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Screening effect and delocalization of interacting Bose-Einstein condensates in random potentials

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We theoretically investigate the physics of interacting Bose-Einstein condensates at equilibrium in a weak (possibly random) potential. We develop a perturbation approach to derive the condensate wavefunction for an amplitude of the potential smaller than the chemical potential of the condensate and for an arbitrary spatial variation scale of the potential. Applying this theory to disordered potentials, we find in particular that, if the healing length is smaller than the latter, the condensate assumes a delocalized Thomas-Fermi profile. In the opposite situation where the typical variation scale of the potential is smaller than the healing length, we show that the random potential can be significantly screened and, in the meanfield regime, the condensate wavefunction can remain delocalized, even for very small correlation lengths of the disorder.

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I. INTRODUCTION

Ultracold atomic gases are currently attracting a lot of attention from both experimental and theoretical viewpoints. Taking advantage of the recent progress in cooling and trapping of neutral atoms [1], dilute atomic Bose-Einstein condensates (BECs) [2] and degenerate Fermi gases (DFGs) [3, 4, 5, 6] are now routinely produced at the laboratory. Using various techniques, space-dependent potentials can be designed almost on demand in these systems. For example, using optical means, one can produce periodic [7, 8], quasi-periodic [9, 10, 11], or random potentials [12, 13, 14, 15, 16, 17]. For these reasons and due to unique control and analysis possibilities, ultracold gases constitute a favorite playground for revisiting standard problems of condensed matter physics (CM) [18, 19, 20, 21].

Most current experiments with BECs lie in the meanfield regime where the Bose gas is described by a single wavefunction, ψ , governed by the (nonlinear) Gross-Pitaevskii equation [22]. Due to the interplay between the kinetic energy term and the interaction term, it is usually difficult to derive the exact solution of this equation. The importance of interactions can be characterized by the *healing length*, ξ , which defines the typical distance below which the spatial variations of ψ significantly contribute to the energy of the BEC, *via* the kinetic energy term [22]. In the Thomas-Fermi regime (TF), *i.e.* when ξ is significantly smaller than the typical variation scale, σ_R , of the potential, $V(\mathbf{r})$, to which the BEC is subjected, the kinetic term is negligible and the BEC density simply follows the spatial variations of the potential¹:

$$|\psi(\mathbf{r})|^2 \propto \mu - V(\mathbf{r}). \quad (1)$$

In the opposite situation ($\xi > \sigma_R$), the kinetic term should be

taken into account and the exact BEC wavefunction usually cannot be found analytically.

Besides a general interest, the question of determining the BEC wavefunction for an arbitrary ratio σ_R/ξ has direct applications to the case where $V(\mathbf{r})$ is a random potential. The physics of quantum systems in the presence of disorder is central in CM [23, 24, 25], owing to unavoidable defects in 'real-life systems'. One of the major paradigms of disordered quantum systems is due to Anderson who has shown that the eigenstates of single quantum particles in arbitrary weak random potentials can be localized, *i.e.* ψ shows an exponential decay at large distances² [26]. Recent experiments have studied the onset of strong or weak localization effects of light-waves [27, 28] and microwaves [29, 30]. Ultracold matter-waves are also widely considered as promising candidates to investigate Anderson localization in random [31, 32, 33] or quasi-random structures [10, 31, 34] and more generally to investigate the effects of disorder in various quantum systems (for a recent review, see Ref. [35] and references therein). It is expected that the dramatic versatility of ultracold gases would allow us for a direct comparison with theoretical studies of quantum disordered systems.

A key peculiarity of BECs is that interactions usually cannot be neglected and interaction-induced delocalization can compete with disorder-induced localization effects [15, 16, 17]. Generally, the interplay between the kinetic energy, the interactions and the disorder is still an open question that has motivated many works [36, 37, 38, 39, 40]. It is clear from Eq. (1) that, in the TF regime ($\sigma_R \gg \xi$), where the interaction forces the wavefunction to adapt to the random potential, a BEC will not localize. Indeed, if $V(\mathbf{r})$ is a *homogeneous* random potential³ [41], so is the BEC wavefunction, ψ , which therefore, cannot decay at large distances. This has been confirmed in recent experiments [15, 16, 17]. The question thus

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¹ This is standard in the case of a harmonic confinement, $V(\mathbf{r}) = m\omega^2\mathbf{r}^2/2$. Although, there is no intrinsic typical variation scale, one can define σ_R as $m\omega^2\sigma_R^2/2 = \mu$, *i.e.* $\sigma_R = L_{TF}$, the usual TF half size of the condensate and the validity of the TF regime reads $\xi \ll L_{TF}$. For periodic, quasi-periodic or random potentials, σ_R is the spatial period or the correlation length (see section III for details).

