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## **Influence of $J_c(B)$ on the full penetration current of superconducting tube**

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### **Abstract**

It is well known that the critical current density  $J_c$  of a superconducting material depends on the magnetic field  $B$ . If magnetic independent  $J_c$  is chosen for analytical calculation of current distribution, the critical current  $I_c$  corresponds to full penetration current  $I_p$ .  $I_c$  is a measured current with  $1\mu\text{V}/\text{cm}$  criterion and  $I_p$  is a calculated current. The aim of this paper is to calculate the influence of the  $J_c(B)$  variation on  $I_p$  of a superconducting tube. To calculate  $I_p$ , which is depending on the material itself, a linear function  $J_c(B)$  is sufficient to obtain realistic values by analytic way. We need to have a linear  $J_c(B)$  law that is close to the measured  $J_c(B)$  characteristics presented in this paper. The linear  $J_c(B)$  law chosen was used for the calculation of the distribution of both magnetic field  $B(r, t)$  and the current density  $J(r, t)$ . These distributions allow the analytical calculation of  $I_p$ . The calculated results of magnetic field distribution and full penetration current with Bean model and linear model are compared. We also present the variation of critical current with the characteristic parameters of the material. The present results, allow to understand the relationship between the full penetration current variation of a sample and the variation of the  $J_c(B)$  characteristics.

**PACS codes: 74.60.Jg or 74.25.Ha**

**Keywords: HTc superconductor, magnetic field dependence, critical current**

## 1. Introduction

To relate the irreversible magnetization to the field and current profiles within the interior of superconducting sample, C.P. Bean [3] introduced the critical state concept and assumed that the critical current density at any point of the sample can only take one constant unique value, the critical current density  $J_c$ , among three different states  $J_c$ ,  $-J_c$  and zero. Thanks to this model, the calculation of the current distribution is possible in case of a superconducting cylinder [1] or tube [2] fed with transport current. With this model, when fed current  $i(t)$  increases into a superconducting sample, the current density penetrates up to the critical current  $I_c$ . For  $I_c$  current density is equals to  $J_c$  everywhere in superconducting material case and the critical current corresponds to the full penetration current  $I_p$ . This is the complete penetration state and the critical current corresponds to the full penetration current  $I_p$ . So the relation between  $J_c$  and  $I_c$  is very simple,  $I_c = J_c \cdot S$ , with  $S$  the section of the sample.  $I_c$  is used to calculate the AC losses [8]. Experimentally  $I_c$  is defined though the critical electric field  $E_c$ . For high temperature superconductors this value is generally fixed to  $1\mu\text{V}/\text{cm}$ . Unfortunately, for superconducting material, the real critical current density varies with magnetic flux density  $B$  [4][5]. In this case  $I_c$  doesn't correspond to  $I_p$ .  $I_c$  remains a measured current with the  $1\mu\text{V}/\text{cm}$  criterion.  $I_p$  is the value of the applied current  $i(t)$  at which the current density arrives to the centre of the sample. It's a calculated current. The aim of this paper is to calculate the influence of  $J_c(B)$  variation on the full penetration current of a superconducting tube.

Considering  $J_c(B)$ , for complete penetration, the current density is not constant in the superconducting material because the magnetic flux density is not constant. As consequence, the relation between  $I_p$  and  $J_c(B)$  is not easy to calculate.

For the calculation of  $I_p$ , the first step is to set  $J_c(B)$  law that must be closed to the measured  $J_c(B)$  characteristics. In a second step, the analytical calculation of the current density, the electric field and the magnetic flux density distributions are presented. Finally the formula of  $I_p$  that takes into account the variation of  $J_c(B)$  is derived.

