



HAL
open science

Propagation of transient acoustic signals in layered porous media: fractional equations for scattering operators

Claude Depollier, Zine El Abiddine E.A. Fellah, Mohamed Fellah

► To cite this version:

Claude Depollier, Zine El Abiddine E.A. Fellah, Mohamed Fellah. Propagation of transient acoustic signals in layered porous media: fractional equations for scattering operators. *Nonlinear Dynamics*, 2005, 38, pp.181-190. 10.1007/s11071-004-3754-8 . hal-00088186

HAL Id: hal-00088186

<https://hal.science/hal-00088186>

Submitted on 23 Mar 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

Propagation of Transient Acoustic Waves in Layered Porous Media: Fractional Equations for the Scattering Operators

CLAUDE DEPOLLIER¹, ZINE EL ABIDINE FELLAH², and MOHAMED FELLAH^{3,*}

¹Laboratoire d'Acoustique de l'Université du Maine CNRS UMR6613, Ave O. Messiaen, F 72085 Le Mans Cedex 9 France;

²INSERM, unité 556, 151 Cours Albert Thomas, F 69424 Lyon Cedex 03, France; ³Laboratoire de Physique Théorique,

Institut de Physique USTHB BP 32 El Alia, Bab Ezzouar 16111, Algeria;

*Author for correspondence (e-mail: fellah1@yahoo.com; fax: +33-2-43833520)

Abstract. Acoustic waves scattering from a rigid air-saturated porous medium is studied in the time domain. The medium is one dimensional and its physical parameters are depth dependent, i.e., the medium is layered. The loss and dispersion properties of the medium are due to the fluid-structure interaction induced by wave propagation. They are modeled by generalized susceptibility functions which express the memory effects in the propagation process. The wave equation is then a fractional telegraphist's equation. The two relevant quantities are the scattering operators – transmission and reflection operators – which give the scattered fields from the incident wave. They are obtained from Volterra equations which are fractional equations for the scattering operators.

Key words: fractional calculus, wave splitting

1. Introduction

When a sound wave travels in a rigid air-saturated porous medium, the dispersive effects are due to the frequency dependence of the complex functions of the physical parameters of the medium thereafter referred to as the generalized susceptibilities which describe the fluid-structure interactions. The analysis of the propagation of transient waves in such media raises two different problems: the direct and the inverse scattering problems. The direct scattering problem is that of determining the scattered fields that arise when a known incident field impinges on a scattering medium with known physical properties. The inverse scattering problem consists of the problem of determining the physical properties from the scattered fields produced by a known incident wave. In most cases, the determination of medium parameters is done in the frequency domain. The reflection and transmission coefficients which contain the information on the physical parameters of the medium are obtained from the Fourier transform of the scattered signals divided by the Fourier transform of the incident signal. Unfortunately, the coefficients of the wave equation are nonlinear functions of the parameters of the medium which are to be solved to achieve the right values of the generalized susceptibilities. Such solution obtained from the Fourier transform procedure might not reproduce important qualitative features of the solutions and produce results which may be difficult to use in applications. So, the frequency domain methods are limited in probing the inhomogeneous media when parameters are depth dependent.

In the present paper a time domain method is presented for finding the dispersion relation from a time domain experiment. The advantages of time domain methods over frequency domain methods are their ability: (i) to solve the inverse scattering problem; (ii) to give causal solution to the direct scattering problem.

In time domain, a dispersive medium is characterized by susceptibility kernels which are the Fourier transforms of the generalized susceptibilities. They act as memory functions on modeling the relaxation processes which develop in the medium: the viscous and the thermal effects.

The time domain method presented in this paper is only approximate [1]. In the time domain, the full wave equation in a dispersive and lossy medium is difficult to be solved. So, we deal with wave equations simpler than the full one that retain the most significant features of the underlying physics and remain analytically tractable.

