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Comparison between Spectral-based Methods for INL Estimation And Feasibility of Their Implantation

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Abstract: *The Integral Non-Linearity (INL) is the most significant static parameter of Analog-to-Digital Converters. This paper presents a comparison between different INL test techniques based on INL estimation from the spectrum of the converted signal. Then, we investigate whether the most accurate technique is applicable in a realistic hardware configuration.*

Keywords: *Analog-to-Digital Converter testing, Integral Non-Linearity, polynomial fitting, Fourier series expansion, Fast Fourier Transform.*

1 Introduction

Because of the explosive evolution of Very Large Scale Integration and the increase of system speed of operation, mixed-signal devices' testing becomes increasingly complex and expensive. Analog-to-Digital Converter (ADC), a fundamental element of any mixed-signal system, is characterized by two different kinds of parameters which cannot be obtained by a single test. Firstly, the static parameters (offset, gain errors and non-linearity) related to the ADC transfer function are measured by the histogram-based test procedure [1]. Then, a Fast Fourier Transform (FFT) test technique [2] is used to evaluate the dynamic performances (SINAD, THD, SFDR...).

In practice, every static parameter has a specific importance and in most cases the evaluation of the sole Integral Non-Linearity gives satisfactory test results. But, as long as the resolution of ADCs and the specified test accuracy are increasing, the INL evaluation through a histogram-based method induces prohibitive testing time. For instance, considering a 14-bit ADC, the test can require several seconds, which is unacceptable in an industrial environment.

Consequently, in the past few years, many published papers have developed alternative methods and solutions to integrate them in order to propose an optimal BIST (Built-In Self Test) architecture.

An attractive idea is to use the link between static and dynamic parameters. In particular, INL strongly influences the dynamic performances through harmonic generation. The information given by the spectral analysis could thus be used to extract INL values with a shorter testing time. This method requires much less samples than the histogram-based one, thus the processing time, and hence the test time, can be significantly shortened. Two very different approaches can be used to evaluate the relationship between static and dynamic specifications: i) a statistical approach proposed in [3] consists in detecting devices exhibiting at least one functional parameter beyond specifications, but this kind of statistical method is not viable in a BIST context; ii) an analytical approach that consists in identifying the Fourier coefficients obtained with the classical FFT, with the parameters that describe the INL curve. Most of the proposed techniques are based on polynomial interpolation of INL [4-7], while a recent one [8] is based on the Fourier series expansion of the INL.

In this paper, we focus on the analytical approach and our objective is to choose the best technique to fit the INL and investigate whether the implantation in a BIST architecture is possible.

In the second section, INL fitting methods are described. The third section is dedicated to the comparison of the test methods based on polynomial fitting or on Fourier series expansion. Then, the last section describes an optimization of the algorithm based on Fourier series expansion to help implementation.

2 Methods of INL extraction from spectral analysis

As mentioned above, the technique consists in determining an analytic expression of the relation

between some spectral information of the converted signal and the Integral Non-Linearity. Indeed, in a case of a sine wave stimulus, INL is closely related to the different harmonics of the converted signal.

In order to define this analytic expression the idea is to describe the INL thanks to a classical mathematical function g defined with α_i coefficients:

$$INL \approx g(\alpha_1, \alpha_1, ... \alpha_{kmax}) \quad (1)$$

Based on this function g, we have to find the direct relationship between the α_i coefficients and the harmonic amplitudes in the converted signal, as illustrated by the following equation:

$$[\alpha_0 ... \alpha_{K\max}] = A \times \begin{bmatrix} S_0 \\ : \\ S_{H\max} \end{bmatrix} \quad (2)$$

where A is the correlation matrix, S_0 is the amplitude of the fundamental and S_i the amplitude of ith harmonic.

The first step of this kind of techniques thus consists in choosing the mathematical form allowing the best fit of the INL.