## 2. Analytical calculation of the flux density distribution in a superconducting tube

### 2.1 Studied sample

The aim of this paper is to calculate the full penetration current  $I_p$  of a tube based on the  $J_C(B)$  variation. To simplify the calculations, without reducing the generality of the problem, the tube applied current  $i(t)$  increases from 0 to  $I_{\max}$  according to (Fig. 1), which is sufficient to determine the critical current. The internal radius  $R_{in}$ , the external radius  $R_e$ . and the length  $h$  of the tube are defined in Fig. 2.

The current  $i(t)$  circulates along  $[Oz]$ . The edge effects are neglected in our calculations;  $E$ ,  $J$  and  $B$  don't depend on  $z$ .

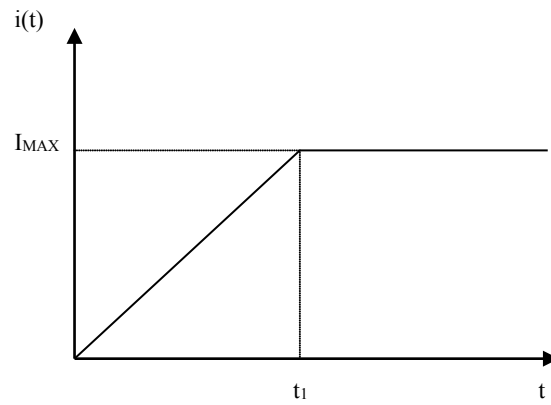


Fig.1: Current supply.

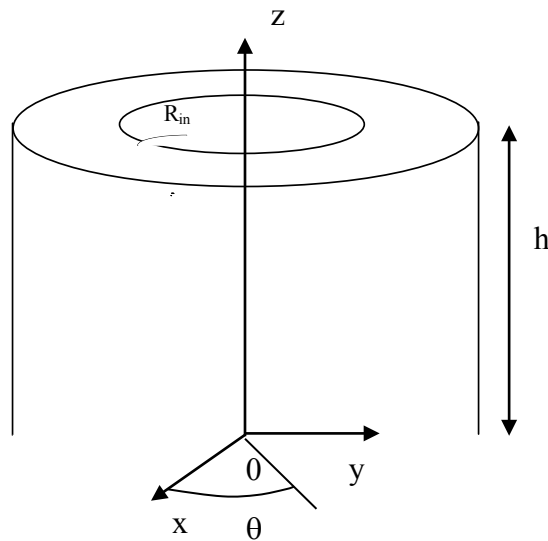


Fig. 2 : Studied superconductor tube.

## 2.2 Modeling of the problem

The electromagnetic behavior of superconductor is governed by the Maxwell equations:

$$\vec{\text{rot}} \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (1)$$

$$\vec{\text{rot}} \vec{B} = \mu_0 \cdot \vec{J}, \quad (2)$$

For the superconducting material, we consider:

$$B = \mu_0 \cdot H. \quad (3)$$

$E(J)$  is given by the critical state model that defines the relation between electric field  $E$  and current density  $J$ :

$$J = \pm J_c \text{ and } E > 0 \quad (4)$$

or

$$E = 0 \text{ and } -J_c < J < +J_c$$

When the current rises, the current density penetrates from the external radius  $R_e$  toward the internal radius  $R_{in}$  and its direction is  $[Oz]$ .

Because of symmetries, the current density being oriented along the  $[Oz]$  axis, because of symmetries, the flux density  $B(r, t)$  has only one component:

$$\vec{B}(r, t) = B(r, t) \cdot \vec{u}_0$$

First we are going to prove that an analytical expression can be found for  $b(r, t)$  with a linear model for  $J_c(B)$ . Considering the relation (2) and (4):

$$\frac{1}{r} \frac{\partial(r \cdot B(r, t))}{\partial r} = \mu_0 \cdot J_c(B) = \mu_0 J_{c0} \left( 1 - \frac{B(r, t)}{B_{j0}} \right) \quad (5)$$

$$|B| = B \text{ because } B > 0$$

### 2.3 Flux density distribution $B(r, t)$

The analytical solution of expression (5) is:

$$B(r, t) = B_{j0} - \frac{B_{j0}^2}{\mu_0 \cdot J_{c0} \cdot r} + K \frac{\exp\left(-\frac{\mu_0 J_{c0} \cdot r}{B_{j0}}\right)}{r} \quad (6)$$

