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Parameterized reduced models for efficient optimization of structural dynamic behavior

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Abstract

Substructure synthesis is a method of model order reduction which is generally more efficient, computationally speaking, than analyzing the complete structural system. However, these methods are not necessarily well adapted for use within an optimization process since they do not preserve the fidelity of the reduced models when structural modifications within the reduced substructure are introduced. As a result, a costly model reduction must be performed at each iteration step.

This paper presents a new method to improve standard reduction methods by taking into account *a priori* knowledge of the potential structural modifications. Indeed, this information proves to be salutary in creating a single enriched model reduction transformation that preserves the precision of the reduced substructure model throughout the optimization process. The proposed approach consists in extending the standard transformation matrix by a set of static residual vectors which are optimized with respect to the design variables to be modified. The proposed method can be used with a variety component mode synthesis approaches with any type of substructure natural modes: free-free, cantilever or hybrid modes. The proposed methodology is illustrated on the basis of a simulated test case taken from the aero-engine industry.

Introduction

The ever increasing demand for faster engineering analysis in the design process has resulted in a substantial amount of research and development on faster and more accurate approximate reanalysis method. The difficulty of any model reduction procedure lies in how to complete the representation basis in order to reduce truncation effects. Two main classes of methodologies can be found in the literature. The first class seeks to generate a set of Ritz vectors capable of representing with precision the structural behavior under a wide variety of structural modifications. For example, Balmès¹⁻¹⁰ studied the possibility of using a constant basis of Ritz vectors to create parametric families of reduced models whereas Bouazzouni A., et al²⁻⁹ developed a method for optimally constructing additional vectors by using the dynamic behavior of the structure before modification combined with the *a priori* knowledge of the design variables. Both of these approaches have already been used effectively in an industrial context. The second class of reduction methods are based on a high order polynomial expansions of model responses about a nominal point in parameter space. This approach proves to be particularly interesting for

small number of design variables and can be used for topological optimization.

In this paper, we will extend the approach developed in² for use with substructure synthesis techniques. The latter represent a economic means for evaluating the structural behavior of complex mechanical assemblies and are known collectively as component mode synthesis methods. This approach represents a structure as an assembly of individually reduced substructures called superelements. For example, the Craig-Bampton (CB) method is one of many techniques of component mode synthesis used intensively in the aerospace industry³⁻⁴. However, these methods are not always well adapted to the industrial problem. This is especially true when parametric studies are to be performed with respect to design variables contained within individual superelements. The designer has the choice of either re-using the nominal model reduction transformation or performing a new superelement analysis for each modified component. The first option generally leads to inaccurate results while the latter is often impracticable due to cost considerations.

We propose in this paper a new method to improve the standard superelement reduction methods by taking into account an *a priori* knowledge of the potentially modifiable design variables with the objective of constructing a unique reduction transformation matrix which will preserve

the predictiveness of the superelement for a wide range of structural modifications. The approach consists in generalizing the procedure proposed in ² to individual structural components or substructures by completing the standard reduction transformation of the component by a set of optimized static residual vectors which depend on the set of design parameters retained for the component in question. The global parameterized model is thus constituted of an assembly of individually reduced and parameterized component models. The potential of the proposed methodology is demonstrated on the basis of a simulated numerical test case taken from the aero-engine industry.

Model reduction problem

The basic philosophy behind the creation of a superelement from the global stiffness and mass matrices of a structural component is briefly described below in the context of a linear conservative elastodynamic substructure. The homogeneous equation of motion for such a substructure is give by:

$$(K - \lambda_v M) y_v = 0 \quad (1)$$

Where M;K represent the discrete mass and stiffness matrices of the system and $y_v; \lambda_v$ are a given pair of eigensolutions. The generic substructure model reduction is performed via a transformation matrix T_0 such that:

$$\phi_v = T_0 q_v \quad (2)$$

Where q_v is a vector of generalized coordinates representing the contribution of each column of T_0 . Note that some of these coordinates may correspond to physical degrees of freedom.

Substituting (2) into (1) and premultiplying by the transpose of T_0 yields:

$$(\hat{K} - \lambda_v \hat{M}) q_v = 0 \quad (3)$$

where $\hat{K}; \hat{M} \in R^{n,n}$ are the reduced order stiffness and mass matrices.

