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SUBSPACE-BASED OPTIMAL IV METHOD FOR CLOSED-LOOP SYSTEM IDENTIFICATION

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Abstract: This paper deals with an optimal instrumental variable method dedicated to subspace-based closed-loop system identification. The presented solution is based on the MOESP technique but requires to modify the original scheme by proposing a new PO MOESP method which uses reconstructed past input and past output data as instrumental variables. The developed approach is then illustrated via a simulation example and a comparison with other subspace-based methods.

Keywords: Closed-loop identification, MIMO systems, subspace methods, instrumental variable, optimal estimation.

1. INTRODUCTION

Identification of dynamical systems operating in closed-loop has recently collected much attention. Indeed, for many industrial production processes, safety and production restrictions are often strong reasons for not allowing identification experiments in open loop. The main difficulty in closed-loop identification is due to the correlation between the disturbances and the control signal, induced by the loop. Many results were established during the last decade in the case of linear transfer function models (Van den Hof, 1998; Forssell and Ljung, 1999; Gilson and Van den Hof, 2005).

In parallel, a large number of subspace identification methods (SIM) have been developed since the 90's (see e.g. (Verhaegen, 1994; Van Overschee and De Moor, 1996) and (Viberg, 1995; Bauer, 2005) for relevant overviews). One of the reasons for the success of SIM lies in the direct correspondence between geometric operations on matrices constructed from input/output data and their implementation in terms of well known, stable and reliable algorithms from the field of numerical linear algebra. Lot of them have been extended to the closed-loop case (Verhaegen, 1993*a*; Van Overschee and De Moor, 1997; Chou and Verhaegen, 1997; Katayama *et al.*, 2005) or directly developed in a closed-loop context (Qin and Ljung, 2003; Oku and Fujii, 2004; Huang *et al.*, 2005).

When looking at methods that can consistently identify plant models of systems operating in closed-loop while relying on simple linear (regression) algorithms, instrumental variable (IV) techniques seem to be rather attractive, but at the same time also not very often applied. Furthermore, when comparing the several available

IV algorithms, the principal question to address should be: how to achieve the smallest variance of the estimate. Concerning extended IV methods, an optimal variance result has been developed in the closed-loop context when a transfer function model is sought (Gilson and Van den Hof, 2005). The goal of this paper is to propose an optimal IV estimator dedicated to subspace closed-loop identification.

The paper is organized as follows. After the preliminaries, the optimal closed-loop instrumental variable problem is briefly presented in section 3 and a subspace-based solution is proposed section 4. Then, the performances of the developed algorithm are illustrated via a simulation example and a comparison with others subspace-based closed-loop algorithms is given.

2. PRELIMINARIES

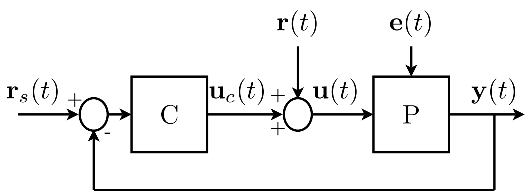


Fig. 1. Closed-loop system setup.

The discrete time closed-loop system configuration that will be considered is sketched in figure 1. The identification setup is the following: $\mathbf{u} \in \mathbb{R}^{n_u \times 1}$ is the process input signal, $\mathbf{y} \in \mathbb{R}^{n_y \times 1}$ the process output signal, $\mathbf{r} \in \mathbb{R}^{n_y \times 1}$ and $\mathbf{r}_s \in \mathbb{R}^{n_u \times 1}$ the reference and setpoint signals respectively. The controller output is denoted by $\mathbf{u}_c \in \mathbb{R}^{n_u \times 1}$, while $\mathbf{e} \in \mathbb{R}^{n_y \times 1}$ represents a white noise signal. It is assumed, without loss of generality, that the setpoint $\mathbf{r}_s = 0$ while an excitation signal is added to the controller output, via $\mathbf{r}$. The process equations are then given by

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{K}\mathbf{e}(t) \quad (1)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{e}(t). \quad (2)$$

The controller equations are defined as

$$\mathbf{x}_c(t+1) = \mathbf{A}_c\mathbf{x}_c(t) - \mathbf{B}_c\mathbf{y}(t) \quad (3)$$
$$\mathbf{u}_c(t) = \mathbf{C}_c\mathbf{x}_c(t) - \mathbf{D}_c\mathbf{y}(t). \quad (4)$$

The process input checks $\mathbf{u}(t) = \mathbf{r}(t) + \mathbf{u}_c(t)$.