² In 1D and 2D systems, all eigenstates are usually localized while in 3D, they are localized below the so-called *mobility edge*.

³ In this context, the term 'homogeneous random potential' means that all local statistical properties of the random potential are independent of the position.

arises as to understand whether, as a naive transcription of the Ioffe-Regel criterion [42] would suggest, localization can happen when $\sigma_r < \xi$.

In this paper, we show that this criterion is actually not sufficient for BECs at equilibrium if the interactions are non-negligible (*i.e.* if $\xi \ll L$, where L is the size of the system). We indeed show that interaction-induced delocalization still overcome localization effects even when $\xi \gg \sigma_r$. In fact, due to the *screening* of the random potential [43], the effect of disorder turns out to be reduced when ξ/σ_r increases.

In the following, we develop a general formalism based on perturbation theory (see section II) to determine the BEC wavefunction in any given weak potential, $V(\mathbf{r})$, for an arbitrary ratio σ_r/ξ . We find that the BEC density, $|\psi|^2$, is still given by Eq. (1), except that the potential $V(\mathbf{r})$ has to be replaced by a *screened potential*, $\tilde{V}(\mathbf{r})$. We derive an exact formula for the screened potential up to first order in the perturbation series. We then apply our results to the case where $V(\mathbf{r})$ is a 1D homogeneous random potential (see section III) and derive the statistical properties of the screened random potential, $\tilde{V}(\mathbf{r})$. From this, we conclude that an interacting BEC remains delocalized, even for $\xi \gg \sigma_r$ (if $\xi \ll L$).

II. SCREENING EFFECT IN INTERACTING BOSE-EINSTEIN CONDENSATES

Consider a low-temperature Bose gas in d dimensions with contact atom-atom interactions, $g_{dD}\delta^{(d)}(\mathbf{r})$, where g_{dD} is the d -dimensional interaction parameter. The Bose gas is assumed to be subjected to a given potential, $V(\mathbf{r})$, with a typical amplitude V_r and a typical variation scale σ_r . Possibly, the potential, $V(\mathbf{r})$, may have various length scales. In this case, we assume that σ_r is the smallest. Assuming weak interactions, *i.e.* $\bar{n}^{2/d-1} \gg mg_{dD}/\hbar^2$, where m is the atomic mass and \bar{n} the mean density [44, 45], we treat the BEC in the mean-field approach [22] and we use the Gross-Pitaevskii equation (GPE):

$$\mu\psi(\mathbf{r}) = \left[\frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r}) + g_{dD}|\psi(\mathbf{r})|^2 \right] \psi(\mathbf{r}), \quad (2)$$

where μ is the BEC chemical potential. Note that ψ minimizes the N-body energy functional so that ψ is necessary a real function (up to a non-physical uniform phase). In 1D and 2D geometries and in the absence of trapping, no true BEC can exist due to significant long-wavelength phase fluctuations [46]. In this case, no macroscopic wavefunction, ψ , can be defined. However, because density fluctuations are strongly suppressed in the presence of interactions, the Bose gas forms a *quasicondensate* [46] and the density, n , can be treated as a classical field. It turns out that $\sqrt{\bar{n}}$ is governed by Eq. (2). Therefore, even though we only refer to BEC wavefunctions in the following, our formalism also apply to quasicondensates, after replacing ψ by $\sqrt{\bar{n}}$.

A. The Thomas-Fermi regime

In the simplest situation, the healing length is much smaller than the typical length scale of the potential ($\xi \ll \sigma_r$). Therefore, the kinetic energy term in the GPE (2) is small and the BEC density, $|\psi|^2$, simply follows the spatial modulations of the potential:

$$\begin{aligned} |\psi(\mathbf{r})|^2 &= [\mu - V(\mathbf{r})]/g_{dD} && \text{for } \mu > V(\mathbf{r}) \\ \text{and } |\psi(\mathbf{r})|^2 &= 0 && \text{otherwise.} \end{aligned} \quad (3)$$

This corresponds to the TF regime. Note that for $V_r \ll \mu$, one has

$$\psi(z) \simeq \psi_0 - \frac{V(\mathbf{r})\psi_0}{2\mu}, \quad (4)$$

with $\psi_0 = \sqrt{\mu/g_{dD}}$ being the BEC wavefunction at $V(\mathbf{r}) = 0$. Therefore, the BEC wavefunction itself follows the modulations of the potential $V(\mathbf{r})$.