For example, in the case of a Craig-Bampton reduction ³, the substructure degrees of freedom (dof) are partitioned into junction and interior dof sets. The junction dof correspond to those dof used to assemble the substructure to the so-called residual structure. The partitioned mass and stiffness matrices of the substructure thus have the following form:

$$K = \begin{bmatrix} K_{jj} & K_{ji} \\ K_{ij} & K_{ii} \end{bmatrix} \quad (4)$$

$$M = \begin{bmatrix} M_{jj} & M_{ji} \\ M_{ij} & M_{ii} \end{bmatrix}$$

the transformation matrix T_0 has the following form:

$$T_0 = \begin{bmatrix} I & 0 \\ G_{ij} & \Phi \end{bmatrix} \quad (5)$$

Where $G_{ij} = -K_{ii}^{-1} K_{ij}$ and Φ are a subset of eigenvectors of the slave eigenvalue problem $[K_{ii} - \sigma_v M_{ii}] \phi_v = 0$. The corresponding superelement matrices of the reduced substructure are given by:

$$\hat{K} = \begin{bmatrix} K_{jj} + K_{ji} G_{ij} & 0 \\ 0 & \Phi^T K_{ii} \Phi \end{bmatrix} \quad (6)$$

$$\hat{M} = \begin{bmatrix} G_{ij}^T M_{ii} G_{ij} + G_{ij}^T M_{ij} + M_{jj} & G_{ij}^T M_{ii} \Phi + M_{ji} \Phi \\ \Phi^T M_{ii} G_{ij} + \Phi^T M_{ij} & \Phi^T M_{ii} \Phi \end{bmatrix}$$

The Craig-Bampton superelement matrices represent the dynamic behavior of the substructure as seen through the junction dof. The static behavior is exact for all junction loads. The question we are concerned with here is: what happens if a design variable (e.g. a plate thickness or beam property) of a superelement is modified? In order to save the cost of calculating a new T_0 , we might be consider using the same transformation (evaluated on the basis of a nominal model) on the modified global matrices of the substructure. Experience shows that the precision of this solution degenerates rapidly with the amplitude of the perturbation. We propose an alternative solution that allows the transformation matrix to be completed with a new basis of vectors which are optimally chosen with respect to the design variables to be modified

Optimal static residual vectors

The strategy described in ² for improving the robustness of the reduction transformation with respect to structural modifications will be briefly reviewed in this section. The novelty of this approach lies in its use of an a priori knowledge of the nature and localization of the potential design modifications while retaining the amplitude of the modification as a variable. The objective is to construct an optimal set of Ritz vectors, using the

nominal model, to complete the standard reduction transformation.

Candidate design modifications

We define a parametric correction of the impedance Z of the substructure model as follows :

$$\Delta Z = \Delta K - \omega^2 \Delta M \quad (7)$$

By introducing notion of zones or groups of finite elements, equation (7) can be written as follows :

$$\Delta Z = \sum_{i=1}^{np} \left[K_i^{zone}(\Delta p_i) - \omega^2 M_i^{zone}(\Delta p_i) \right] \quad (8)$$

where Δp_i is the variation of the np design parameter p_i .

In general, the correction matrix is a nonlinear function of the design parameters p_i and, for example, the stiffness modification of a zone can be expressed as :

$$\Delta K_i^{zone}(\Delta p_i) = \frac{\Delta p_i}{p_i} \sum_{\alpha} \alpha(p_i)^{\alpha} K_{i\alpha}^{zone} \quad (9)$$

The homogeneous equations of motion of the modified substructure is given by :

$$\left[Z(\omega) + \Delta Z(\omega) \right] y(\omega) = 0 \quad (10)$$

Force basis associated to a structural modification

By introducing the notion of force $f_{\Delta}(\omega)$ associated with structural modifications $\Delta Z(\omega)$, equation (10) can be rewritten:

$$Z(\omega) y(\omega) = f_{\Delta}(\omega) \quad (11)$$

where:

$$f_{\Delta}(\omega) = -\Delta Z(\omega) y(\omega) \quad (12)$$

The vector $f_{\Delta}(\omega)$ represents the forces acting on the nominal structure due to the structural modifications. Given that the response vector y is unknown, the force vector cannot be determined exactly either. The essential idea of the methodology proposed in ² is expressed in the following steps:

1. use (12) to generate a basis of force vectors that, if it does not contain the exact force vector

with respect to a specific design modification, will at least represent a space containing these vectors. This is accomplished by injecting known nominal model response vectors into (12).