2.1 Assumptions

The following assumptions are made on the closed-loop system (Verhaegen, 1993*a*):

- The identification problem is supposed to be well-posed in a sense that the output is uniquely determined by the state of the plant and the controller and by the disturbances and excitation signal. This generic condition is satisfied when $(\mathbf{I}_{n_y} + \mathbf{D}\mathbf{D}_c)$ is non singular (Van Overschee and De Moor, 1997).
- The closed-loop system is internally stable.
- The excitation signal $\mathbf{r}$ and noise $\mathbf{e}$ are mutually uncorrelated.

2.2 Notations

In order to simplify the subspace solution presented further, some notations are introduced. The identification framework considered hereafter is the subspace identification algorithms family gathered under the acronym `MOESP` (Verhaegen, 1994) (see (Haverkamp, 2001) for an interesting overview of the `MOESP` identification schemes). The key problem of this approach consists in consistently estimating the column space of the extended observability matrix of order i defined as

$$\mathbf{\Gamma}_i = \left[\mathbf{C}^T \ (\mathbf{C}\mathbf{A})^T \ \cdots \ \left(\mathbf{C}\mathbf{A}^{i-1}\right)^T\right]^T \quad (5)$$

from measured input/output (I/O) samples. Indeed, from this matrix, it is relatively straightforward to derive (up to a similarity transformation) the state space matrices $[\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{K}]$ by exploiting particular properties of the observability matrix (see e.g. (Viberg, 1995)). For that, let us introduce the following generic notations. Given a sampled data sequence $\{w\}$ with $w \in \mathbb{R}^{n_w \times 1}$, the past and future Hankel matrices are defined as

$$\mathbf{W}_p^-(\bar{t}) = \left[\mathbf{w}_p^-(t) \ \cdots \ \mathbf{w}_p^-(\bar{t})\right] \in \mathbb{R}^{n_w p \times M} \quad (6)$$
$$\mathbf{W}_f^+(\bar{t}) = \left[\mathbf{w}_f^+(t) \ \cdots \ \mathbf{w}_f^+(\bar{t})\right] \in \mathbb{R}^{n_w f \times M} \quad (7)$$

with $\bar{t} = t + M - 1$ and

$$\mathbf{w}_p^-(t) = \left[\mathbf{w}(t-p) \ \cdots \ \mathbf{w}(t-1)\right] \in \mathbb{R}^{n_w p \times 1} \quad (8)$$
$$\mathbf{w}_f^+(t) = \left[\mathbf{w}^T(t) \ \cdots \ \mathbf{w}^T(t+f-1)\right]^T \in \mathbb{R}^{n_w f \times 1} \quad (9)$$

where p and f are user defined integers such as $M >> f, p > n$. Furthermore, let us denote by $\mathbf{H}_i^{\mathbf{u}}$ the block Toeplitz matrix of the impulse responses from $\mathbf{u}$ to $\mathbf{y}$ $\mathbf{H}_i^{\mathbf{e}}$ the block Toeplitz matrix of the impulse responses from $\mathbf{e}$ to $\mathbf{y}$ (Viberg, 1995).

2.3 Problem formulation

Given input, output and reference data $\{\mathbf{u}\}$, $\{\mathbf{y}\}$ and $\{\mathbf{r}\}$ of a closed-loop identification problem, estimate a nominal state space model.