$K$  is a constant which can be calculated thanks to the boundary condition at  $r = R_e$ . The Ampere law enables to write:

$$B(r = R_e, t) = b_e(t) = \frac{\mu_0 \cdot i(t)}{2\pi R_e} \quad (7)$$

Considering (6) and (7), we can deduce:

$$B(r, t) = \frac{B_{j0}(\mu_0 \cdot J_{c0} \cdot r - B_{j0}) + \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_e - r)}{B_{j0}}\right)(B_{j0}^2 + \mu_0 \cdot J_{c0} \cdot R_e(b_e(t) - B_{j0}))}{\mu_0 \cdot J_{c0} \cdot r} \quad (8)$$

This expression is valid for  $B(r, t) > 0$ . So we have to determine the point where  $B(r, t) = 0$ .

To do that, one can remark that it is equivalent to solve  $B(r = d(t), t) = 0$  with  $d(t) = R_e - pd(t)$ , where  $pd(t)$  is the penetration depth. Using the relation (8) and mathematical software, we obtain:

$$\frac{B_{j0}(\mu_0 \cdot J_{c0} \cdot d(t) - B_{j0}) + \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_e - d(t))}{B_{j0}}\right)(B_{j0}^2 + \mu_0 \cdot J_{c0} \cdot R_e(b_e(t) - B_{j0}))}{\mu_0 \cdot J_{c0} \cdot d(t)} = 0$$

$$d(t) = \frac{B_{j0}}{\mu_0 \cdot J_{c0}} (1 + W(X_1)) \quad (9)$$

$$X_1(t) = -\frac{(B_{j0}^2 + \mu_0 J_{c0} R_e (b_e(t) - B_{j0}))}{B_{j0}^2} \exp\left(\frac{\mu_0 J_{c0} R_e}{B_{j0}} - 1\right), \text{ where } X_1 > 0. W(X_1) \text{ is Lambert's } W\text{-function and}$$

is well-known in many mathematical libraries.

### 3. Experimental $J_c(B)$ of the sample

The critical current density  $J_c$  varies with magnetic flux density  $B$ . Several variation laws  $J_c(B)$  are presented in Table 1. The experimental tests are made with a cylindrical current lead of BiSCCO. The dimensions of this sample are:  $R_{in} = 3.8$  mm,  $R_e = 5$  mm, tube section  $S = 33\text{mm}^2$  and  $h = 11.7$  cm. Without external magnetic field, with a  $1\mu\text{V}/\text{cm}$  criterion, the measured critical current  $I_{mc0} = 96$  A. To obtain the experimental curve  $J_c(B)$  of this tube, it was fed with direct current  $I$  and plunged in an external magnetic flux density  $B_{ext}$  parallel to the axis  $[Oz]$  (Fig. 3). The sample voltage drop  $U$  versus  $I$  is measured for different external magnetic flux densities  $B_{ext}$  (Fig. 4).

Table 1 :  $J_c(B)$  laws,  $J_c$ ,  $J_{K0}$ ,  $B_K$ ,  $J_{c0}$ , and  $B_{j0}$  are constants and depend on the superconducting material.

