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# HOW CRACKS MODIFY THE SPECTRUM OF A BEAM

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## Introduction

In recent years several authors have tried to answer to this simple question: if a structure is cracked, can this be observed from qualitative modifications in the dynamics, and in particular from modifications in the frequency of the periodic vibrations of the system (see the theoretical works of Conley and Zehnder, (1984), Ekeland and Hofer, (1987), Dell Antonio et al., (1995), Kunze, (2000), and the more practical work of Lan Zuo, (1992)). From an experimental point of view, it seems clear that a modification appears in the spectrum of a cracked beam (see Ovigne et al. (2001), (2003a), (2003b), (2003c)). In particular, one can observe an asymmetry and a large decay of the spectrum.

The aim of this paper is to try to modelize numerically the spectrum of a cracked beam. In the first part of the paper general mathematical results due to M. Schatzman are presented. In the second part of the paper, the mechanical problem is presented. In the third part an original finite element is proposed in order to take into account a crack in the beam. The fourth part is devoted to numerical results. Conclusion are given at the end of the paper.

## The mathematical results due to M. Schatzman

We consider the following perturbed problem in finite dimension:

$$\ddot{Mu} + Ku + \varepsilon f(u) = 0 \quad (1)$$

in the neighborhood of harmonic vibrations. We denote respectively by  $M$  and  $K$  the mass and stiffness matrices; they are symmetric positive definite. Let  $\omega_j$  be the square roots of the eigenvalues of  $M^{-1}K$ , with

$$0 < \omega_1 < \dots < \omega_d, \quad (2)$$

with corresponding  $M$ -orthonormal eigenvectors  $v_j$ :

$$K v_j = \omega_j^2 M v_j. \quad (3)$$

We assume that  $f$  is a Lipschitz continuous function from  $\mathbb{R}^d$  to itself, which is positively homogeneous of degree 1. The dynamical system can be written:

$$\ddot{Mu} + Ku + \varepsilon f(u) = 0. \quad (4)$$

The only periodic solutions of (4) for  $\varepsilon = 0$  are given by:

$$A \cos(\omega_j t + \phi_j) v_j. \quad (5)$$

We seek periodic solutions which are close to the ones defined by (5). We suppose that our solutions are of the form:

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$$(i) u(t) = w(\rho t), (ii) w = w_0 + \varepsilon w_1 + \dots, (iii) \rho = 1 + \varepsilon \rho_1 + \dots \quad (6)$$

We substitute (6ii) in (4) and we perform the change of variable  $\rho t = s$ . Equation (4) becomes:

$$\rho^2 \ddot{M}w^\varepsilon + K w^\varepsilon + \varepsilon f(w^\varepsilon) = 0. \quad (7)$$

We substitute (6ii) and (6iii) in (7) and we equate the terms of equal degree in  $\varepsilon$ . We find at order 0:

$$\ddot{M}w_0 + K w_0 = 0, \quad (8)$$

and at order 1:

$$2\rho_1 \ddot{M}w_0 + \ddot{M}w_1 + K w_1 + f(w_0) = 0. \quad (9)$$

We observe that:

$$w_0(t) = A \cos(\omega_j t + \phi_j) v_j. \quad (10)$$

Equation (9) can have a periodic solution if and only if  $2\rho_1 \ddot{M}w_0 + f(w_0)$  is orthogonal to  $\cos(\omega_j t) v_j$  and to  $\sin(\omega_j t) v_j$ .  $\rho_1$  is determined by the following relation:

$$2\rho_1 \omega_j^2 \int_0^{2\pi/\omega_j} \cos(\omega_j t) v_j^T M w_0 dt = \int_0^{2\pi/\omega_j} \cos(\omega_j t) v_j^T f(w_0) dt \quad (11)$$

$$\text{or} \quad \rho_1 = \frac{1}{\pi \omega_j} \left( \int_0^{\pi/2\omega_j} \cos^2(\omega_j t) v_j^T f(v_j) dt - \int_0^{2\pi/\omega_j} \cos^2(\omega_j t) v_j^T f(-v_j) dt \right) \quad (12)$$

Equation (12) gives the perturbation of the eigenfrequency  $w_j$  due to the perturbation  $\varepsilon f$ . This result can be proved rigorously using a bifurcation analysis.