3. OPTIMAL CLOSED-LOOP INSTRUMENTAL VARIABLE METHOD

The direct application of the classical open loop subspace identification algorithms to the I/O measurements $\mathbf{u}, \mathbf{y}$ leads to a biased estimate when the data are collected under feedback. More precisely, the use of past input and/or past output as instrumental variable (IV) gives unreliable estimates because of the correlation between the disturbances and the input signal (see (Ljung and

McKelvey, 1996) for the proof). To overcome this drawback, a particular IV based on the extended instrumental variable techniques (Ljung, 1999) is proposed. To apply this latter, the user has to choose the instruments $\boldsymbol{\xi}$, the number of instruments n_ξ to be used, a weighting matrix $\mathbf{Q}$ and a prefilter $\mathbf{L}(z)$. Moreover, to be used in the subspace identification framework, this instrumental variable has to be uncorrelated with the noise but correlated with the noise free I/O signals. As proved in (Gilson and Van den Hof, 2005), the choice of these design variables may have a considerable effect on the resulting covariance matrix. It has been more particularly shown herein that, in the closed-loop transfer function identification case, there exists a minimum value of the covariance matrix as a function of the design variables under the restriction that $\boldsymbol{\xi}$ is only a function of the external signal $\mathbf{r}$. This optimal IV estimator can be achieved by the following choice of the design variables:

- $n_\xi = 2n_x$, where n_x is the model order,
- $\mathbf{Q} = \mathbf{I}$,
- the instruments are chosen to be equal to the noise free I/O data,
- the filter $\mathbf{L}(z)$ is equal to the inverse of the true noise model.

By analysing these conditions, it is obvious that the optimal IV estimator can only be obtained if the true noise model is exactly known and if the noise free data are available. Since the knowledge of the noise is impossible in the considered noisy environment, optimal accuracy cannot be achieved in practice. Therefore, estimation techniques of the noise free signals and the stochastic part of the system are developed in the following in order to approach this optimality at best.

4. PROPOSITION OF A CLOSED-LOOP SUBSPACE (SUB)OPTIMAL IV METHOD

Since the optimal IV method cannot be achieved in practice, approximate implementations will be considered. For this purpose, it is necessary to take care that

- a noise model is available to construct the prefilter $\mathbf{L}(z)$ and the instruments $\boldsymbol{\xi}$,
- a first model of the process has been estimated to compute the noise free part of the data.

The choice of the instruments and prefilter in the IV method affects the asymptotic variance, while consistency properties are generically secured. This suggests that minor deviations from the optimal value (which is not available in practice) will only cause second order effects in the resulting accuracy. Therefore it is considered to be sufficient to use consistent, but not necessarily efficient estimates of the dynamics and of the noise when constituting the instrument and the prefilter (Ljung, 1999). Additionally, for obtaining the necessary preliminary models, a restriction is made to linear regression estimates in order to keep computational procedures simple and tractable.
Before introducing in details the various algorithms used for the estimation, a quick description of the steps composing the closed-loop subspace identification developed in this article is proposed in the following subsection. Due to the lack of space, it is assumed that the controller operating in the loop is known *a priori*. The case of an unknown controller is technically similar but requires the estimation of the closed-loop system instead of the open-loop one in the first step.

4.1 Overview of the closed-loop subspace identification method

To get an accurate model of the system (1)-(2), the following six steps can be considered:

step 1: estimate a subspace model of the process described in (1)-(2) by using the `Ordinary MOESP` (Verhaegen and Dewilde, 1992) and get the first (biased) estimates $\left\{\hat{\mathbf{A}}_1, \hat{\mathbf{B}}_1, \hat{\mathbf{C}}_1, \hat{\mathbf{D}}_1\right\}$,

step 2: compute the I/O data from the simulation of the model estimated in step 1, on the basis of the reference signal $\mathbf{r}$ and the knowledge of the controller. The instruments $\{\boldsymbol{\xi}_1\}$ are then generated from the past simulated I/O data,