2. The resulting force basis is then used to generate static response vectors, once again on the basis of the nominal model.
3. The first two steps are repeated for each candidate design parameter.

In practice, many different types of model response vectors may be injected into (12), including global substructure modes or sensitivity vectors, slave system modes, static vectors, etc. These forces are obviously frequency dependent and are generally evaluated at the frequency corresponding to the response vector in question. Let $B \in R^{N,m}$ denote the basis of response vectors used for this purpose, hence:

$$y(\omega) = Bc(\omega) \quad (13)$$

For a modification of the design parameter p_i , equation (12) becomes:

$$F_{\Delta}^i = \left[-\Delta Z^i(\omega_1) b_1 \quad \dots \quad -\Delta Z^i(\omega_m) b_m \right] \quad (14)$$

The final force basis, representative of the set of global set of candidate structural modifications, is then obtained by concatenation of the series of bases generated for each zone:

$$F_{\Delta} = \left[F_{\Delta}^1 \quad F_{\Delta}^2 \quad \dots \quad F_{\Delta}^{np} \right] \quad (15)$$

Enriched transformation matrix

Once we have built our force basis, we can compute a series of response vectors which will be used to complete the standard model reduction transformation matrix. These responses are obviously frequency dependent and in some cases it is worthwhile. Once again, these responses are generally frequency dependent and in some cases it is worthwhile to calculate them explicitly as in ⁹. However, in order to avoid re-decomposing the global stiffness matrix of the substructure, the static solution is often evaluated instead:

$$S_{\Delta} = K^{-1} F_{\Delta} \quad (16)$$

Practically speaking, the basis S_{Δ} is rarely of full rank and must be reconditioned before use. The form this reconditioning takes depends on both the contents of both S_{Δ} and T_0 as will be discussed in the next section. Letting $Q \in R^{N,q}$ be the

reconditioned matrix, the improved transformation matrix T is given by:

$$T = [T_0 \quad Q] \quad (17)$$

where T_0 is a classical transformation matrix. In the case where T_0 is the Craig-Bampton transformation, the enriched superelement has the form:

$$\hat{K} = \begin{bmatrix} K_{jj} + K_j G_j & 0 & 0 \\ 0 & \Phi^T K_i \Phi & \Phi^T K_i Q \\ 0 & Q^T K_i \Phi & Q^T K_i Q \end{bmatrix} \quad (18)$$

$$\hat{M} = \begin{bmatrix} G_j^T M_i G_j + G_j^T M_j + M_{jj} & G_j^T M_i \Phi + M_{ji} \Phi & 0 \\ \Phi^T M_i G_j + \Phi^T M_j & \Phi^T M_i \Phi & \Phi^T M_i Q \\ 0 & Q^T M_i \Phi & Q^T M_i Q \end{bmatrix}$$

In the case of the Craig-Bampton reduction, the complementary basis Q is calculated with respect to the slave system. Hence, the displacements on the junction dof are zero.

Remarks

Several practical remarks are in order concerning the practical application of equation (17):

- The static responses S_A are often linearly dependent and it is important to perform an adequate reconditioning of this matrix before use. The form of this pre-processing depends on the contents of T_0 . For example, if part or all of T_0 the modal matrix of the substructure, then it is advantageous to replace S_A by the corresponding static residual matrix formed by removing the static contribution of the retained modes.
- When possible, it is important not to combine the contributions of several zones to construct a single (sub)basis. This has the effect of coupling otherwise independent parameters.

Industrial application

Description of test case

The proposed methodology will be illustrated on a subassembly of an aero-engine model (figure 1) comprising four Craig-Bampton superelements and a residual structure containing, by definition, the ensemble of junction dof for all superelements as well as physical dof corresponding to unreduced structural elements.