$J_c(B) = J_{cB}$ (10)	Bean Model [3]
$J_c(B) = \frac{J_{K0}}{1 + \frac{ B }{B_K}}$ (11)	Kim Model [7]
$J_c(B) = \frac{J_{c0}(B_{j0} -  B )}{B_{j0}}$ (12)	Linear Model [5]

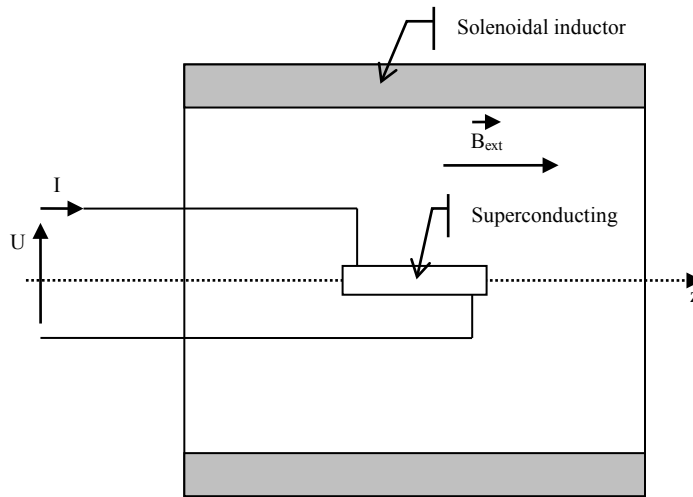


Fig. 3:  $J_c(B)$  measurement device.

From the measured critical current  $I_{mc}(B_{ext})$  corresponding to voltage drop equal to  $1\mu V/cm \cdot h$  (Fig. 4) we

can deduce the  $J_{mc}(B_{ext}) = \frac{I_{mc}(B_{ext})}{S}$ . Fig. 5 shows the curve related to this function.

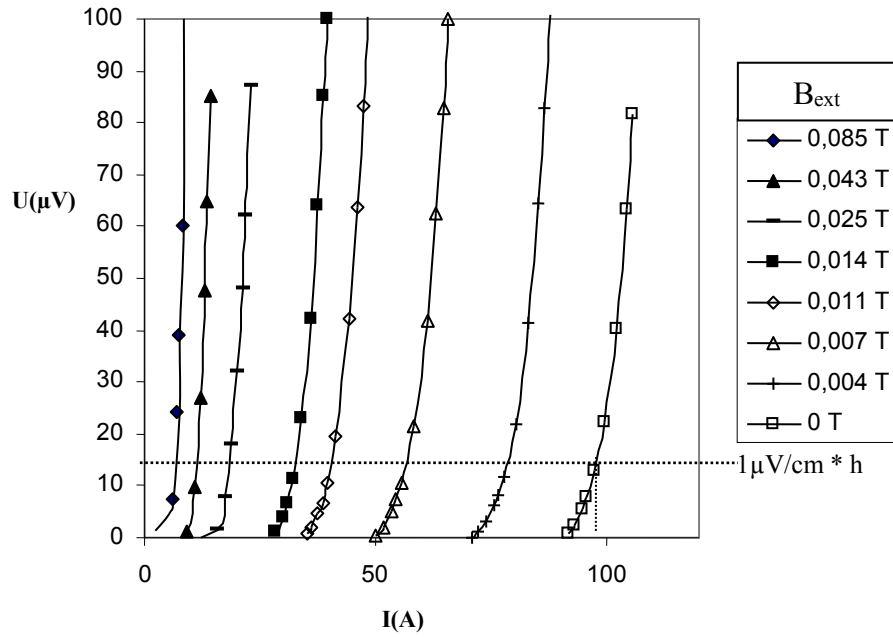


Fig. 4 : Measure of sample voltage drop versus direct current for different external magnetic flux density.