Step 3: determine the new estimate of the process $\left\{\hat{\mathbf{A}}_2, \hat{\mathbf{B}}_2, \hat{\mathbf{C}}_2, \hat{\mathbf{D}}_2, \hat{\mathbf{K}}_2\right\}$ from a modified version of the `PO MOESP` algorithm (Verhaegen, 1994), named `POopt MOESP`, with $\{\boldsymbol{\xi}_1\}$ as instrumental variable,

step 4: compute the noise model
- from $\mathbf{L}(z^{-1}) = \hat{\mathbf{C}}_2\left(z\mathbf{I} - \hat{\mathbf{A}}_2\right)^{-1}\hat{\mathbf{K}}_2 + \mathbf{I}$,
- from the reconstructed input and output,

step 5: generate new instruments $\{\boldsymbol{\xi}_2\}$ following the same method as in step 1 but with the estimates $\left\{\hat{\mathbf{A}}_2, \hat{\mathbf{B}}_2, \hat{\mathbf{C}}_2, \hat{\mathbf{D}}_2\right\}$ and filter the I/O data with the noise model $\hat{\mathbf{L}}(z)$,

step 6: use these instruments $\{\boldsymbol{\xi}_2\}$ and the filtered data to determine the IV estimate of the process $\left\{\hat{\mathbf{A}}_3, \hat{\mathbf{B}}_3, \hat{\mathbf{C}}_3, \hat{\mathbf{D}}_3\right\}$ from the `POopt MOESP` algorithm.

A presentation of the main contributions of this approach is proposed in the following subsections. A particular attention is devoted to the new `POopt MOESP` algorithm.

4.2 The `POopt MOESP` algorithm

The use of the `MOESP` class of methods in a closed-loop framework has received a special care since the beginning of the 90's (Verhaegen, 1993*a*; Chou and Verhaegen, 1997; Chou and Verhaegen, 1999; Zhao and Westwick, 2003; Oku and Fujii, 2004). The study realised in (Chou and Verhaegen, 1997) has more precisely proved that the `PO MOESP` algorithm could be directly applied from data collected in closed-loop if and only if $\mathbf{u}$ is a white noise, property which can't be checked in practice. In this paper, it is proposed to modify the original `PO MOESP` method by introducing reconstructed past input and output $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{y}}$ as instrumental variable. The following theorem is also formulated

Theorem 4.1. Let $\{\mathbf{u}, \mathbf{y}\}$ the I/O data of the system (1)-(2) and $\{\tilde{\mathbf{u}}, \tilde{\mathbf{y}}\}$ the reconstructed I/O sequence computed from the following simulation of the closed-loop system

$$\mathbf{x}(t+1) = \hat{\mathbf{A}}\mathbf{x}(t) + \hat{\mathbf{B}}\tilde{\mathbf{u}}(t) \quad (10)$$

$$\tilde{\mathbf{y}}(t) = \hat{\mathbf{C}}\mathbf{x}(t) + \hat{\mathbf{D}}\tilde{\mathbf{u}}(t) \quad (11)$$

$$\mathbf{x}_c(t+1) = \mathbf{A}_c\mathbf{x}_c(t) - \mathbf{B}_c\tilde{\mathbf{y}}(t) \quad (12)$$

$$\mathbf{u}_c(t) = \mathbf{C}_c\mathbf{x}_c(t) - \mathbf{D}_c\tilde{\mathbf{y}}(t). \quad (13)$$

Furthermore, consider the following QR factorisation

$$\begin{bmatrix} \mathbf{U}_f^+ \\ \tilde{\mathbf{U}}_p^- \\ \tilde{\mathbf{Y}}_p^- \\ \mathbf{Y}_f^+ \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \mathbf{0} & \mathbf{0} \\ \mathbf{R}_{31} & \mathbf{R}_{32} & \mathbf{R}_{33} & \mathbf{0} \\ \mathbf{R}_{41} & \mathbf{R}_{42} & \mathbf{R}_{43} & \mathbf{R}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \\ \mathbf{Q}_3 \\ \mathbf{Q}_4 \end{bmatrix}. \quad (14)$$

Then

$$\lim_{M\to\infty} \frac{1}{\sqrt{M}} \begin{bmatrix} \mathbf{R}_{42} & \mathbf{R}_{43} \end{bmatrix} = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \boldsymbol{\Gamma}_f \mathbf{X}_f \begin{bmatrix} \mathbf{Q}_2^T & \mathbf{Q}_3^T \end{bmatrix}. \quad (15)$$