B. Beyond the Thomas-Fermi regime: the screening effect

The situation changes when the healing length is of the order of, or larger than, the typical length scale of the potential ($\xi > \sigma_r$). Indeed, the kinetic contribution limits the smallest variation length of the spatial modulations of a BEC wavefunction to a finite value of the order of the healing length [22]. Therefore, the BEC can only follow modulations of the potential on a length scale typically larger than ξ and Eq. (3) no longer holds.

For a weak amplitude of the potential⁴, we can use perturbation theory techniques. We thus write the BEC wavefunction as $\psi(\mathbf{r}) = \psi_0 + \delta\psi(\mathbf{r})$ where we assume that $\delta\psi \ll \psi_0$, and ψ_0 is the zero order solution of the GPE (2):

$$\mu\psi_0 = -\frac{\hbar^2}{2m}\nabla^2\psi_0 + g_{dD}\psi_0^3. \quad (5)$$

Since the BEC is homogeneous at zero order, one has $\psi_0 = \sqrt{\mu/g_{dD}}$. Then, the first order term of the perturbation series is given by

$$-\frac{\hbar^2}{2m}\nabla^2(\delta\psi) - [\mu - 3g_{dD}\psi_0^2] \delta\psi = -V(\mathbf{r})\psi_0. \quad (6)$$

Since $\mu - 3g_{dD}\psi_0^2 = -2\mu$, we are left with the equation

$$-\frac{\xi^2}{2}\nabla^2(\delta\psi) + \delta\psi = -\frac{V(\mathbf{r})\psi_0}{2\mu}, \quad (7)$$

where $\xi = \hbar/\sqrt{2m\mu}$ is the healing length of the BEC. We straightforwardly find the solution of Eq. (7), which reads

$$\delta\psi(\mathbf{r}) = -\int d\mathbf{r}' G(\mathbf{r} - \mathbf{r}') \frac{V(\mathbf{r}')\psi_0}{2\mu}, \quad (8)$$

⁴ A precise condition for the validity of the perturbative approach will be given later [see Eq. (16)].

where $G(\mathbf{r})$ is the Green function of Eq. (7), defined as the solution of

$$\left[-\frac{\xi^2}{2}\nabla^2 + 1\right] G(\mathbf{r}) = \delta^{(d)}(\mathbf{r}), \quad (9)$$

or equivalently, in Fourier space

$$\left[\frac{\xi^2}{2}|\mathbf{k}|^2 + 1\right] \widehat{G}(\mathbf{k}) = 1/(2\pi)^{d/2}, \quad (10)$$

where, $\widehat{G}(\mathbf{k}) = \frac{1}{(2\pi)^{d/2}} \int d\mathbf{r} G(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}$ is the Fourier transform of G . In contrast to the case of single particles, the Green function, $\widehat{G}(\mathbf{k})$, has no singularity point so that the perturbative approach can be safely applied for any wavevector \mathbf{k} .

The explicit formula for the Green function, G , depends on the dimension of the system. After some simple algebra, we find

$$\text{in } 1D, \quad G(z) = \frac{1}{\sqrt{2\xi}} \exp\left(-\frac{|z|}{\xi/\sqrt{2}}\right) \quad (11)$$

$$\text{in } 2D, \quad G(\rho) = \frac{1}{\pi\xi^2} K_0\left(\frac{|\rho|}{\xi/\sqrt{2}}\right) \quad (12)$$

$$\text{in } 3D, \quad G(\mathbf{r}) = \frac{1}{2\pi\xi^2|\mathbf{r}|} \exp\left(-\frac{|\mathbf{r}|}{\xi/\sqrt{2}}\right). \quad (13)$$

where K_0 is the modified Bessel function. Finally, up to first order in the perturbation series, the BEC wavefunction reads

$$\psi(\mathbf{r}) \simeq \psi_0 - \frac{\widetilde{V}(\mathbf{r})\psi_0}{2\mu} \quad (14)$$

with

$$\widetilde{V}(\mathbf{r}) = \int d\mathbf{r}' G(\mathbf{r}') V(\mathbf{r} - \mathbf{r}'). \quad (15)$$