2.1 Polynomial-fitting technique

The most widely used technique is the polynomial fitting method [4-7]. This technique consists in describing the INL (or the transfer function of the ADC) as an n-order polynomial:

$$INL(x) = \sum_{k=0}^{n} \alpha_k x^k \quad (3)$$

where $x=0,\ldots,2^N-1$ is the digital output code of the N-bit converter. In [4] and [7], a low order polynomial is proposed. In [7], the wobbling technique is used to stabilize the harmonic bins during the correlation phase between polynomial coefficients and harmonic amplitudes. In [4], to identify the polynomial coefficients a piece-wise approximation is used to reduce the processing and thus optimize the required hardware resources in a view of integration. The methods described in [5] and [6] are based on high order polynomial. The extraction of the polynomial coefficients is performed by identification with the output signal spectrum [5] or by using an analytical expression based on Chebycheff polynomials [6].

All these methods are based on polynomial interpolation. Obviously, the higher the polynomial degree, the more accurate the fitting. In section 3, we evaluate the efficiency of this kind of interpolation in a case a real INL measured by the histogram-based test procedure.

2.2 Discrete Fourier series expansion

More recently, a method based on discrete Fourier series has been proposed [8]. A curve must be periodic to be expressed with a discrete Fourier series. In order to make the INL periodic, we use an extension of INL curve given by the following equations:

$$\begin{aligned} &INL(x) && 0 \le x \le 2^N - 1 \\ &INL(x + p.2^N) = INL(x) && p \varepsilon Z \end{aligned} \quad (4)$$

where $x=0,\ldots,2^N-1$ is the digital output code of the converter.

The periodic extension of the INL function leads to the following expression of the discrete Fourier series expansion:

$$INL(x) \approx \frac{a_0}{2} + \sum_{k=1}^{2^N-1} \left[a_k \cos\left(2\pi \frac{k}{2^N} x\right) + b_k \sin\left(2\pi \frac{k}{2^N} x\right) \right] \quad (5)$$

The coefficients a_k and b_k can be computed from the kth complex Fourier coefficient α_k given by

$$\alpha_k = \sum_{x=0}^{2^N-1} INL(x).e^{\left(-j\frac{2\pi}{2^N-1}kx\right)} \quad (6)$$

with

$$a_k = \frac{2}{2^N-1} Re[\alpha_k] \quad \text{and} \quad b_k = \frac{-2}{2^N-1} Im[\alpha_k] \quad (7)$$

where $Re[\alpha_k]$ and $Im[\alpha_k]$ are respectively the real and imaginary parts of the complex Fourier coefficient α_k.

The expression of the relationship between the Fourier coefficients a_k, b_k of the INL interpolation and spectral components of the converted signal, which is more complex than in the case of polynomial interpolation, will be described in the following.

