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A Thermodynamic Analysis of Wear

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8.5.1 INTRODUCTION

Wear phenomena due to contact and relative motion between two solids depend on the loading conditions and material mechanical behaviors and are characterized by a loss of materials. Particles are detached from sound solids Ω_1, Ω_2 when some criteria are satisfied at the boundaries Γ_i ($i = 1, 2$) between Ω_i and the complex medium Ω_3 , called hereafter the “interface”, with some wear rates ϕ_i (normal component of the velocity). The interface is a complex medium made of detached particles, eventually a lubricant fluid, and damaged zones. Macroscopic descriptions of the interface are known in the literature [1–4]. Recently, Dragon-Louiset [5, 6] and Stupkiewicz and

Mróz [7] introduced microscopic scale models in order to derive macroscopic laws. A thermodynamical analysis of the evolution of the moving surfaces Γ_i is provided by Zmitrowicz [8], Dragon-Louiset and Stolz [9], and Stolz and Pradeilles-Duval [12]. The evolutions of the damaged surfaces are complex, particularly in the transient phase of the interface life. However, for particular geometry of the interface, and for the steady state case, with constant fluxes of materials, continuous wear states can be studied experimentally. There are useful wear criteria, such as Archard's law [10], but most of them depend on experimental conditions and cannot be used for other geometry or for different loading conditions.

In contrast with the eulerian description used in References [5,6], we reconsider the model of the interface given in References [5,6,11], using rather the lagrangian approach. In this paper, the interface description is based on given macroscopic laws and differs from the one derived from microscopic considerations in Dragon-Louiset [5,6]. The local approach allows us to make a distinction between mechanical quantities evaluated on a given geometry and for specific loading conditions and intrinsic ones associated with any moving wear surfaces Γ_i . Our approach to wear criteria is based on the energy release rate like quantities, similar to Griffith's theory in fracture mechanics. All these quantities, some of them accessible to experiments, make it possible to better define wear criteria and wear rates.

Throughout this article, we make the small perturbations assumption.

8.5.2 GENERAL THERMODYNAMICAL CONSIDERATIONS ON WEAR PHENOMENA

We consider a system consisting of two sliding contacting solids (Ω_1 and Ω_2) and the contact interface zone Ω_3 , having some mechanical properties which are assumed to be known and described by usual laws of continuous media (Fig. 8.5.1). Such interface laws may be understood in a macroscopic sense, as average or homogenized through the thickness of Ω_3 (for a model of macroscopic laws issued from microscopic considerations, see Dragon-Louiset [5,6]). The interface thickness is small compared to the size of the tribological system; it results from damage mechanisms occurring in sound solids at Γ_i . Outside the damaged process zones in both solids the behavior laws are described by the free energies ψ_i and the dissipation potentials d_i . As a result of wear mechanisms, the boundaries Γ_i move with the velocities $\phi_i \mathbf{n}_i$ in the reference configuration. Along the boundary, we define $[f]_{\Gamma_i} = f_i^+ - f_i^-$ with the (+) sign in Ω_i , (-) sign in Ω_3 .

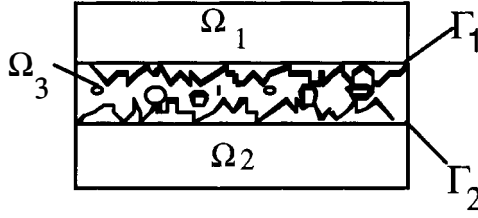


FIGURE 8.5.1 The contact interface.

By expressing the conservation laws in Ω_i and across Γ_i as in References [6, 9, 12], we get the following equations for the quasi-static case:

- Mass conservation (v : velocity; μ_i : mass flux; ρ : density; σ : stress):

$$\dot{\rho} + \text{div}(\rho v) = 0 \quad \text{in } \Omega_i, \quad \mu_i = \rho_i \phi_i n_i \quad \text{on } \Gamma_i \quad (1)$$

- Momentum conservation:

$$\text{div} 2pt\sigma = 0 \quad \text{in } \Omega_i, \quad n_i \cdot [\sigma]_{\Gamma_i} = 0 \quad \text{on } \Gamma_i, \quad (2)$$