Interestingly enough, the Green function in any dimension shows an exponential decay, with a typical attenuation length, ξ , and is normalized to unity⁵, $\int d\mathbf{r} G(\mathbf{r}) = 1$. Therefore, $G(\mathbf{r})$ can be seen as a *screening function* with a typical width ξ . Indeed, it should be noted that Eq. (14) is similar to Eq. (4), except that the potential $V(\mathbf{r})$ which is relevant in the case $\xi \ll \sigma_r$, changes to the potential $\widetilde{V}(\mathbf{r})$ for $\xi > \sigma_r$. The potential $\widetilde{V}(\mathbf{r})$ is a convolution of $V(\mathbf{r})$ with a function which has a typical width ξ and thus corresponds to a *screened potential* with an amplitude smaller than V_r and a variation scale of the order of $\max(\sigma_r, \xi)$.

Note that, for $\xi \ll \sigma_r$, $G(\mathbf{r})$ can be approximated by $\delta^{(d)}(\mathbf{r})$ in Eq. (15), and $\widetilde{V}(\mathbf{r}) \simeq V(\mathbf{r})$. We thus recover the results of section II A, valid for the TF regime.

The validity condition of the perturbation approach directly follows from Eq. (14):

$$\widetilde{V}(\mathbf{r}) \ll \mu. \quad (16)$$

Note that if $\xi \gg \sigma_r$, the potential can be significantly screened so that the above condition can be less restrictive than the *a priori condition*, $V(\mathbf{r}) \ll \mu$.

The results of this section show that the potential, $V(\mathbf{r})$, can be significantly screened in interacting BECs. We stress that this applies to any kind of potentials provided that $\xi \ll L$ and $\widetilde{V}(\mathbf{r}) \ll \mu$. In the next section, we present an illustration of the screening effect in the case of a random potential.

III. APPLICATION TO A TRAPPED INTERACTING BOSE-EINSTEIN CONDENSATE IN A 1D RANDOM POTENTIAL

A. Trapped 1D Bose-Einstein condensate in a random potential

In this section, we consider a 1D Bose gas subjected to a weak homogeneous random potential, $V(z)$, with a vanishing average value ($\langle V \rangle = 0$), a standard deviation, V_r , and a spatial correlation length, σ_r , significantly smaller than the size of the system. In addition, we assume that the gas is trapped in a confining harmonic trap⁶, $V_h(z) = m\omega^2 z^2/2$ as in almost all current experiments on disordered BECs [14, 15, 16, 17]. We consider a situation such that $\hbar\omega \ll ng_{1D} \ll \hbar^2 n^2/m$, *i.e.* the Bose gas lies in the meanfield regime, and in the absence of disorder, the interactions dominate over the kinetic energy⁷. The situation mimics the experimental conditions of Ref. [15, 17]. The presence of the harmonic confinement introduces a low-momentum cut-off for the phase fluctuations so that the 1D Bose gas forms a true condensate at low temperatures [44, 45]. In this case, the BEC wavefunction is

$$\psi_0 = \sqrt{\mu_0(z)/g_{1D}}, \quad (17)$$

where $\mu_0(z) = \mu - m\omega^2 z^2/2$ is the local chemical potential. This corresponds to an inverted parabolic density profile with a half-size, $L_{TF} = \sqrt{2\mu/m\omega^2}$, where the chemical potential is $\mu = \mu_{TF} = \frac{\hbar\omega}{2} \left(\frac{3Nm g_{1D} l}{2\hbar^2}\right)^{2/3}$, with $l = \sqrt{\hbar/m\omega}$ being the extension of the ground state of the harmonic oscillator.

As $L_{TF} \gg (\xi, \sigma_r)$, it is legitimate to use the local density approximation (LDA) [22], *i.e.* in a region significantly smaller than L_{TF} , the quantities ψ_0 and μ_0 can be considered as uniform. We can thus directly apply the results of section II B. From Eqs. (14)-(17), we immediately find that

$$n(z) \simeq n_0(z) - \frac{\widetilde{V}(z)}{g_{1D}}, \quad (18)$$

⁵ This property follows directly from the definition (10) of the Green function. Indeed, $\int d\mathbf{r} G(\mathbf{r}) = (2\pi)^{d/2} \widehat{G}(\mathbf{k} = 0) = 1$.

⁶ All results also apply if there is no trapping. In this case, all zero-order terms simply do not depend on z .