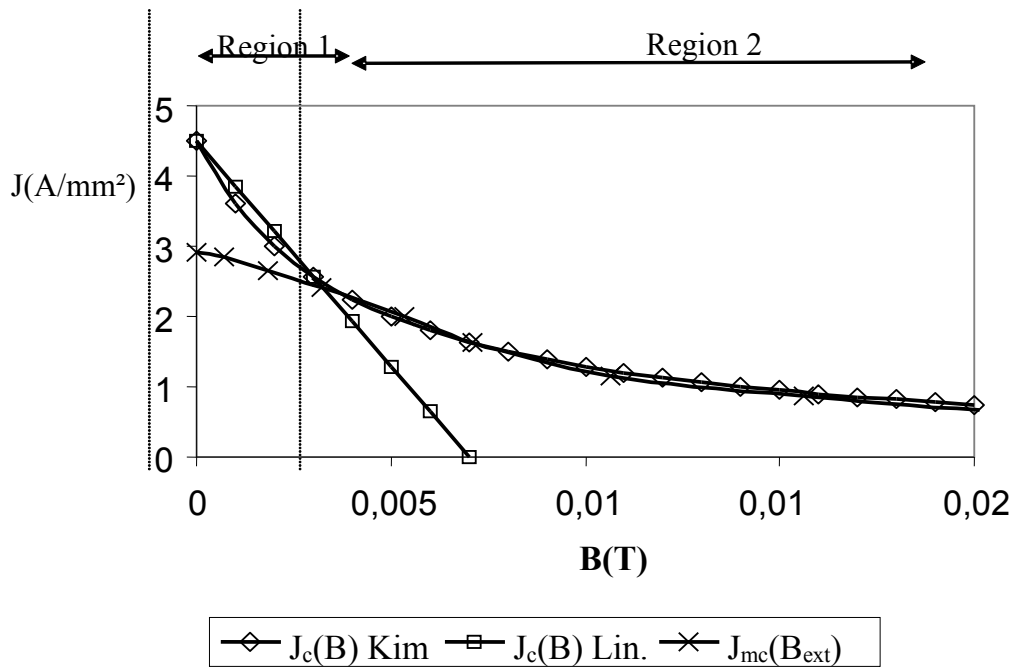


Fig. 5 : Measured  $J_c(B_{ext})$ ,  $J_c(B)$  with Kim model and linear model.



$J_{mc}(B_{ext})$  and  $J_c(B)$  are different because :

- $J_c(B)$  is a locally law and  $J_{MC}(B_{ext})$  is a macroscopic law
- $B$  is not equal to  $B_{ext}$  because of self magnetic field  $B_{SF}$  :  $\vec{B} = \vec{B}_{SF} + \vec{B}_{ext}$

When the value of  $B_{ext}$  is sufficiently large,  $B_{SF}$  becomes negligible and the values of  $J_{mc}$  are very close to  $J_c(B)$ . Among the laws represented in Table 1, Kim model is the most suitable to extrapolate  $J_{mc}(B)$ . With the appropriate values of  $J_{K0}$  and  $B_K$ , we propose the following expression of the Kim law:

$$J_c(B) = \frac{4.6 \cdot 10^6}{1 + \frac{|B|}{0.004}} \quad (13)$$

As shown in Fig. 5, values of  $J_c(B)$  provided by the Kim model are close to measurements for high values of  $B$ . An important discussion has to be set before going on. We have to find  $I_c$  in the region where the magnetic field is low, so we have to eliminate the self field effect. To do that we allow that the Kim law continues to be true near  $B = 0$ . To develop an analytical study in the previous region, we approximate the function by a linear one, with  $J_{c0} = 4.6 \text{ A/mm}^2$  and  $B_{j0} = 7 \text{ mT}$ .

$$J_c(B) = \frac{4.6 \cdot 10^6 (0.007 - |B|)}{0.007} \quad (14)$$

Fig. 5 represents the different results of the previous approaches. The following part deals with the study of the flux density in the region 1.

#### 4. Calculation of the magnetic flux density and current density penetration

In this part we present the calculations of  $B(r, t)$  and  $J(r, t)$  distributions. Results provided by both the Bean model and the linear model are compared. For  $B(r, t)$  distribution with linear model, relation (8) is used:

$$B(r, t) = \frac{B_{j0} (\mu_0 \cdot J_{c0} \cdot r - B_{j0}) + \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_e - r)}{B_{j0}}\right) (B_{j0}^2 + \mu_0 \cdot J_{c0} \cdot R_e (b_e(t) - B_{j0}))}{\mu_0 \cdot J_{c0} \cdot r}$$

$$\text{with } J_c(B) = \frac{4.6 \cdot 10^6 (0.007 - B)}{0.007}, J_{c0} = 4.6 \text{ A/mm}^2, B_{j0} = 7 \text{ mT}.$$

From now, the Bean model full penetration current is named  $I_{pB}$  and the linear model one remains named  $I_p$ .