The numerical model contains a total of about 40000 dof and is under free-free boundary conditions. Substructure 1, a casing, contains 9000 dof with 300 interface dof while substructures 2 and 3 are both stators containing respectively 4200 dof with 600 interface degrees of freedom and 4400 dof

with 600 interface dof. Substructure 4 contains 6000 dof with 60 interface dof and is also a casing. A analysis range is defined which contains 6 rigid body modes and 12 flexible modes in the initial configuration. Superelements for all four components have been created using the Craig-Bampton technique.

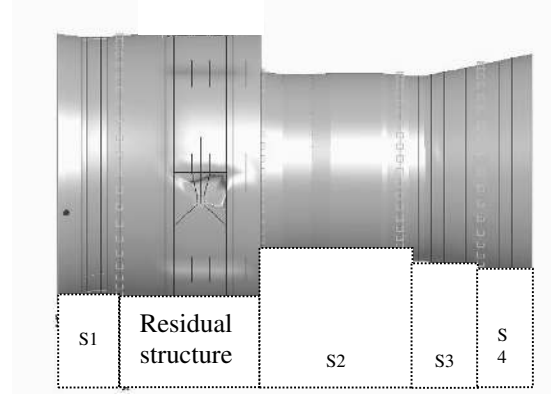


Figure 1

In order to illustrate the proposed methodology, we have perturbed the nominal substructure models as follows:

- concerning the first substructure, we define two zones where the Young's modulus are modified by a factor 2.
- concerning the second and the third substructures which are two tubes we choose to introduce local modifications. We define two zones for each substructure where the Young's modulus are modified respectively by a factor 2 and 0.5.
- and for the last substructure, we define a zone where the thickness is the modification parameter. The nominal values of the shell element thickness are divided by 2.

These structural perturbations lead to maximum frequency shifts on the order of 20%.

We will compare the precision between two types of calculation:

1. Re-using the standard Craig-Bampton transformation of each the nominal substructure models on the corresponding perturbed models.
2. Enrichment of the standard Craig-Bampton transformation by a set of optimally selected Ritz vectors.

In order to demonstrate the improved convergence properties optimal basis vectors, the dimensions of the two transformation matrices are chosen to be identical, that is to say, the number of slave eigenmodes included in Φ are increased for the standard Craig-Bampton reduction. The dimension

of the reduction bases for both methods are reported in table 1 and 2.

	Substructure			
	1	2	3	4
Constraint modes	300	600	600	60
Normal modes	20	40	40	30
Optimal vectors	0	0	0	0
Reduced model size	320	640	640	90

Table 1: Composition of the standard Craig-Bampton reduction

	Substructure			
	1	2	3	4
Constraint modes	300	600	600	60
Normal modes	15	20	20	20
Optimal vectors	5	20	20	10
Reduced model size	320	640	640	90

Table 2: Composition of the enriched Craig-Bampton reduction

Comparison criteria

The relative precision of the two analyses will be compared on the basis of three criteria:

1. Relative frequency error:

$$\mathcal{E}_f = \frac{|f_R - f_{exact}|}{f_{exact}} * 100$$

where:

- f_{exact} is an eigenfrequency of the assembled perturbed system system calculated using an exact update of the modified superelements.
- f_R is an eigenfrequency of the assembled system system calculated using either the standard or enriched Craig-Bampton transformation on the perturbed model.

2. Modal assurance criteria (MAC):

$$\mathcal{E}_{MAC} = \frac{(y_{exact}^T y_R)^2}{(y_{exact}^T y_{exact})(y_R^T y_R)} * 100$$

where:

- $y_{exact} \in \mathbb{R}^{N,1}$ an eigenvector of the assembled perturbed system calculated using an exact update of the modified superelements.
- $y_R \in \mathbb{R}^{N,1}$ an eigenvector of the assembly system calculated using either the standard or enriched Craig-Bampton transformation on the perturbed model.

3. Transfer function

This criteria is used in a qualitative way only and allows the impact the predictive errors to be interpreted in terms of amplitude envelopes.