⁷ This corresponds to the usual TF regime for confined BECs in the absence of disorder [22]. However, no restriction is imposed for the ratio σ_r/ξ , so that the BEC can be out of the TF regime as defined in section I.

where

$$\tilde{V}(z) = \int dz' \frac{\exp\left(\frac{-|z'|}{\xi_0(z)/\sqrt{2}}\right)}{\sqrt{2}\xi_0(z)} V(z-z'), \quad (19)$$

is the screened potential, with $\xi_0(z) = \hbar/\sqrt{2m\mu_0(z)}$ being the local healing length. The density profile of the BEC is thus expected to follow the modulations of a *screened random potential*.

Note that the total number of condensed atoms is $N = \int dz |\sqrt{n_0(z)} + \delta\psi|^2 \simeq \int dz (n_0(z) - \tilde{V}(z)/g_{1D})$ up to first order in \tilde{V}/μ . Since $\langle \tilde{V} \rangle = 0$, one has $\langle N \rangle \simeq \int dz n_0(z)$, owing to the assumed self-averaging property of the potential [41]. In addition, we have $\mu = \mu_{TF}$.

We now compare our predictions to the exact solutions of the GPE (2) as obtained numerically. For the sake of concreteness, we consider a speckle random potential [47] similar to the one used in the recent experiments [14, 15, 16, 17] (see Fig. 1). Briefly, a speckle pattern consists in a random intensity distribution and is characterized by its statistical properties. First, the single-point amplitude distribution is a negative exponential

$$\mathcal{P}[V(z)] = \frac{\exp[-(V(z) + V_R)/V_R]}{|V_R|} \quad \text{if } \frac{V(z)}{V_R} \geq -1$$

and $\mathcal{P}[V(z)] = 0$ otherwise, (20)

corresponding to the average value $\langle V \rangle = 0$ and the standard deviation $\Delta V = \sqrt{\langle [V(z) - \langle V \rangle]^2 \rangle} = |V_R|$. Second, the spatial correlations are characterized by the autocorrelation function $C(\Delta z) = \langle V(\Delta z)V(0) \rangle$ which correlation length is denoted σ_R and can be chosen at will [17, 47]. For the numerical calculations, we numerically generate a 1D speckle pattern using a method similar to the one described in Ref. [48] in 1D and corresponding to the correlation function

$$C(\Delta z) = V_R^2 \left| \text{sinc}(\sqrt{3}\Delta z/\sqrt{2}\sigma_R) \right|^2, \quad (21)$$

where $\text{sinc}(x) = \sin(x)/x$. For the sake of simplicity, it is useful to approximate $C(z)$ to a Gaussian function (see for example section III B). Up to second order in $\Delta z/\sigma_R$, we have $C(\Delta z) \simeq V_R^2 \exp(-\Delta z^2/2\sigma_R)$.

Numerical solutions of the GPE (2) are presented in Fig. 2 for two values of the ratio σ_R/ξ . In the first case (Fig. 2a), we have $\xi \ll \sigma_R$, and the density simply follows the modulations of the bare random potential, according to Eq. (3). In the second case (Fig. 2b), we have $\xi > \sigma_R$, and as expected, the BEC wavefunction does not follow the modulations of the bare random potential $V(z)$ but actually follows smoother modulations of the screened potential $\tilde{V}(z)$.

B. Statistical properties of the screened random potential

It is useful to compute the statistical properties of the screened random potential $\tilde{V}(z)$ as they will be imprinted on

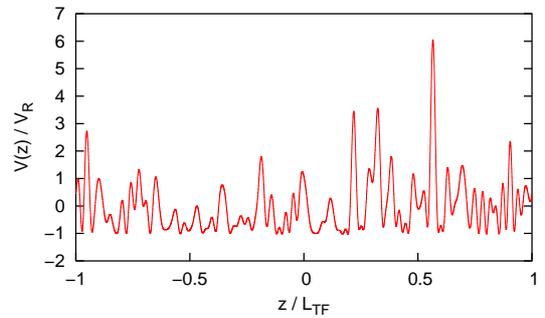


Figure 1: (color online) Example of the realization of a speckle random potential with $\sigma_R \simeq 10^{-2}L_{TF}$.