Proof 1. By using the definition of the Hankel matrices and the equations (1)-(2), it is straightforward to find the following data equation (Viberg, 1995)

$$\mathbf{Y}_f^+ = \boldsymbol{\Gamma}_f \mathbf{X}^+ + \mathbf{H}_f^{\mathbf{u}} \mathbf{U}_f^+ + \mathbf{H}_f^{\mathbf{e}} \mathbf{E}_f^+. \quad (16)$$

Furthermore, from (14), we know that

$$\mathbf{Y}_f^+ = \mathbf{R}_{41}\mathbf{Q}_1 + \mathbf{R}_{42}\mathbf{Q}_2 + \mathbf{R}_{43}\mathbf{Q}_3 + \mathbf{R}_{44}\mathbf{Q}_4. \quad (17)$$

Thus, we get

$$\boldsymbol{\Gamma}_f \mathbf{X}^+ + \mathbf{H}_f^{\mathbf{u}} \mathbf{U}_f^+ + \mathbf{H}_f^{\mathbf{e}} \mathbf{E}_f^+ = \mathbf{R}_{41}\mathbf{Q}_1 + \mathbf{R}_{42}\mathbf{Q}_2 + \mathbf{R}_{43}\mathbf{Q}_3 + \mathbf{R}_{44}\mathbf{Q}_4. \quad (18)$$

By post multiplying both sides of this expression by $\frac{1}{\sqrt{M}} \begin{bmatrix} \mathbf{Q}_2^T & \mathbf{Q}_3^T \end{bmatrix}$, it holds that

$$\frac{1}{\sqrt{M}} \begin{bmatrix} \mathbf{R}_{42} & \mathbf{R}_{43} \end{bmatrix} = \frac{1}{\sqrt{M}} \boldsymbol{\Gamma}_f \mathbf{X}^+ \begin{bmatrix} \mathbf{Q}_2^T & \mathbf{Q}_3^T \end{bmatrix} + \frac{1}{\sqrt{M}} \mathbf{H}_f^{\mathbf{e}} \mathbf{E}_f^+ \begin{bmatrix} \mathbf{Q}_2^T & \mathbf{Q}_3^T \end{bmatrix} \quad (19)$$

since $\mathbf{U}_f^+ = \mathbf{R}_{11}\mathbf{Q}_1$ and $\mathbf{Q}_i\mathbf{Q}_j^T = \mathbf{0}$ for $i \neq j$. The proof will be completed if we succeed in showing that the last term of the right hand side of (19) vanishes when $M \to \infty$. For that purpose, note first of all that, by construction, the noise free past input and output data are uncorrelated with the future innovation:

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \tilde{\mathbf{U}}_p^{-\,T} = \lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \left(\mathbf{Q}_1^T \mathbf{R}_{21}^T + \mathbf{Q}_2^T \mathbf{R}_{22}^T \right) = \mathbf{0} \quad (20)$$

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \tilde{\mathbf{Y}}_p^{-\,T} = \lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \left(\mathbf{Q}_1^T \mathbf{R}_{31}^T + \mathbf{Q}_2^T \mathbf{R}_{32}^T + \mathbf{Q}_3^T \mathbf{R}_{33}^T \right) = \mathbf{0}. \quad (21)$$

Furthermore, the past simulated I/O data (the instruments) are uncorrelated with the future system input measurements. Then

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{U}_f^+ \tilde{\mathbf{U}}_p^{-\,T} = \mathbf{0}, \quad \lim_{M\to\infty} \frac{1}{M} \mathbf{U}_f^+ \tilde{\mathbf{Y}}_p^{-\,T} = \mathbf{0}. \quad (22)$$

By using the factorisation (14), we can rewrite the previous equations as follows

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{R}_{11} \mathbf{R}_{21}^T = \mathbf{0}, \lim_{M\to\infty} \frac{1}{M} \mathbf{R}_{11} \mathbf{R}_{31}^T = \mathbf{0} \quad (23)$$

since $\mathbf{Q}_i\mathbf{Q}_j^T = \mathbf{0}$ for $i \neq j$ and $\mathbf{Q}_i\mathbf{Q}_i^T = \mathbf{I}$. Then, by assuming that the reference signal is persistently excited, it can be supposed that the input $\mathbf{u}$ asymptotically verifies the same property (Zhao and Westwick, 2003). From this hypothesis, it can be asserted that the matrix $\lim_{M\to\infty} \frac{1}{\sqrt{M}} \mathbf{R}_{11}$ is invertible. By combining this particularity with equations (23), we have

$$\lim_{M\to\infty} \frac{1}{\sqrt{M}} \mathbf{R}_{21} = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \mathbf{R}_{31} = \mathbf{0}. \quad (24)$$