Results

Table 3 reports the relative frequency error and MAC criteria for the two analyses. We note a general improvement in the precision of all eigenfrequencies and eigenvectors. The remaining modes are local to individual substructures. Figures 2 and 3 show the exact (nominal and perturbed) and approximate transfer functions for two different collocated observation-excitation points. In both cases, the modal damping coefficient was defined to be 2%. Note that axis labels have been purposely omitted. The dotted and dashed lines represent the exact transfer function of the perturbed and nominal models, respectively, while the dot-dashed and continuous lines represent the standard and enriched Craig-Bampton results, respectively. We can observe that the standard Craig-Bampton transformation largely underestimates the magnitude of the frequency shifts of the perturbed structure while the proposed methodology yields a significantly more accurate prediction.

Mode number	Standard Craig-Bampton		Enriched Craig-Bampton	
	\mathcal{E}_f	\mathcal{E}_{MAC}	\mathcal{E}_f	\mathcal{E}_{MAC}
1	0.64	1.0	0.60	1.0
2	0.63	1.0	0.60	1.0
3	4.30	1.0	0.06	1.0
4	0.03	1.0	0.03	1.0
5	1.40	1.0	1.06	1.0
6	1.76	1.0	1.03	1.0
7	1.05	1.0	0.43	1.0
8	3.28	0.7	1.36	1.0
9	3.57	0.5	0.62	1.0
10	5.07	0.3	1.30	1.0
11	12.82	0.2	2.86	1.0
12	3.82	0.7	2.08	1.0