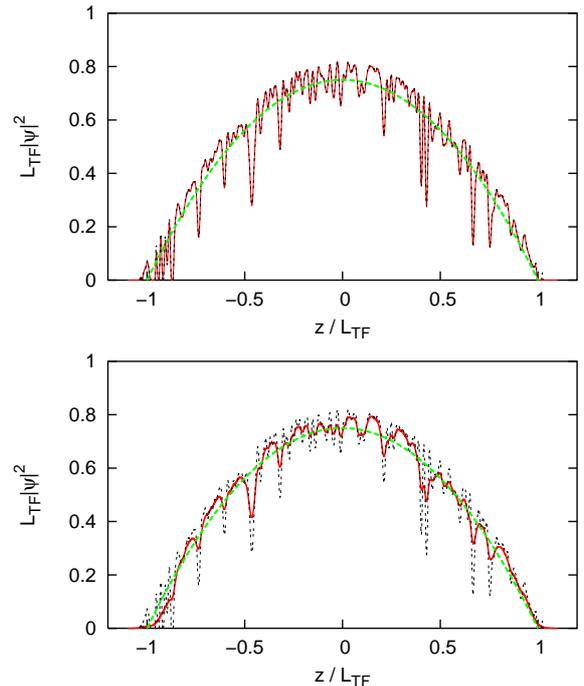


Figure 2: (color online) Density profiles of a BEC confined in a combined harmonic plus random potential ($V_R = 0.1\mu$, $\sigma_R = 7.5 \times 10^{-3}L_{TF}$). The solid (red online) line corresponds to the numerically computed BEC wavefunction; the dashed (green online) line is the TF profile in the absence of disorder; and the black dotted line is a plot of the disordered TF profile [Eq. (3)]. a) Case where the healing length ξ is smaller than the typical length scale of the random potential: $\sigma_R/\xi \simeq 10$. In this case, the density profile follows the modulations of the random potential according to Eq. (3). b) Opposite situation: $\sigma_R/\xi \simeq 0.5$. In this case, the BEC density profile significantly differs from Eq. (3), showing evidence of the screening effect.

the BEC density profile according to Eq. (18). From Eq. (15), we immediately find that, (i) $\tilde{V}(z)$ is a random *homogeneous* potential, (ii) the average value of \tilde{V} vanishes,

$$\langle \tilde{V} \rangle = \langle V \rangle = 0, \quad (22)$$

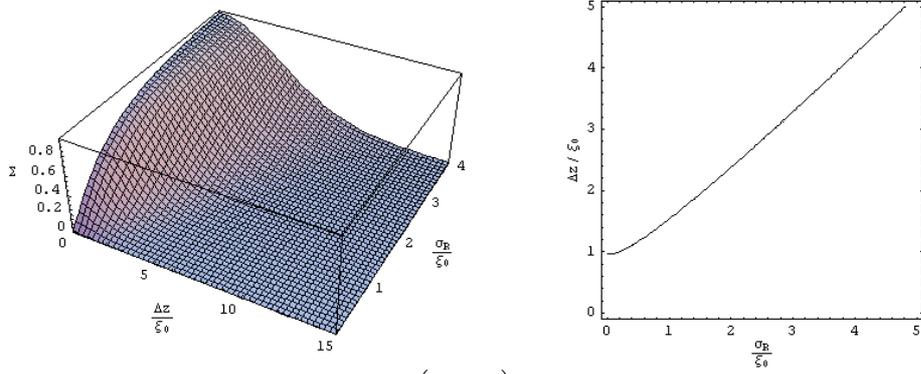


Figure 3: (color online) Left: Plot of the correlation function $\Sigma\left(\frac{\sigma_R}{\xi_0}, \frac{\Delta z}{\xi_0}\right)$. Right: Width at $1/\sqrt{e}$ of the normalized correlation function $\Sigma\left(\frac{\sigma_R}{\xi_0}, \frac{\Delta z}{\xi_0}\right)/\Sigma\left(\frac{\sigma_R}{\xi_0}, 0\right)$.