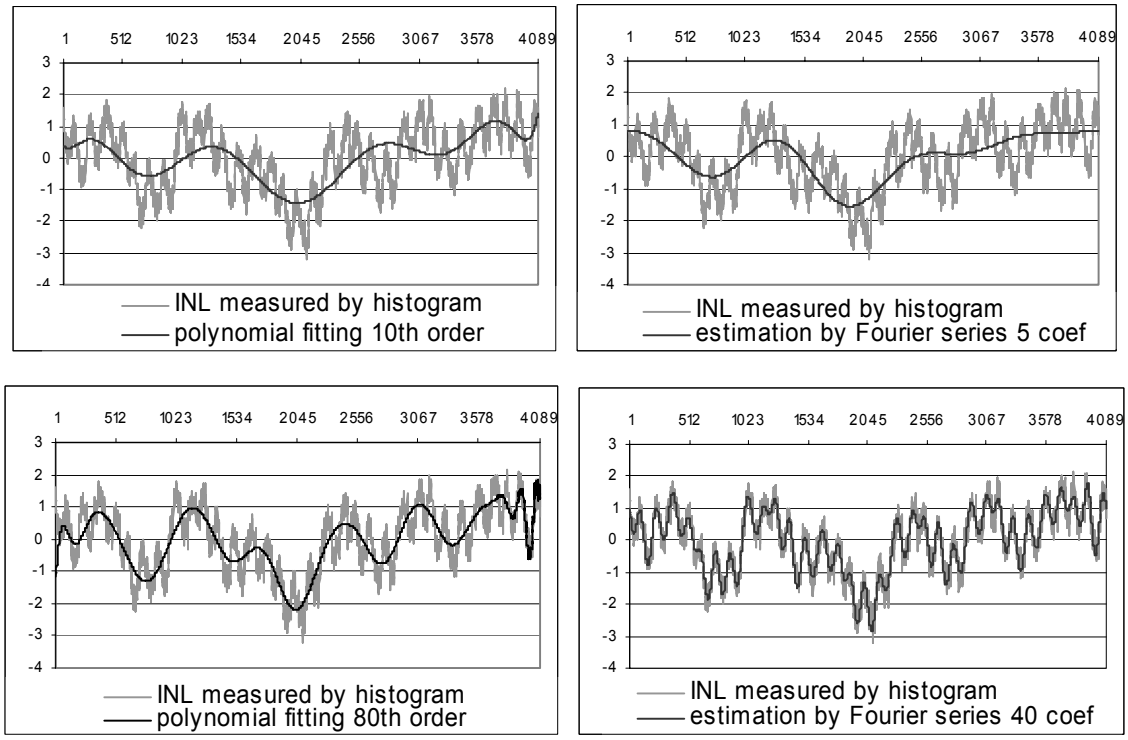
3 Fitting methods vs. real INL

The methods presented in the previous section are based on an INL interpolation by a polynomial or a discrete Fourier series expansion. Before further optimization of the technique, we have to choose the best interpolation technique to fit a real INL.

3.1 Comparison between the two fitting methods

Our objective is to compare the INL measured by the histogram-based test technique with the estimated INL obtained by interpolation. The techniques proposed in [4-7] have been validated on low resolution ADC or with additional noise to smooth the INL. In this study, we focus on a real 12-bit ADC TDA8769 from Philips [9].

In order to compare estimation methods based on polynomials or Fourier series expansion in terms of hardware resource requirements, we consider N/2 complex coefficients (that correspond to N/2 real and N/2 imaginary coefficients) to be compared with a Nth-order polynomial interpolation. We have conducted several experiments varying the values of N. Figure 1 gives some examples (N=10 and N=80) of comparison between INL estimations and INL measured using a histogram-based test procedure.



a) b)

Figure 1. a) comparison between the INL curve from the histogram test achieved on the TDA8769 and the polynomial interpolation b) comparison between the INL curve from the histogram test achieved on the TDA8769 and the estimation by Fourier series expansion

Two main observations arise from the analysis of these results. Firstly, for a low number of coefficients (N=10), the results obtained with polynomial interpolation or discrete Fourier expansion are similar. The two methods allow one to fit the smooth feature of INL curve, but from a test point of view the results of the two methods in this case are not accurate enough to give a valuable evaluation of the extreme values INL_{max} and INL_{min} required for the ADC test.

Secondly, when the number of coefficients increases, we can observe a great change of efficiency of the two interpolation techniques. The polynomial interpolation only fits the smooth INL variations due to low order harmonics whereas the Fourier series expansion gives in addition a good estimation of the sharp variations of the INL curve.

Indeed, with a brief analysis, a Nth-order polynomial has only a maximum of N-1 local extremes that corresponds to a limited number of possible sharp variations. In contrast, a Fourier series with N/2 complex coefficients is composed (by definition) of N sine waves exhibiting different amplitudes and phases; the sum of these sine waves allows a larger variety of possible variations.

To conclude, the polynomial model leads to a satisfying representation in the case of a smooth curve, but as the actual INL curves have sharp transitions, the Fourier series expansion seems to be the best solution to evaluate precisely extreme values of the INL, INL_{max} and INL_{min}. Indeed, these are the values to be compared to the datasheet specifications. In the following section we carry out further evaluation of this technique.

3.2 Quantitative evaluation of the fitting technique based on discrete Fourier series

We have seen that fitting techniques based on discrete Fourier series achieve significantly better results than the

method based on polynomials. Now, we have to evaluate precisely the estimation error for a real INL. Figure 2 shows the INL measured by histogram-based test technique on the TDA8769 converter and estimated with a very extended discrete Fourier series (800 complex coefficients).