- Energy balance (q : heat flux; ε : strain; χ : internal energy; s : entropy; T : temperature):

$$\rho \dot{\chi} = \sigma : \dot{\varepsilon} - \text{div } q \quad \text{in } \Omega_i, \quad (3)$$

$$\mu_i [\psi + sT]_{\Gamma_i} - n_i \cdot \sigma \cdot [v]_{\Gamma_i} + [q]_{\Gamma_i} \cdot n_i = 0 \quad \text{on } \Gamma_i.$$

On each moving surface, using the second conservation law (Eq. 3), one can show that there exists a heat source along Γ_i , with the density

$$\mu_i G_{th} = \mu_i [\psi + sT]_{\Gamma_i} - n_i \cdot \sigma \cdot [v]_{\Gamma_i} \quad (4)$$

8.5.2.1 ANALYSIS OF ENTROPY PRODUCTION

The internal entropy production is positive and consists of different kinds of separately positive contributions:

- volumic thermal conduction:

$$D_{th} = -q \cdot \nabla T / T^2 \geq 0 \quad (5)$$

- volumic intrinsic mechanical irreversibility:

$$D_m = \frac{1}{T} \left(\sigma : \text{grad } v - \rho \left(\dot{\psi} + s\dot{T} \right) \right) \geq 0 \quad (6)$$

- and surface (or line in two-dimensional problems) irreversibility terms:

$$D_{\Gamma} = \frac{1}{T} (\mu_i [\dot{\psi}]_{\Gamma_i} - \mathbf{n}_i \cdot \boldsymbol{\sigma} \cdot [\mathbf{v}]_{\Gamma_i}) \geq 0$$

If at any point of Γ_i the mass flux vanishes $\mu_i = 0$, the displacement \mathbf{u} is continuous, the velocity jump is $[\mathbf{v}]_{\Gamma_i} = 0$, and then $D_{\Gamma} = 0$. This means that the last dissipation D_{Γ} is a characteristic of material loss, i.e, the part of dissipation due to wear.

8.5.3 GLOBAL APPROACH OF THE INTERFACE BEHAVIOR LAWS

At each front Γ_i , the normal \mathbf{n}_i is directed toward the sound solid. The interface of thickness $e = 2h(x, t)$ is defined by its mean surface Γ . The total dissipation per unit area of Γ is given by

$$D = \int_e D_3 dz + \sum_i \left(\frac{1}{T_i} (\mu_i [\dot{\psi}]_{\Gamma_i} - \mathbf{n}_i \cdot \boldsymbol{\sigma} \cdot [\mathbf{v}]_{\Gamma_i}) \right) \geq 0 \quad (8)$$

where D_3 is the volumic dissipation of the interface medium:

$$D_3 = \left\{ -\mathbf{q} \cdot \nabla T / T^2 + \frac{1}{T} (\boldsymbol{\sigma} : \text{grad } \mathbf{v} - \rho (\dot{\psi} + s\dot{T})) \right\} \Omega_3 \quad (9)$$

The total global dissipation associated with each surface contact element contains two distinct terms, D_3 due to friction associated with relative movements of solids, and terms on Γ_i due to the fluxes of material losses (μ_1 and μ_2).

For a given mechanical behavior of the interface Ω_3 , issued either by experiments or by micromechanical considerations as was done in References [5, 6], the resolution of the evolution equations of a strip of height $e(x)$, subjected to uniform loadings, with the surfaces $z = \pm h$ sliding with the relative velocity V , may give separately the dissipation rates by friction and by wear.

8.5.3.1 INTERFACE STUDY

The displacement is assumed to be continuous across the sound solids and the interface. Let us describe the tribological system Ω_1 , Ω_2 , Ω_3 by

considering the interface Ω_3 as an abstract medium with some surface energy ψ_5 and dissipation potential D_5 to be defined hereafter. The mean surface Γ is defined by $S(\mathbf{x}, t) = 0$. At each point \mathbf{x} of Γ , with the normal $\mathbf{n}(\mathbf{x})$, we define the surfaces Γ_1 and Γ_2 , respectively, by $\mathbf{x} + h(\mathbf{x}, t)\mathbf{n}(\mathbf{x})$ and $\mathbf{x} - h(\mathbf{x}, t)\mathbf{n}(\mathbf{x})$.