### Statement of the mechanical problem

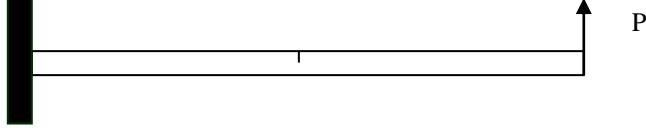


Figure 1: an example of cracked beam

We consider a cracked beam with only one crack (figure 1). In the numerical applications, the length of the beam is 77 cm, the dimension of its section are 4 cm and 8.5 cm, the beam is supposed to beam embedded on its left part and loaded on its right part during a short time, the density is equal to  $2400 \text{ N/ms}^2$  and the Young modulus is equal to 30 GPa (concrete). The crack is supposed to be close to the middle of the beam. The real position of the crack and the value of the loading will be précis in the following. The problem is to analyze the spectrum of this beam and to compare it to the spectrum of a safe beam.

### Numerical method

In order to eliminate instabilities and percussions effects, we have developed an original finite element which takes into account the crack **in the element**. Classically, the finite element approximation uses cubic Hermite elements. We have developed a piecewise quadratic Hermite element (see the two first Hermite functions on figure 2). This element takes into account the discontinuity of the rotation at a given point. The safe part of the structure is meshed by classical elements, the cracked part by the new element. In order to model the opening and the closing of the crack, we have proceeded in two steps. If the crack is open, we suppose that the stiffness of the element is small (10 or 100 times lower than the stiffness of the safe beam). If the crack is closed, we consider that the stiffness is equal to the safe one. This process can be performed implicitly or

explicitly by a test on the curvature or on the gap of rotation. Note that integration is done by an implicit second order Euler schema. The algorithm is given in table 1.

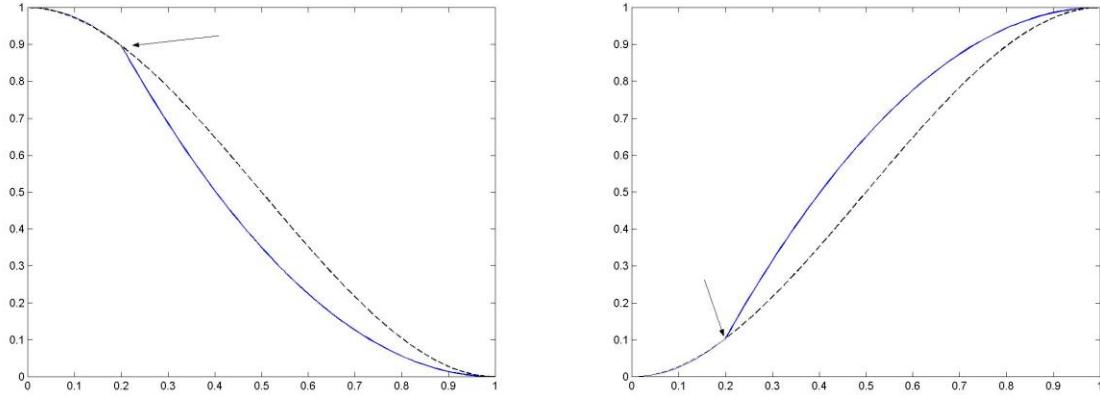


Figure 2: The two first piecewise quadratic Hermite functions (compared to the classical ones)