As a consequence of this fact, equations (20) and (21) can be simplified as follows

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \mathbf{Q}_2^T \mathbf{R}_{22}^T = \mathbf{0} \quad (25)$$

$$\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \left(\mathbf{Q}_2^T \mathbf{R}_{32}^T + \mathbf{Q}_3^T \mathbf{R}_{33}^T \right) = \mathbf{0}. \quad (26)$$

By using a similar argument for $\tilde{\mathbf{u}}$, it can be supposed that the matrix $\lim_{M\to\infty} \frac{1}{\sqrt{M}} \mathbf{R}_{22}$ is invertible. Thus, equation (25) can be rewritten as $\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \mathbf{Q}_2^T = \mathbf{0}$. Consequently, this relation implies that $\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \mathbf{Q}_3^T \mathbf{R}_{33}^T = \mathbf{0}$ (cf. equ. (26)). Because of the noise contribution, the matrix $\lim_{M\to\infty} \frac{1}{\sqrt{M}} \mathbf{R}_{33}$ is invertible. We also obtain $\lim_{M\to\infty} \frac{1}{M} \mathbf{E}_f^+ \mathbf{Q}_3^T = \mathbf{0}$, which completes the proof.

It is interesting to notice that, according to the fact that the input $\mathbf{u}$ is not white, the most natural approach would consist in considering the following modified version of the QR factorisation developed in paragraph 4.2 of (Chou and Verhaegen, 1997) (see e.g. (Zhao and Westwick, 2003) for a second contribution)

$$\begin{bmatrix}\mathbf{U}_f^+ \\ \mathbf{Y}_f^+\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{U}}_p^{-\,T} & \tilde{\mathbf{Y}}_p^{-\,T}\end{bmatrix} = \begin{bmatrix}\mathbf{R}_{11} & \mathbf{0} \\ \mathbf{R}_{21} & \mathbf{R}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{Q}_1 \\ \mathbf{Q}_2\end{bmatrix}. \quad (27)$$

However, in our case, such an implementation would give ill conditioned estimates since the past noise free I/O data are asymptotically uncorrelated with the future I/O measurements.

5. SIMULATION EXAMPLE

The following numerical example is used to illustrate the performances of the proposed method. The example is partially borrowed from (Hakvoort, 1990) and is also used in (Verhaegen, 1993*b*) and (Van Overschee and De Moor, 1997). The plant corresponds to a discrete-time model of a laboratory plant set-up of two circular plates rotated by an electrical servo motor with flexible shafts. The closed-loop system set-up is the one displayed on figure 1. The plant has a state-space description as in (1) and (2) with

$$\mathbf{A} = \begin{pmatrix} 4.40 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4.00 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.0033 \\ -0.00002 \end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbf{K}^T = \begin{pmatrix} 2.3 & -6.64 & 7.515 & -4.0146 & 0.86336 \end{pmatrix}$$

$D = 0$ and e_k is a Gaussian white noise sequence. Note that the plant has one integrator, and therefore is only marginally stable. The controller has a state-space description as in (3) and (4) with

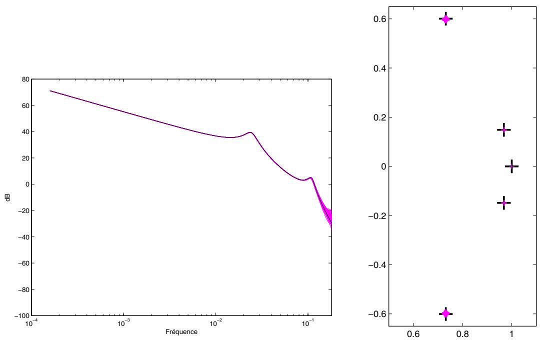
$$\mathbf{A}_c = \begin{pmatrix} 2.65 & -3.11 & 1.75 & -0.39 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{B}_c = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\mathbf{C}_c = (-0.4135 \ 0.8629 \ -0.7625 \ 0.2521), \ D_c = 0.61.$$