Two of us (Z.E.A. F. and C.D.) have developed a model of memory functions which describe the loss and the dispersion properties of the medium on a relevant frequency interval [1]. In high frequency range, dispersion and attenuation of waves are well described by a fractional power of the frequency. So, by Fourier transform, we have shown that the behavior of the waves is determined by a Fractional Telegraph Equation (FTE). To investigate the wave scattering by a layered medium, the relevant quantities are the operators which map the incident wave in the reflected and transmitted ones. Reflection and transmission operators are defined by their kernels which are the solutions of differential equations. In an inhomogeneous porous medium, we show that the scattering kernels satisfy coupled fractional differential equations. The paper is organized as follows. In Section 2 the basic elements of the physical derivation of the FTE are presented. The fluid equivalent model of the propagation in porous media is introduced and the FTE is derived. Section 3 describes the layered porous medium and the wave splitting. In Section 4 the scattering operators are defined and their fractional differential equations are worked out.

2. Derivation of the FTE

2.1. THE FLUID EQUIVALENT MODEL OF POROUS MEDIA

We consider a slab of homogeneous porous material which occupies the region between $x = 0$ and $x = l$ (x is the Cartesian coordinate in the direction perpendicular to the slab interfaces). In our model the slab is characterized by constant parameters. The incident wave is planar and normally incident. It propagates in the positive x direction and impinges the slab at $x = 0$.

In air saturated porous media, the structure is assumed to be motionless: the acoustic waves travel only in the fluid filling the pores. The wave propagation is described by the equivalent fluid model which is a particular case of the Biot's theory. In this model, the interactions between the fluid and the structure are taken into account in two frequency dependent response factors which are the generalized susceptibilities: the dynamical tortuosity of the medium $\alpha(\omega)$ [2] and the dynamical compressibility of the air included in the medium $\beta(\omega)$ [3]. These two response factors are complex functions which heavily depend on the frequency $f = \omega/2\pi$. These functions represent the deviation from the behavior of the fluid in the free space as the frequency increases. Their theoretical expressions are given by Johnson [2], Allard [3] and Lafarge [4]:

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{\eta\phi}{j\omega\alpha_\infty\rho_f k_0} \sqrt{1 + j \frac{4\alpha_\infty^2 k_0^2 \rho_f \omega}{\eta\Lambda^2 \phi^2}} \right), \quad (1)$$

$$\beta(\omega) = \gamma - (\gamma - 1) \left(1 + \frac{\eta\phi}{j\omega\rho_f k_0' Pr} \sqrt{1 + j \frac{4k_0^2 \rho_f \omega Pr}{\eta\phi^2 \Lambda^2}} \right)^{-1/2}, \quad (2)$$

where $j^2 = -1$, γ represents the adiabatic constant, Pr the Prandtl number, α_∞ the tortuosity, k_0 the static permeability, k'_0 the thermal permeability and Λ and Λ' the viscous and thermal characteristic lengths [5, 6]. This model was initially developed by Johnson, Koplik and Dashen [5], and completed by Champoux and Allard by adding the description of thermal effects [6]. Later on, Lafarge has introduced the parameter k'_0 which describes the additional damping of sound waves due to the thermal exchanges between fluid and structure at the surface of the pores [4].

The functions $\alpha(\omega)$ and $\beta(\omega)$ express the viscous and thermal exchanges between the air and the structure which are responsible of the sound damping in acoustic materials. These exchanges are due on the one hand to the fluid-structure relative motion and on the other hand to the air compressions-dilatations produced by the wave motion. The part of the fluid affected by these exchanges can be estimated by the ratio of a microscopic characteristic length of the media, as for example the sizes of the pores, to the viscous and thermal skin depth thickness $\delta = (2\eta/\omega\rho_0)^{1/2}$ and $\delta' = (2\eta/\omega\rho_0 Pr)^{1/2}$. For the viscous effects this domain corresponds to the region of the fluid in which the velocity distribution is perturbed by the frictional forces at the interface between the viscous fluid and the motionless structure. For the thermal effects, it is the fluid volume affected by the heat exchanges between the two phases of the porous medium, the solid skeleton being seen as a thermostat.