Table 3 : Comparison of relative prediction errors

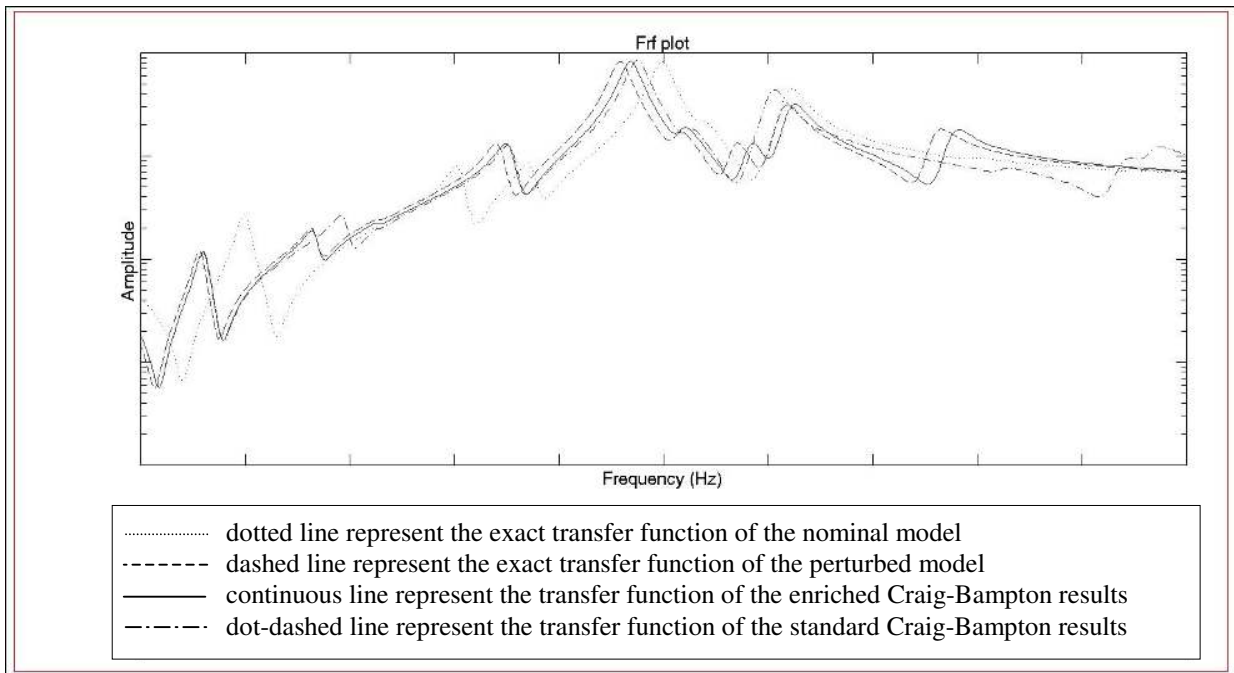


Figure 2 : Frequency Response Function

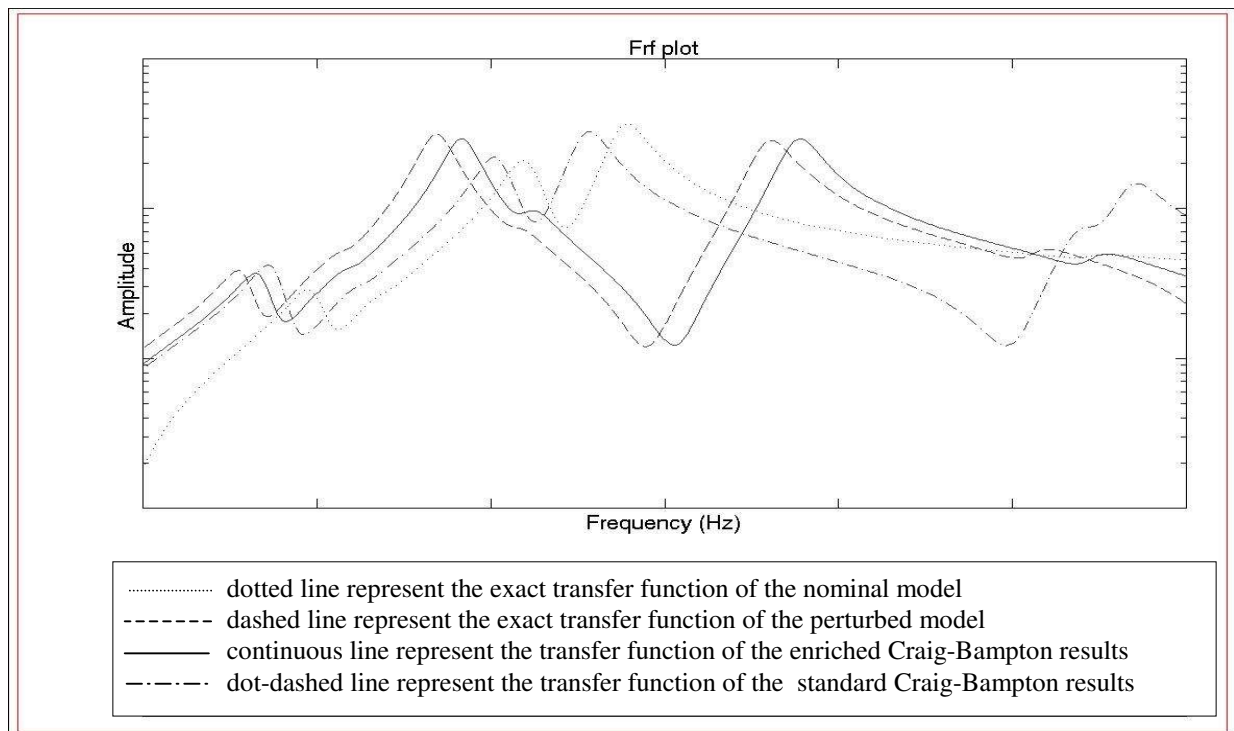


Figure 3 : Frequency Response Function

Conclusions

In this paper, we have extended an existing methodology for the approximate reanalysis of linear elastodynamic behavior to superelement technology. This approach allows design parameters of a superelement to be modified, for example in the context of an optimization algorithm, without

having to perform a complete superelement analysis at each point in parameter space. The method is based on the enrichment of the standard superelement reduction transformation by a basis of representation which is optimized with respect to the design parameters to be modified. The proposed methodology can be integrated in a variety of

component mode synthesis techniques and we have illustrated its use in the context of the Craig-Bampton superelement. An industrial test case derived from the aero-engine industry was used to illustrate the efficiency of the optimized basis in representing the modified behavior of a perturbed structure. The proposed approach is particularly effective when dealing with very large models having a large number of subassemblies. The improved model reduction transformation can be prepared in advance for each substructure and then used directly during the optimization procedure, thus avoiding the exact recalculation of the modified superelements. Current work is in progress to take into account local non-linearities at the interfaces of superelements.

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