We introduce the convected differential $D_\Gamma f$ of functions f (scalar, vector) with respect to Γ ,

$$D_\Gamma f(\mathbf{x}, t) := \lim_{dt \rightarrow 0} \frac{f(\mathbf{x} + \phi \mathbf{n} dt, t + dt) - f(\mathbf{x}, t)}{dt}$$

In particular, we have for the evolution of the middle surface

$$D_\Gamma S(\mathbf{x}, t) = 0,$$

$$D_\Gamma(\mathbf{x} + h(\mathbf{x}, t)\mathbf{n}(\mathbf{x})) = \phi_1 \mathbf{n}, \quad D_\Gamma(\mathbf{x} - h(\mathbf{x}, t)\mathbf{n}(\mathbf{x})) = \phi_2 \mathbf{n}$$

$$D_\Gamma(\mathbf{x}) = \phi \mathbf{n}, \quad D_\Gamma(\mathbf{n}(\mathbf{x})) = \text{div}_\Gamma(\phi \mathbf{n})$$

At each Γ_i , the displacement \mathbf{u} is continuous, and the velocity fields \mathbf{v} satisfy the Hadamard condition $D_\Gamma[\mathbf{u}]_i = 0$, or equivalently

$$[\mathbf{v}]_{\Gamma_i} + \phi_i [\nabla \mathbf{u}]_{\Gamma_i} \cdot \mathbf{n}_i = 0 \quad (10)$$

The total dissipation takes the form

$$D = \int_e D_3 dz + \sum_i \frac{\phi_i}{T_i} (G_i - G_{3i}) \geq 0 \quad (11)$$

where G_i and G_{3i} are the same expressions as given in References [5, 6]:

$$G_i = \rho_i \psi_i - \mathbf{n}_i \cdot \boldsymbol{\sigma}_i \cdot \nabla \mathbf{u}_i \cdot \mathbf{n}_i \quad (12)$$

$$G_{3i} = \rho_{3i} \psi_{3i} - \mathbf{n}_i \cdot \boldsymbol{\sigma}_{3i} \cdot \nabla \mathbf{u}_i \cdot \mathbf{n}_i \quad (13)$$

In Eq. 11, the two dissipation terms are of different natures. The first one, volumic at the origin (or at a microscopic scale in Reference [5]), becomes a surface dissipation by the thickness average process, per unit contact area (the macroscopic scale in Reference [5]). It corresponds to friction, which depends on the characteristics of Ω_3 . The complementary terms in Eq. 11 are due to wear rates ϕ_i , which depend on the characteristics of both media Ω_i and Ω_3 .

Let us consider the isothermal evolution of the interface under uniform temperature. The global dissipation is written in the following form:

$$D = \frac{1}{T} \left\{ \int_e \left(\boldsymbol{\sigma}_3 : \text{grad } \mathbf{v}_3 - \rho_3 \dot{\psi}_3 \right) dz + \sum_i \phi_i (G_i - G_{3i}) \right\} \geq 0 \quad (14)$$

It follows that a global approach of wear can be done by means of surface-free energy defined a priori on Γ . The interface of thickness $2h$ is characterized by the free energy per unit volume ψ_3 and the local strain ε . We define the surface energy by

$$\psi_S = (2\rho_S(\mathbf{x})h(\mathbf{x}))^{-1} \int_{2h} \rho(\mathbf{x}, z) \psi_3(\varepsilon(\mathbf{x} + z\mathbf{n})) j(\mathbf{x}, z) dz \quad (15)$$

where $j(\mathbf{x}, z)$ is a scalar which takes account of the curvature of Γ (case of wear in rotors, for example). The associated mass ρ_S is defined as

$$\rho_S = (2h(\mathbf{x}))^{-1} \int_{2h} \rho(\mathbf{x}, z) j(\mathbf{x}, z) dz \quad (16)$$