2.2. THE FRACTIONAL TELEGRAPH EQUATION

The high frequency approximations of $\alpha(\omega)$ and $\beta(\omega)$ ($\omega \rightarrow \infty$) are given by the relations:

$$\alpha(\omega) \approx \alpha_\infty \left(1 + \frac{2}{\Lambda} \left(\frac{\eta}{j\omega\rho_f} \right)^{1/2} \right), \quad (3)$$

$$\beta(\omega) \approx 1 + \frac{2(\gamma - 1)}{\Lambda'} \left(\frac{\eta}{j\omega Pr\rho_f} \right)^{1/2}. \quad (4)$$

In these equations, the factor $(j\omega)^{-1/2}$, expresses in the frequency domain the semiintegral operator defined as [7]

$$I^{1/2}\psi(x) = \frac{1}{\sqrt{\pi}} \int_a^x \frac{\psi(\tau)}{\sqrt{x-\tau}} d\tau, \quad (5)$$

where a is real valued. In the time domain, the susceptibilities are operators [7] and their asymptotic expressions are given in [1] as:

$$\tilde{\alpha}(t) = \alpha_\infty \delta(t) + \frac{2\alpha_\infty}{\Lambda} \left(\frac{\eta}{\rho_f} \right)^{1/2} t^{-1/2}, \quad (6)$$

$$\tilde{\beta}(t) = \delta(t) + \frac{2(\gamma - 1)}{\Lambda'} \left(\frac{\eta}{Pr\rho_f} \right)^{1/2} t^{-1/2}. \quad (7)$$

In each of these equations the first term in the right hand side is the instantaneous response of the medium while the second term is the memory function. In electromagnetism, the instantaneous response is called optical response. It describes all the processes which cannot be resolved by the signal. In this framework,

the basic equations of the acoustic waves propagation along the ox axis are:

$$\rho_f \tilde{\alpha}(t) * \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} \quad (8)$$

$$\frac{\tilde{\beta}(t)}{K_a} * \frac{\partial p}{\partial t} = -\frac{\partial v_x}{\partial x}, \quad (9)$$

where K_a is the bulk modulus of the air, p is the acoustic pressure and v the particle velocity. $*$ is the shorthand notation for the time convolution

$$(f * g)(t) = \int_0^t f(t-t')g(t')dt'. \quad (10)$$

For a causal function $v(x, t)$, the wave equation is deduced from these equations as the FTE:

$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 v}{\partial t^2} + A \int_0^t \frac{\partial^2 v / \partial t'^2}{\sqrt{t-t'}} dt' - B \frac{\partial v}{\partial t} = 0, \quad (11)$$

where A and B are the following constants:

$$A = \frac{2\alpha_\infty}{K_a} \sqrt{\frac{\rho_f \eta}{\pi}} \left(\frac{1}{\Lambda} + \frac{\gamma-1}{\sqrt{Pr} \Lambda'} \right) \quad (12)$$

$$B = \frac{4\alpha_\infty(\gamma-1)\eta}{K_a \Lambda \Lambda' \sqrt{Pr}} \quad (13)$$

c is the velocity $c = c_0 / \sqrt{\alpha_\infty}$ in the porous medium, and $c_0 = \sqrt{\rho_f / K_a}$ is the wave velocity in the free space. The other coefficients govern the spreading and the attenuation of the waves. During the last decade, FTE has been investigated by many authors [8, 9]. Fractional differential equations appear more and more frequently in different research areas and engineering applications. They are derived by the generalization of classical equations and they seem to be an essential complementary tool in the description of a lot of phenomena.

3. Layered Slab of Porous Material

3.1. THE BASIC EQUATION

We consider now a slab of inhomogeneous porous material between $x = 0$ and $x = l$. The slab parameters are x -dependent, i.e., the medium is layered. However, in $0 < x < l$, it is assumed that the parameters of the medium are continuous to avoid jump discontinuities in the operators kernels. In a layered medium, the coefficients A , B and c of Equation (11) are no more constant. They are written as $A = a(x)$, $B = b(x)$ and $c = c(x)$. Then the FTE has the following form

$$(\partial_x^2 - c(x)^{-2} \partial_t^2 - b(x) \partial_t - a(x) \partial_t^{3/2}) v(x, t) = 0, \quad (14)$$

where $\partial_x v(x, t) = \partial v(x, t) / \partial x \dots$

3.2. WAVE SPLITTING

To split the wave, we put the Equation (14) in the matrix form:

$$\partial_x \begin{pmatrix} v(x, t) \\ \partial_x v(x, t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ c^{-2}(x)\partial_t^2 & 0 \end{pmatrix} \begin{pmatrix} v(x, t) \\ \partial_x v(x, t) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ b(x)\partial_t + a(x)\partial_t^{3/2} & 0 \end{pmatrix} \begin{pmatrix} v(x, t) \\ \partial_x v(x, t) \end{pmatrix} \quad (15)$$

