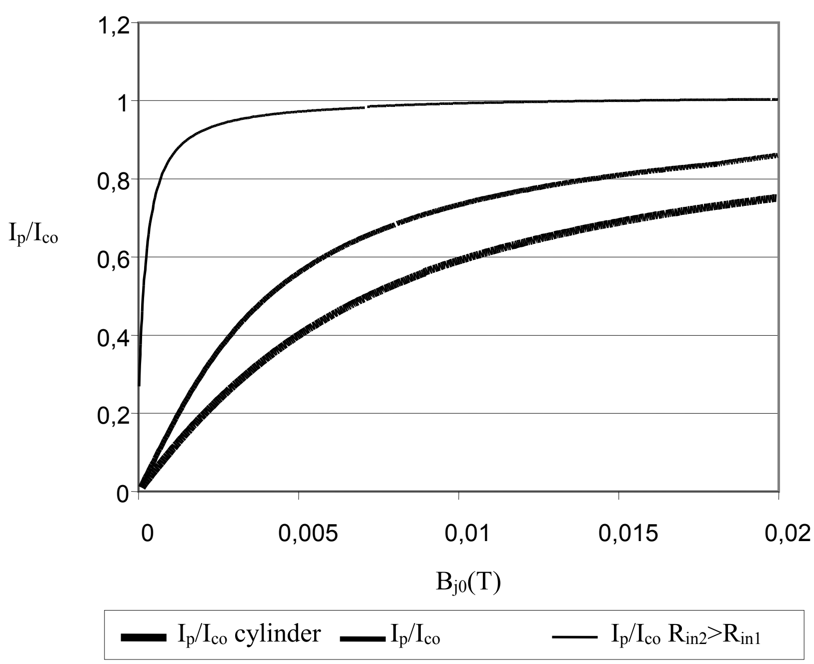

Fig. 8 : Ratio I_p/I_{c0} versus B_{j0}.

6. Conclusion

In this article the influence of $J_c(B)$ on the current, electric field and magnetic field distributions in a superconductor tube fed by a current i(t), was studied. These distributions were calculated using the linear model.

The influence of $J_c(B)$ is important for distributions of B(r,t), J(r,t) and E(r,t). It was found that the current penetrates deeper in the case of the linear model than for the Bean model. An analytical calculation of the influence of $J_c(B)$ on the full penetration current I_p is given. For the Bean model, $I_{pB} = I_c = J_{cB}.S$, while for the linear model $I_p < J_{c0}.S$. It follows that with the linear model there is full current penetration for smaller value of i(t) than for the Bean model. Thanks to these results, one is able to understand the relationship of the I_p variation with the variation of the $J_c(B)$ characteristic.

7. References

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