The section of the interface of thickness $2h$ is in equilibrium with external loads. Since the displacement fields are continuous on Γ_i , the surface energy ψ_S is a function of the displacement fields and internal parameters α , discussed later. The global potential energy of the tribological system can be written as

$$P(\mathbf{u}) = \int_{\Omega_i} \rho_i \psi_i(\varepsilon(\mathbf{u})) d\omega + \int_{\Gamma} 2\rho_S h \psi_S(\mathbf{u}_1, \mathbf{u}_2; \alpha) dA$$

where $\mathbf{u}_1 = \mathbf{u}(\mathbf{x} + h\mathbf{n}, t)$ and $\mathbf{u}_2 = \mathbf{u}(\mathbf{x} - h\mathbf{n}, t)$. At equilibrium, $\boldsymbol{\sigma} = \rho_i \partial \psi_i / \partial \varepsilon$ and $\text{div } \boldsymbol{\sigma} = 0$ on Ω_i .

In the case of the nonviscous interface, one has $\boldsymbol{\sigma} \cdot \mathbf{n}_i = 2\rho_S h \partial \psi_S / \partial \mathbf{u}_i$ on Γ_i .

In the case of the viscous interface, for a given dissipation potential $D_3 = d(\dot{\varepsilon}; \alpha)$ a similar average through the thickness can be done giving the surface dissipation $D_S(\mathbf{v}_1, \mathbf{v}_2; \alpha)$:

$$2\rho_S h D_S(\mathbf{v}_1, \mathbf{v}_2) = \int_{2h} \rho d(\dot{\varepsilon}(\mathbf{x} + z\mathbf{n}); \alpha) j(z) dz \quad (17)$$

Equation 17 yields

$$\boldsymbol{\sigma} \cdot \mathbf{n}_i = 2\rho_S h (\partial \psi_S / \partial \mathbf{u}_i + \partial D_S / \partial \mathbf{v}_i) \quad \text{on } \Gamma_i$$

The macroscopic description given here ignores the details on the microscopic level, so that the thickness $h(x)$ is the relevant parameter geometrically defining the interface, while α describes the given physical properties of the interface. In a microscopic analysis of the interface, as given in References [5, 6, 11], the parameter α also contains a description of the

constituents, i.e., the set of volume fractions of debris $\{\varphi_1, \varphi_2\}$ or the mass fractions $\{m_1, m_2\}$, together with their physical properties, etc. It can be shown that in the steady state case when only one solid, for example, Ω_2 , is subjected to wear and when damage occurs in an elastic-brittle manner, both φ_2 and m_2 are equivalent to the interface thickness $2h(x)$. In a microscopic approach to wear, the set of internal variables for general constitutive laws of the constituents is $\{\varphi_1, \varphi_2, \alpha_1, \alpha_2, \dots\}$.

The variation of the potential energy can be written as

$$\frac{dP}{dt} = \frac{d}{dt} \int_{\Omega} \rho_i \dot{\psi}_i [\varepsilon(\mathbf{u})] d\omega + \frac{d}{dt} \left[\int_{\Gamma} 2\rho_S h \psi_S(\mathbf{u}_1, \mathbf{u}_2, \alpha) dA \right]$$

which becomes after simple rearrangements

$$\frac{dP}{dt} = \int_{\Omega} \rho_i \dot{\psi}_i [\varepsilon(\mathbf{u})] d\omega - \int_{\Gamma_i} \rho_i \psi_i \phi_i dA + \int_{\Gamma} \{D_{\Gamma}(2\rho_S h \psi_S) + 2\rho_S h \psi_S \phi \gamma\} dA$$

where γ is the curvature. Similarly, the average process through the thickness of the mass conservation law yields

$$\int_{\Gamma} \{D_{\Gamma}(2\rho_S h) + 2\rho_S h \phi \gamma\} dA = -\rho_1 \phi_1 + \rho_2 \phi_2 \quad (18)$$

Equation 18 makes it possible to relate the internal variables α , h , or other relevant variables defining the interface properties appearing in its left-hand side to the wear rates ϕ_1 , ϕ_2 . Equation 18 is another form of the mass conservation law, more suitable for the interface description. Its right-hand side may be considered as the source terms feeding the contact interface.