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Begin
Step 0 Initialization
  u(0) and u'(0) given
  Fixed matrix computation
  Non-fixed matrix computation (safe case)
  Resolution (one step of Euler schema)
Step 1 to k Normal step
  Test on the crack status (opening or closing)
  If (crack closed) then
    Non-fixed matrix computation (safe case)
  Else (crack open)
    Non-fixed matrix computation (unsafe case)
  End if
  Resolution (one step of Euler schema)
  If (k < Kmax) then k = k + 1 goto Step k
End

```

Table 1: Algorithm (explicit version)

### Numerical Examples

In this section, we present only a small part of our numerical results. We present on figure 3 the beam with an “equivalent” stiffness of the crack equal to 3 GPa. In the first computation (figure 3a) we have supposed that the crack is placed to a distance of 37.5 cm of the left of the beam. In the second computation, we have changed this distance (37.5 cm, 38.5 cm, 39.5 cm). We observe a dissymmetry in the amplitude of the displacement and the importance of the position of the crack. The differences between the safe and the unsafe beams are magnified observing the Fourier transform (figure 4).

### Conclusions

In this paper, we have developed a numerical procedure to study the modifications of the spectrum of a cracked beam. Our numerical results show in particular, that the response is less smooth and the value of the Fourier transform is not equal to zero in zero.

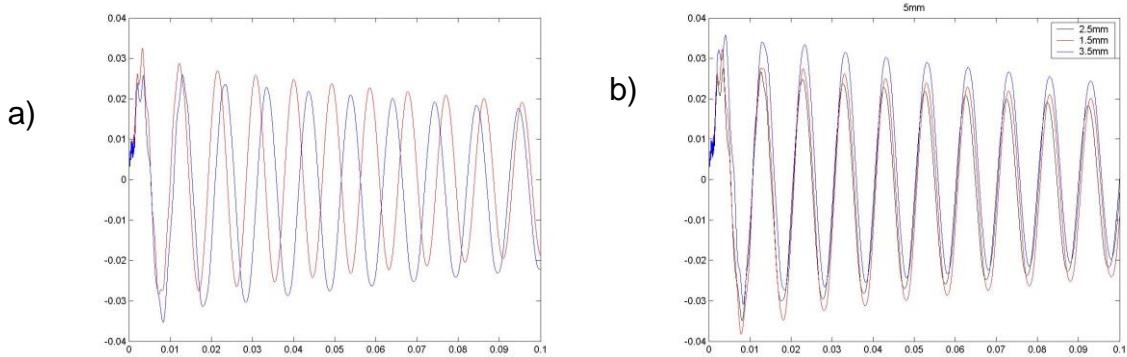


Figure 3: modification of the displacement response a) comparison safe/unsafe b) three positions of the crack

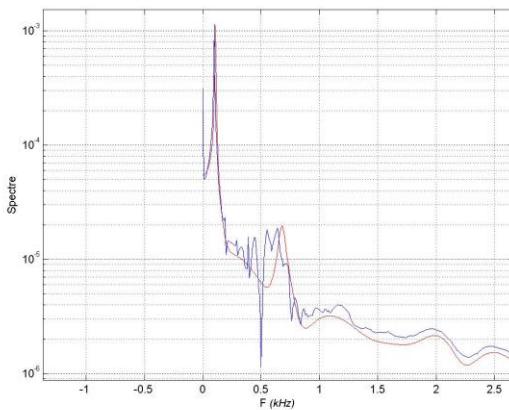


Figure 4: modification of the spectrum (comparison safe/unsafe)

## References

Conley, C. & Zehnder, E. (1984) Morse-type index theory for flows and periodic solutions for Hamiltonian equations, *Com. Pure Appl. Math.*, 37(2), 207-253.

Dell'Antonio, G., D'Onofrio, B., & Ekeland, I. (1995) Periodic solutions of elliptic type for strongly nonlinear Hamiltonian systems, In *The Floer memorial volume*, Birkhauser, Basel, 327-333.

Ekeland, I. & Hofer, H. (1987) Convex Hamiltonian energy surfaces and their periodic trajectories, *Comm. Math. Phys.* 113(3), 419-469.

Kunze, M. (2000) *Non-smooth dynamical systems*, Springer-Verlag, Berlin.

Lan Zuo (1992) Etude du comportement dynamique des systèmes linéaires par morceaux – Contribution à la détection des fissures dans les arbres de machines tournantes, PhD Thesis, Ecole Polytechnique Fédérale de Lausanne.

Ovigne, P.A., Massenzio, M., Jacquelain, E., & Hamelin, P. (2001) Analytical model for the prediction of the natural frequencies of a cracked beam with composite reinforcement, *Composites in Construction International Conference, CCC 2001*, Porto, Portugal, 10-12 octobre 2001.

Ovigne, P.A., Massenzio, M., Jacquelain, E., & Hamelin, P. (2003a) Analytical model for the prediction of the eigen modes of a beam with open cracks and external strengthening, *Steel & Comp. Struct.*, 3(1), 33-46.

Ovigne, P.A., Massenzio, M., Jacquelain, E., & Hamelin, P. (2003b) Crack-bridging force transfer of composite strengthening: a dynamical point of view, *Struct. Engng & Mech.*, 15(4), 437-449.

Ovigne, P.A., Massenzio, M., Jacquelain, E., & Hamelin, P. (2003c) Eigenfrequencies of a cracked RC-beam with composite strengthening, *ACI (submitted)*.