For a  $B(r,t)$  distribution with the Bean model [9], on which case  $J_c = J_{cB}$  and  $I_{pB} = I_c = J_{cB}.S$ , there is :

$$B(r, t) = \frac{\mu_0 \cdot J_{cB}}{2} \cdot \left[ r - \frac{R_e^2}{r} \left( 1 - \left( 1 - \frac{R_e^2}{R_{in}^2} \right) \frac{i(t)}{I_{pB}} \right) \right]$$

To compare the linear model and the Bean model for the  $B(r, t)$  distributions, the critical current density  $J_{cB}$  has to be equal to  $J_{c0}$  and so  $I_{pB} = I_c = J_{c0}.S = 156 \text{ A}$ .

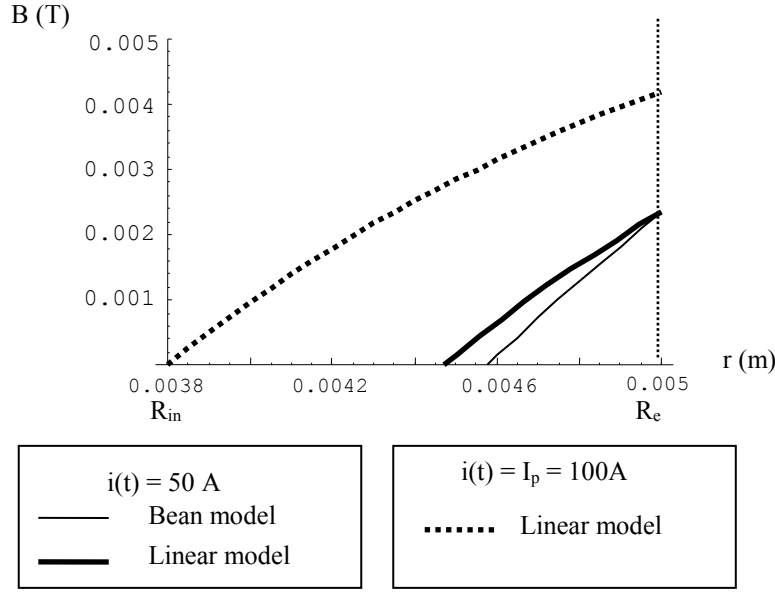


Fig. 6 :  $B(r)$  distributions with Bean model and linear model.

Fig. 6 represents  $B(r)$  for two values of current. For  $i(t) = 50 \text{ A}$  the penetration is incomplete for both, the Bean model and the linear model. For  $i(t) = 100 \text{ A}$  with the linear model there is complete penetration, so  $i(t) = I_p = 100 \text{ A}$ . There is complete penetration with the Bean model for  $i(t) = I_p = I_c = 156 \text{ A}$ . It follows that the magnetic flux density penetrates faster into material with the linear model than with the Bean model.

From  $B(r,t)$ , the maximum penetration depth ( $R_e - r_s$ ) and  $B_{\max}(r)$  are deduced, where  $B(r,t)$  for  $i(t) = I_{\max}$  :

$$r_s = \frac{B_{j0}}{\mu_0 \cdot J_{c0}} (1 + W(X_1(t_1)))$$

$$r_s < r < R_e : B_{\max}(r) = \frac{B_{j0}(\mu_0 \cdot J_{c0} \cdot r - B_{j0}) + (B_{j0}^2 + \mu_0 \cdot J_{c0} \cdot R_e (B_{e\max} - B_{j0})) \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_e - r)}{B_{j0}}\right)}{\mu_0 \cdot J_{c0} \cdot r}, \quad (15)$$

where  $B_{e\max} = B_{\max}(r = R_e)$ .