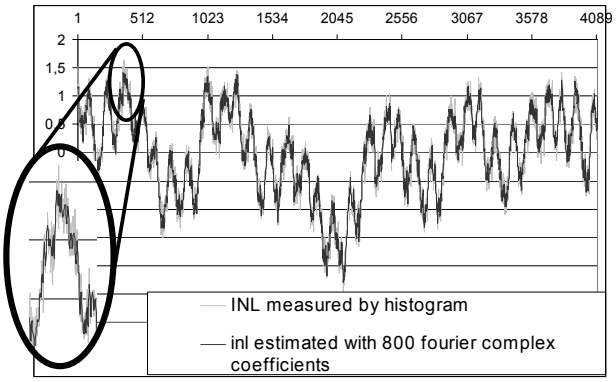


Figure 2. Comparison between INL measured by the histogram test and INL estimated by Fourier series expansion with 800 complex coefficients

From a rough observation, the INL seems to fit perfectly the measured INL, but when we zoom in the curve (figure 2), we can observe that the estimated INL cannot follow every INL sharp variation despite the use of a very extended Fourier series. This might involve a large error on the estimated INL maximal value required for the test. In our study case, the standard deviation is as low as 0.12LSB but the error on the estimated INL maximal value is quite large. The table 1 summarizes these results.

	Measurement. (LSB)	Estimation (LSB)	Error (%)
INL_{max}	1.64	1.42	13.4
INL_{min}	-2.45	-2.29	6.5

Table 1. Measured and estimated INL maximal values

The error of estimation on the INL maximal value might be of 13.4%, which may seem inappropriate in a test context.

In practice, the histogram-based test technique is sensitive to noise [10]. The measured INL is not the real INL but the real INL plus noise; that is why the results obtained with histogram-based test technique are not perfectly repeatable. To avoid this drawback, some solutions based on dithering or averaging are generally used.

In our case, if we perform a large number of histogram-based test procedures on the same device and with the same test conditions, we obtain a measurement dispersion of INL. Let us consider this dispersion as a measurement uncertainty.

Figure 3 shows the extreme values of measured and estimated INL for each ADC code in case of 100 successive test procedures:

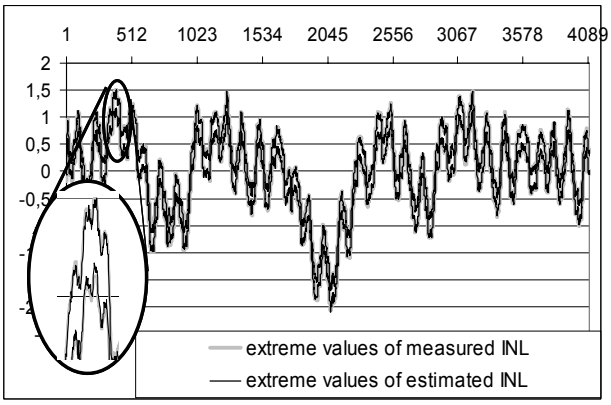


Figure 3. Comparison between extreme values of INL measured by the histogram test and estimated by Fourier series expansion with 800 complex coefficients

We can observe that when we take into account the measurement uncertainty, the estimated INL is globally surrounded by the measurement uncertainty. Now, if we focus on the maximal values of INL we obtain the following results:

	Measure. (LSB)	Measure. uncertainty (LSB)	Estimation (LSB)	Estimation uncertainty (LSB)
INL_{max}	1.30	0.19	1.34	0.16
INL_{min}	-2.37	0.20	-2.38	0.18.

Table 2. Measured and estimated INL and uncertainties

The Fourier series expansion acts as an averaging of the histogram-based measurements. In other words, the Fourier series expansion offers a better repeatability of the INL estimation than the measurement itself by the histogram-based test.

4 Fourier series expansion-based test

4.1 Theory

The technique described in [8] consists in finding the analytic expression linking the interpolated INL based on Fourier series with the spectral information of the converted signal. Let us consider a sine wave as stimulus:

$$x(t) = V_0 \cos(2\pi f_0 t + \theta_0) + \frac{V_{FS}}{2} \qquad (8)$$

where f_0 is the frequency, V_0 the amplitude and θ_0 the initial phase of the input signal, and V_{FS} the Full Scale amplitude of the converter. The analytical expression linking the interpolated INL with the spectral information of the converted signal can be expressed as follows:

$$\left[a_0 ... a_{K\max}, b_1 ... b_{K\max}\right] = T_{KMAX}^{-1} \begin{bmatrix} S_0 \\ : \\ S_{H\max} \end{bmatrix} \quad (9)$$

where $\left[S_0 ... S_{H\max}\right]^T$ is the vector composed of the fundamental and the harmonics of the output signal, the vector $\left[a_0 ... a_{K\max}, b_1 ... b_{K\max}\right]$ corresponds to the coefficients of the Fourier series expansion (5) of the INL, K_{max} is the order of the Fourier series expansion, H_{max} is the number of harmonics extracted from FFT and T_{KMAX} is the following matrix:

$$T_{KMAX}=\begin{pmatrix} 1/2 & A_1^0 & ... & A_{K\max}^0 & B_1^0 & .... & B_{K\max}^0 \\ 0 & A_1^1 & ... & A_{K\max}^1 & B_1^1 & .... & B_{K\max}^1 \\ : & : & : & : & : & : & : \\ 0 & A_1^{H\max} & ... & A_{K\max}^{H\max} & B_{H\max}^1 & .... & B_{K\max}^{H\max} \end{pmatrix} \quad (10)$$

where

$$\begin{aligned} A_k^0 &= (-1)^k . J_0(k\alpha_1) \\ A_k^{2.p+1} &= 0 && \forall p \geq 0 \\ A_k^{2.p} &= (-1)^{p+k} . 2 . J_{2.p}(k\alpha_1) && \forall p \geq 1 \\ B_k^{2.p} &= 0 && \forall p \geq 0 \\ B_k^{2.p+1} &= (-1)^{p+k} . 2 . J_{2.p+1}(k\alpha_1) && \forall p \geq 0 \end{aligned} \quad (11)$$

$J_p(x)$ is the Bessel function of the pth order and $\alpha_1 = 2\pi \frac{V_0}{V_{FS}}$.

We do not want to describe the theory more in detail. Our objective is to evaluate the feasibility of the integration of this technique in terms of hardware resource requirements.

4.2 Optimization

An INL BIST architecture based on the Fourier series expansion technique involves a pre-processing of the T_{KMAX}^{-1} and the storage of this matrix. Then, we have to extract spectral bins by FFT processing and perform the multiplication with T_{KMAX}^{-1}.

The two main hardware requirements are thus the FFT processing (or equivalent) and the storage of the matrix T_{KMAX}^{-1} on-chip. In this study we only focus on the memory issue.

If we consider a Fourier series expansion of INL with 800 complex coefficients and if we use 800 spectral bins (corresponding to the 800 first harmonics of the converted signal), the matrix T_{KMAX} would be composed of 641,200 different terms to be stored in N-bit words. This corresponds to an unacceptable memory size for a BIST solution.

Our objective is to simplify the matrix while preserving the estimation accuracy. In practice, we consider that the estimation is satisfying as long as the estimated INL remains within the measurement uncertainty of the histogram-based test.

4.2.1 Size reduction of T_{Kmax}^{-1} matrix

The first step of the matrix optimization concerns its size, defined by the number of harmonics (H_{max}) and the number of coefficients (K_{max}) in the Fourier series expansion.

As described in section 3, the more Fourier series coefficients used, the better INL estimation. But the estimation also depends directly on the number of harmonics used in the spectrum of the ADC output signal. In practice, we have only access to the harmonics with frequency lower than the Nyquist frequency. In this study, we consider 250 harmonics, which is quite realistic.

In this context, we have conducted several experiments varying the values of K_{max} and H_{max} from 70 to 250. Figure 4 gives the estimation and measurement dispersions of INL_{max} and INL_{min} versus couples H_{max}=K_{max}.