and the correlation function of \tilde{V} is given by

$$\tilde{C}_z(\Delta z) = \int dudv C[\Delta z + (v-u)]G_z(u)G_{z+\Delta z}(v), \quad (23)$$

where $C(\Delta z) = \langle V(\Delta z)V(0) \rangle$ is the correlation function of the bare potential V and $G_z(u)$ is given by Eq. (11) with ξ replaced by $\xi_0(z)$. In the following, we assume that $\Delta z \ll L_{TF}$ so that $G_z \simeq G_{z+\Delta z}$ and we omit the subscripts. Assuming

for simplicity a Gaussian correlation function for the bare random potential, $C(\Delta z) \simeq V_R^2 \exp(-\Delta z^2/2\sigma_R^2)$, we find after some algebra

$$\tilde{C}(\Delta z) = V_R^2 \Sigma\left(\frac{\sigma_R}{\xi_0}, \frac{\Delta z}{\xi_0}\right), \quad (24)$$

with

$$\begin{aligned} \Sigma(\bar{\sigma}_R, \bar{\Delta z}) &= \bar{\sigma}_R^2 \exp\left(-\frac{\bar{\Delta z}^2}{2\bar{\sigma}_R^2}\right) \\ &+ \frac{\sqrt{\pi}}{4} \bar{\sigma}_R \left(1 - 2\bar{\sigma}_R^2 - \sqrt{2} \bar{\Delta z}\right) \exp\left(\bar{\sigma}_R^2 + \sqrt{2} \bar{\Delta z}\right) \operatorname{erfc}\left(\frac{2\bar{\sigma}_R^2 + \sqrt{2} \bar{\Delta z}}{2\bar{\sigma}_R}\right) \\ &+ \frac{\sqrt{\pi}}{4} \bar{\sigma}_R \left(1 - 2\bar{\sigma}_R^2 + \sqrt{2} \bar{\Delta z}\right) \exp\left(\bar{\sigma}_R^2 - \sqrt{2} \bar{\Delta z}\right) \operatorname{erfc}\left(\frac{2\bar{\sigma}_R^2 - \sqrt{2} \bar{\Delta z}}{2\bar{\sigma}_R}\right) \end{aligned} \quad (25)$$

where $\bar{\sigma}_R = \sigma_R/\xi_0$, $\bar{\Delta z} = \Delta z/\xi_0$ and $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty dt e^{-t^2}$ is the complementary error function. The correlation function $\Sigma\left(\frac{\sigma_R}{\xi_0}, \frac{\Delta z}{\xi_0}\right)$ is plotted in Fig. 3.

This function Σ clearly decreases with σ_R/ξ_0 , indicating the onset of an increasing screening effect. At $\Delta z = 0$, we have a simple asymptotic expression for $\sigma_R \gg \xi_0$:

$$\Sigma(\sigma_R/\xi_0, 0) \simeq 1 - \left(\frac{\xi_0}{\sigma_R}\right)^2, \quad \sigma_R \gg \xi_0. \quad (26)$$

So, as expected, $\Sigma(\sigma_R/\xi_0, 0) \rightarrow 1$ as $\sigma_R/\xi_0 \rightarrow \infty$, *i.e.* the random potential is hardly screened. For $\sigma_R \ll \xi_0$:

$$\Sigma(\sigma_R/\xi_0, 0) \simeq \frac{\sqrt{\pi}}{2} \frac{\sigma_R}{\xi_0}, \quad \sigma_R \ll \xi_0. \quad (27)$$

So, $\Sigma(\sigma_R/\xi_0, 0) \rightarrow 0$ as $\sigma_R/\xi_0 \rightarrow 0$, *i.e.* the amplitude of the screened random potential is significantly reduced compared to the amplitude of the bare random potential. Generally speaking, from Eq. (24), we have $\langle \tilde{V}^2 \rangle = \tilde{C}(0) = V_R^2 \Sigma(\sigma_R/\xi_0, 0)$. It follows that $\langle \tilde{V}^2 \rangle$ is an increasing function of σ_R/ξ_0 and that $\langle \tilde{V}^2 \rangle \leq V_R^2$. This is consistent with the idea of a *screening* of the random potential.

In addition, the correlation length, $\tilde{\sigma}_R$, of the screened random potential, \tilde{V} , is given by the width at $1/\sqrt{e}$ of the function $\Delta z \rightarrow \Sigma(\bar{\sigma}_R/\xi_0, \Delta z/\xi_0)$. At $\sigma_R \gg \xi_0$, the screening is weak and $\tilde{\sigma}_R \simeq \sigma_R$. For $\sigma_R < \xi_0$, the screening is significant, so that $\tilde{\sigma}_R$ saturates at $\tilde{\sigma}_R \simeq \xi_0$, as expected. Roughly speaking, we have $\tilde{\sigma}_R \sim \max(\sigma_R, \xi_0)$ [see Fig. 3].