The excitation signal r_k is a Gaussian white noise sequence with variance 1. Firstly, the example is used to validate the performance of the proposed closed-loop subspace optimal IV (`sivcl`) algorithm. The process model is estimated on the basis of closed-loop data sequences of length $N = 1000$. Monte Carlo simulation of 100 experiments has been performed for a signal to noise ratio

$$SNR = 10 \log \left(\frac{P_{y_d}}{P_e}\right) = 15 \quad \text{dB}, \qquad (28)$$

where P_x denotes the power of the signal x and y_d is the noise-free output signal.
In figure 2(a), the Bode diagrams of the 100 models identified by the `sivcl` algorithm are represented and the poles of these estimated models are gathered in figure 2(b).
It can be seen from both the Bode diagrams and pole plots that the developed method gives indeed unbiased and accurate results.



(*a*) Bode diagram (*b*) Poles

Fig. 2. Real (black) and estimated (magenta) Bode diagrams and poles over the 100 Monte Carlo simulation runs, `sivcl` algorithm, $SNR = 15dB$.

The proposed algorithm is now compared with two other subspace techniques:

- the closed-loop subspace method proposed by Ljung and McKelvey (1996), referenced here as `sscl`,
- the closed-loop `4sid` solution proposed by Van Overschee and De Moor (1997), referenced here as `4sidcl`.

The process parameters are estimated on the basis of closed-loop data sequences of length $N = 1000$. Monte Carlo simulation of 100 experiments has been performed for $SNR = 25$dB.
In figure 3, the pole of the 100 models identified by the three methods are represented. It can be seen that in presence of relatively high noise, the `sscl` algorithm fails to give unbiased results, while the `4sidcl` and `sivcl` methods give unbiased and good results. The `sivcl` algorithm seems to give estimates with smaller covariance compare to the `4sidcl` algorithm. However, it can be noted that 2 poles are difficult to estimate, since they operate in high frequency and with very low weight.

6. CONCLUSION

An optimal IV algorithm dedicated to the closed-loop subspace-based identification problem has been developed. Moreover, a new `PO MOESP` algorithm which makes use of reconstructed past input and past output data as instrumental variables has also been proposed. This method has then been compared to other subspace-based closed-loop techniques which are known to lead to unbiased plant estimates in closed-loop. However, for arriving at estimates with attractive variance properties, it is preferably to apply bootstrap IV methods as considered in this paper.

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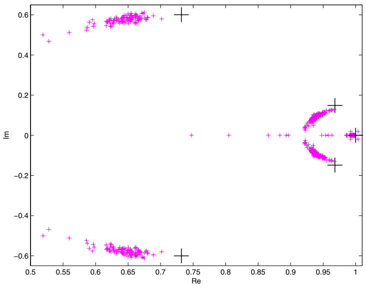
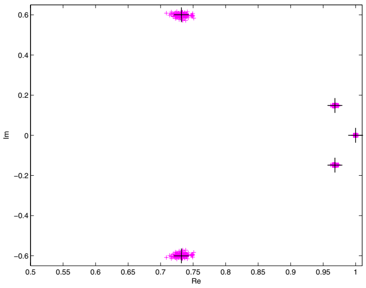
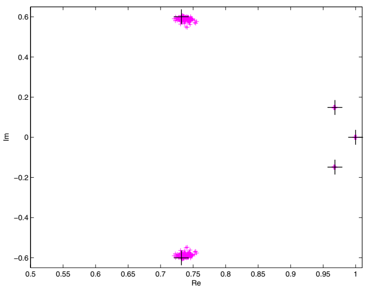


(a) `sscl`



(b) `4sidcl`



(c) `sivcl`

Fig. 3. Real (black) and estimated (magenta) poles over the 100 Monte Carlo simulation runs, $nsr = 25dB$

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