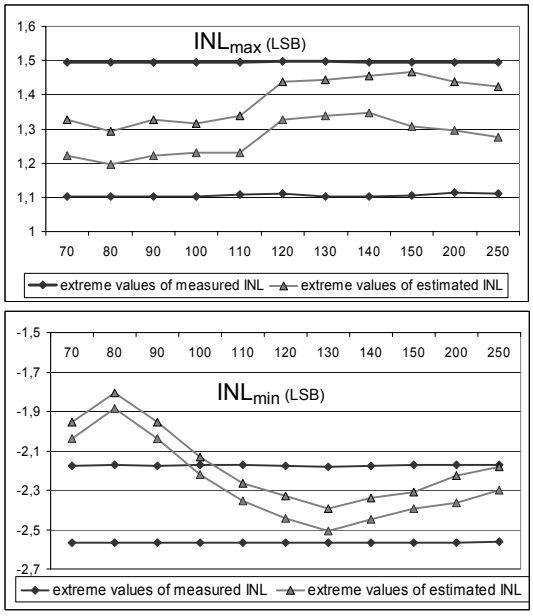


Figure 4. Extreme values of both measured and estimated INL_{max} and INL_{min} versus H_{max}=K_{max}

As expected, we can observe that the INL estimation is not included in the measurement uncertainty when few coefficients are considered. But from these results, we can conclude that a number of H_{max}=K_{max}=110 complex coefficients and spectral bins ensures a satisfactory estimation accuracy in a test context. This corresponds to a 111*221 matrix and 12,100 coefficients to be stored. In comparison to the initial situation, a significant reduction

is achieved. But the memory size also depends on the size of stored words.

4.2.2 Size reduction of stored words

Until now, all the computations were performed with floating point number with 64bit IEEE double precision. In embedded circuitry context, we can use only fixed decimal with N bit resolution. In order to reduce the memory size we have to optimize N, the length of memory words. In the same way as previously, we search the minimal word length for which the estimated INL dispersion is within the measurement uncertainties. Figure 5 shows dispersions of INL_{max} and INL_{min} according to word size from 4 bits to 32 bits.

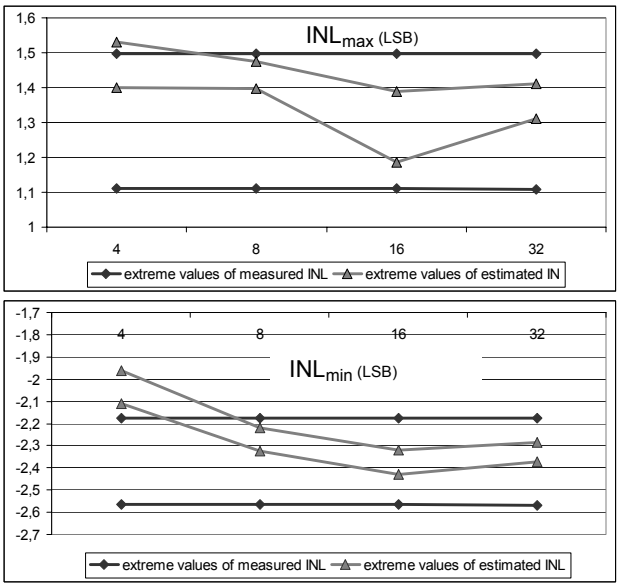


Figure 5. Extreme values of both measured and estimated INL_{max} and INL_{min} versus word length

We can observe that 8-bit words are sufficient to store the matrix. Thus finally, we need to store 12,100 bytes.

In practice, because we use low precision words, very few different values are needed in our process. It might be possible to store only these values and to use additional process to decompress the data for the INL estimation procedure. In our case study, there are only 126 different values in the matrix. Therefore, in an extreme case, we need a very small memory of 126 bytes to store values required for the INL estimation from the spectral analysis of the ADC output signal. In this case, the reduction of the memory size might be at the price of a more complex decompression circuitry.

5 Conclusion and outlook

We have shown that the polynomial fitting technique does not allow an efficient INL estimation in the case of real mildly or high resolution converters. This kind of techniques gives a good estimation of a smooth feature of INL variations but is not able to follow sharp variations of a real INL. In contrast, we have shown that the INL estimation based on Fourier series is suitable for test application and we have optimized the size of memory required for the storage of the matrix used to extract INL characteristics from the spectral domain. Finally, we need to store only 126 different values of 8 bits for a 111*221 matrix.

We are presently in the process of validating the matrix simplification on several types (resolution, architecture...) of real ADCs. Moreover, we are investigating which solution is the most appropriate for an on-chip extraction of spectral characteristics from a temporal ADC output signal.

6 References

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