The variation of the potential energy P can be written in the form

$$\begin{aligned} \frac{dP}{dt} = & \int_{\Omega_i/\Gamma_i} \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{v} dA - \int_{\Gamma_i} \rho_i \psi_i \phi_i dA + \int_{\Gamma} \psi_S [\rho \phi]_1^2 dA - \int_{\Gamma_i} \mathbf{n}_i \cdot \boldsymbol{\sigma}_i \cdot \mathbf{v}_i dA \\ & + \int_{\Gamma} \left\{ 2\rho_S h \frac{\partial \psi_S}{\partial \mathbf{u}_i} \cdot (\mathbf{v}_i + \nabla \mathbf{u}_i \cdot \phi_i \mathbf{n}_i) + 2\rho_S h \frac{\partial \psi_S}{\partial \alpha} \cdot D_{\Gamma}(\alpha) \right\} dA \end{aligned} \quad (19)$$

We obtain the dissipation

$$\begin{aligned} \frac{dP}{dt} - \int_{\Omega_i/\Gamma_i} \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{v} dA = & - \int_{\Gamma_i} \{ \rho_i \psi_i \mathbf{n}_i - \mathbf{n}_i \cdot \boldsymbol{\sigma}_i \cdot \nabla \mathbf{u}_i \} \cdot \phi_i \mathbf{n}_i dA \\ & + \int_{\Gamma} \left\{ 2\rho_S h \frac{\partial \psi_S}{\partial \alpha} \cdot D_{\Gamma}(\alpha) - \psi_S (\rho \phi)_1^2 - 2\rho_S h \frac{\partial D_S}{\partial \mathbf{v}_i} \cdot D_{\Gamma}(\mathbf{u}_i) \right\} dA \end{aligned}$$

We recover the dissipation because of the interface movement. The first term corresponds exactly to G_i , and the complementary ones correspond to the interface contribution.

In particular, if $2h$ is the internal parameter α , since $D_{\Gamma}(2h)\mathbf{n} + 2hD_{\Gamma}(\mathbf{n}) = (\phi_1 - \phi_2)\mathbf{n}$, the previous expression takes a form similar to the

global dissipation in which G_{3i} is given by

$$G_{3i} = G_{si} = \psi_s \rho_i - 2\rho_s h \frac{\partial \psi_s}{\partial \alpha}$$

In this approach, the interface is in equilibrium at any time. This expression shows that G_{si} are parameters directly associated with the interface model.

8.5.3.2 WEAR CRITERIA

8.5.3.2.1 Local Model

A wear criterion can be defined as a relationship between the normal velocity ϕ_i and the associated thermodynamic force $\gamma_i = G_i - G_{3i}$. One can define a dissipation potential d , a convex function of γ_i , such that $\phi_i = \partial d / \partial \gamma_i$. The simplest criterion is similar to Griffith's criterion in fracture mechanics, for which $d(\gamma_i)$ is the supporting function of the convex $\gamma_i \leq \gamma_{ic}$. The wear criterion of medium (i) depends on τ_c , γ_{ic} , G_{3i} , i.e., on five parameters.

8.5.3.2.2 Interface Model

There are simplifications when the free energy and the potential dissipation have simple forms

$$\psi_s(\mathbf{u}_1, \mathbf{u}_2, \alpha) := f(\mathbf{u}_1 - \mathbf{u}_2, \alpha), D_s(\mathbf{v}_1, \mathbf{v}_2, \alpha) := g(\mathbf{v}_1 - \mathbf{v}_2, \alpha)$$

We can show that, at first order, $[\boldsymbol{\sigma}] \cdot \mathbf{n} = 0$ and the viscous dissipation term is reduced to $2\rho_s h (\partial D_s / \partial \mathbf{v}) \cdot [D_\Gamma(\mathbf{u}_i)]_2^1$. Since the velocities ϕ_i are small compared to the relative velocity \mathbf{V} , we have $2\rho_s h (\partial D_s / \partial \mathbf{v}) \cdot [D_\Gamma(\mathbf{u}_i)]_2^1 \cong 2\rho_s h (\partial D_s / \partial \mathbf{v}) \cdot \mathbf{V}$, which means that the dissipation is essentially due to friction. Such an approximation is assumed in References [5, 6, 11].

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