From  $J_c(B)$  and  $B(r,t)$ ,  $J(r,t)$  can be deduced:

$$J(r,t) = J_c(B(r,t)) = \frac{4.6 \cdot 10^6 (0.007 - B(r,t))}{0.007}$$

We represent  $J(r)$  for the two models at the same instant for  $i(t) = 50A$  (Fig. 7). We understand that  $J(r)$  is not the same for the models because for linear model  $J(r)$  is weaker than for Bean model except where  $B(r)=0$ . It follows that the current penetrates deeper in the case of linear model.

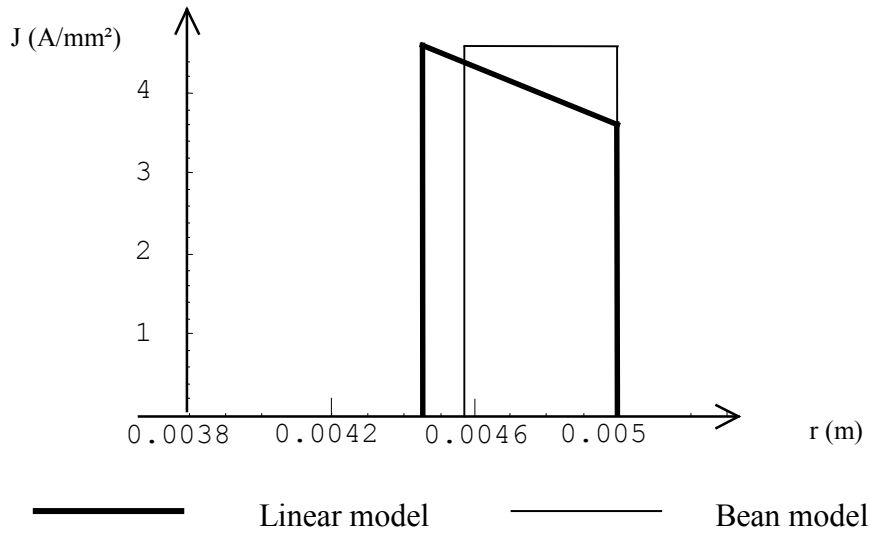


Fig. 7 : Distribution of  $J(r)$  with the Bean model and the linear model for  $i(t) = 50A$ .

## 5. Influence of flux density on the full penetration current $I_p$

### 5.1 Analytical calculation of $I_p$

This part shows the most important difference between the linear model and the Bean model. With the Bean model, the current corresponding to complete penetration of current density is  $I_{pB}$  with  $I_{pB} = I_c = J_{cB} \cdot S$ .

With the linear model, the current density penetrates faster into the tube, as presented above. The full penetration current  $I_p$  can be calculated with the linear model with:

$$B(r = R_{in}, i(t) = I_p) = 0 =$$

$$\frac{B_{j0}(\mu_0 \cdot J_{c0} \cdot R_{in} - B_{j0}) + \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_e - R_{in})}{B_{j0}}\right) \left( B_{j0}^2 + \mu_0 \cdot J_{c0} \cdot R_e \left( \frac{\mu_0 \cdot I_p}{2\pi R_e} - B_{c0} \right) \right)}{\mu_0 \cdot J_{c0} \cdot R_{in}}$$

So :

$$I_p = \frac{2\pi \cdot B_{j0}}{\mu_0^2 \cdot J_{c0}} \cdot \left( \mu_0 \cdot J_{c0} \cdot R_e - B_{j0} + (B_{j0} - \mu_0 \cdot J_{c0} \cdot R_{in}) \exp\left(\frac{\mu_0 \cdot J_{c0} \cdot (R_{in} - R_e)}{B_{j0}}\right) \right), \quad (16)$$

$I_p$  is smaller than  $I_{c0}$  because the current penetrates deeper for the linear model than for the Bean model (for the same current).