C. On the effect of disorder in interacting Bose-Einstein condensates

We finally discuss the properties of the BEC wavefunction in the presence of disorder. It follows from Eq. (18) that the BEC density follows the modulations of a random potential \tilde{V} . In the TF regime ($\xi \ll \sigma_r$), $\tilde{V} \simeq V$ while when $\xi > \sigma_r$, \tilde{V} is screened. Since \tilde{V} is a homogeneous random potential, there is no decay of the wavefunction. In particular, Anderson localization does not occur, even for $\xi \gg \sigma_r$. In the case when $\xi > \sigma_r$, it turns out that the BEC density is actually less affected by the random potential than in the TF regime ($\xi \ll \sigma_r$). This is in striking contrast with the case of non-interacting particles where localization effects are usually stronger at low energy [41].

More quantitatively, using the statistical properties of the screened random potential, \tilde{V} , one can easily compute the fluctuations $\Delta n(z) = \sqrt{\langle [n(z) - n_0(z)]^2 \rangle}$ of the BEC density around the average value $n_0(z) = [\mu - m\omega^2 z^2/2]/g_{1D}$. From Eq. (18), we find $\Delta n^2 \simeq \tilde{C}(0)/g_{1D}^2$. Note that Δn^2 depends on the displacement from the trap center through the dependence of ξ_0 on z . At the trap center, we find

$$\Delta n_c = \frac{V_R}{g_{1D}} \sqrt{\Sigma(\sigma_r/\xi, 0)} \quad (28)$$

where $\xi = \xi_0(0) = \hbar/\sqrt{2m\mu}$ is the BEC healing length in the trap center.

As shown in Fig. 4, numerical calculations of the fluctuations of the density in the center of the BEC perfectly agree with Eq. (28) over a large range of the ratio ξ/σ_r . The numerical calculations are performed for the speckle potential described in section III B and no fitting parameter has been used.

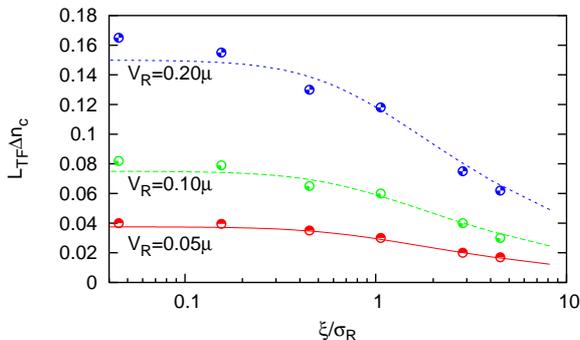


Figure 4: (color online) Amplitude of the fluctuations of the BEC density at the trap center, Δn_c , in the combined harmonic plus random potential as a function of the ratio of the healing length to the correlation length of the disorder for several amplitudes of the random potential. The dots correspond to exact numerical results in the Gross-Pitaevskii approach [Eq. (2)] and the lines show the theoretical prediction [Eq. (28)].

Finally, we find from Eq. (16) that the perturbative approach that we have performed is valid whenever $\Delta n \ll n_0$, *i.e.* whenever

$$V_R \sqrt{\Sigma(\sigma_r/\xi_0, 0)} \ll \mu. \quad (29)$$

Note that this effect is more restrictive in the trap center where ξ_0 is minimum.

IV. CONCLUSION

In summary, we have presented an analytical technique, based on perturbation theory, to compute the static wavefunction of an interacting BEC subjected to a weak potential in the case where the healing length is much smaller than the size of the system. This applies to an arbitrary variation space scale of the potential. In particular, we have shown that when the healing length is larger than the latter, the BEC is sensitive to a *screened potential* which can be determined within our framework.

Applying these results to the case of a 1D random potential, we have shown that the wavefunction of a static interacting BEC is delocalized, similarly as in the TF regime [15]. This is confirmed by numerical calculations. The results of this analysis show that, for an interacting BEC at equilibrium, the larger the healing length, the smaller the perturbation induced by the disorder. It is worth noting that this conclusion holds for *static* BECs in the *meanfield regime* and when the interaction energy dominates over the kinetic energy in the absence of disorder, *i.e.* when the healing length is significantly smaller than the BEC half size ($\xi \ll L$). In contrast, Anderson localization can be expected in very weakly interacting static BECs or in the transport of interacting BECs. For example, localization has been studied in matterwave beams [33] and in the expansion of an interacting BEC [15, 16, 17]. In the latter two cases, localization indeed does occur although non-negligible interactions can modify the usual picture of localization [33, 49].

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