### 5.2 $I_p$ variation with $B_{j0}$

Fig. 8 represents the ratio  $I_p / I_{c0}$  according to  $B_{j0}$  for different internal radii for the same section. It shows that  $I_p$  is closer to  $I_{c0}$  for large internal radii because the magnetic flux density is weaker and the influence of  $J_c(B)$  is weaker. It also shows that  $I_p$  is close to  $I_{c0}$  for large value of  $B_{j0}$ . On the other hand,  $I_p$  is much smaller than  $I_{c0}$  for small values of  $B_{j0}$ . For this small values of  $B_{j0}$ , the self field creates by the tube (around 5mT) is close to  $B_{j0}$ . Now, as shown in part 3, one has to use only the linear model for  $B$  much smaller than  $B_{j0}$ . So there is no sense to use this  $I_p$  formula (16) for very small values of  $B_{j0}$ , where  $B > B_{j0}$ .

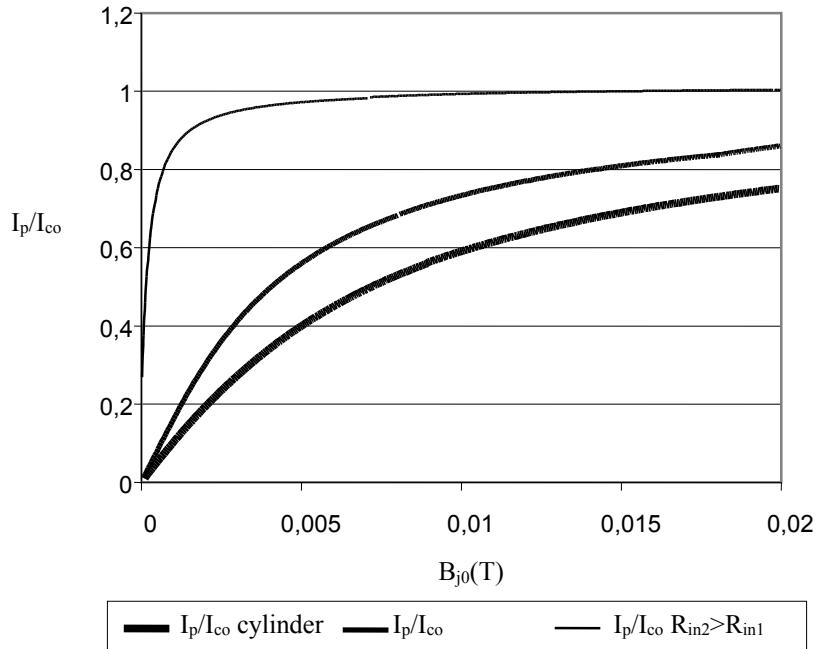


Fig. 8 : Ratio  $I_p/I_{c0}$  versus  $B_{j0}$ .

## 6. Conclusion

In this article the influence of  $J_c(B)$  on the current, electric field and magnetic field distributions in a superconductor tube fed by a current  $i(t)$ , was studied. These distributions were calculated using the linear model.

The influence of  $J_c(B)$  is important for distributions of  $B(r,t)$ ,  $J(r,t)$  and  $E(r,t)$ . It was found that the current penetrates deeper in the case of the linear model than for the Bean model. An analytical calculation of the influence of  $J_c(B)$  on the full penetration current  $I_p$  is given. For the Bean model,  $I_{pB} = I_c = J_{cB} \cdot S$ , while for the linear model  $I_p < J_{c0} \cdot S$ . It follows that with the linear model there is full current penetration for smaller value of  $i(t)$  than for the Bean model. Thanks to these results, one is able to understand the relationship of the  $I_p$  variation with the variation of the  $J_c(B)$  characteristic.

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