Equation (15) describes the evolution of vector $(v, \partial_x v)^t$ (t denotes the transpose) when x varies on the direction of propagation. Since $v(x, t)$ and $\partial_x v(x, t)$ are linearly independent quantities, they form a basis of a two-dimensional space from which any propagating wave can be generated. In the wave splitting, quantities of interest are two independent linear combinations of $v(x, t)$ and $\partial_x v(x, t)$: i) which form a new basis of the space; ii) which are convenient for physical interpretations. A particular choice is $\{v^+, v^-\}$ defined by the transformation:

$$\begin{pmatrix} v^+(x, t) \\ v^-(x, t) \end{pmatrix} = S \begin{pmatrix} v(x, t) \\ \partial_x v(x, t) \end{pmatrix} \quad \text{where } S = \begin{pmatrix} 1 & -c^{-1}(x)\partial_t^{-1} \\ 1 & c^{-1}(x)\partial_t^{-1} \end{pmatrix}.$$

$v^\pm(x, t)$ are the split fields which satisfy the relation $v(x, t) = v^+(x, t) + v^-(x, t)$. The dynamical equations for these fields are deduced from Equation (15) in the following form:

$$\partial_x \begin{pmatrix} v^+(x, t) \\ v^-(x, t) \end{pmatrix} = \mathcal{M} \begin{pmatrix} v^+(x, t) \\ v^-(x, t) \end{pmatrix}, \quad (16)$$

where

$$\mathcal{M} = SMS^{-1} - S(\partial_x S^{-1}) + SNS^{-1}, \quad (17)$$

$$M = \begin{pmatrix} 0 & 1 \\ c^{-2}(x)\partial_t^2 & 0 \end{pmatrix}, \quad (18)$$

$$N = \begin{pmatrix} 0 & 0 \\ b(x)\partial_t + a(x)\sqrt{\pi}\partial_t^{3/2} & 0 \end{pmatrix}. \quad (19)$$

The wave splitting converts the matrix M into diagonal form. So, during their travel through the material, the split fields are mixed by the $S(\partial_x S^{-1})$ and SNS^{-1} matrices i.e., by the losses and dispersion processes:

$$SMS^{-1} = \frac{\partial}{c\partial t} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (20)$$

$$\begin{aligned} S(\partial_x S^{-1}) &= -S^{-1}(\partial_x S) \\ &= \frac{1}{2} \frac{c'(x)}{c(x)} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \end{aligned} \quad (21)$$

$$SNS^{-1} = \frac{c(x)}{2} [b(x) + a(x)\sqrt{\pi}\partial_t^{1/2}] \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \quad (22)$$

So, the dynamic equation of the split fields is

$$\frac{\partial}{\partial x} \begin{pmatrix} v^+ \\ v^- \end{pmatrix} = \left[\begin{pmatrix} -\frac{\partial_t}{c(x)} - \frac{1}{2}c(x)a(x)\sqrt{\pi}\partial_t^{1/2} - \frac{1}{2}c(x)a(x)\sqrt{\pi}\partial_t^{1/2} \\ \frac{1}{2}c(x)a(x)\sqrt{\pi}\partial_t^{1/2} \frac{\partial_t}{c(x)} + \frac{1}{2}c(x)a(x)\sqrt{\pi}\partial_t^{1/2} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \right] \begin{pmatrix} v^+ \\ v^- \end{pmatrix}, \quad (23)$$

with

$$\delta_{11} = -\delta_{21} = -\frac{1}{2} \frac{c'(x)}{c(x)} - \frac{b(x)c(x)}{2}, \quad (24)$$

$$\delta_{12} = -\delta_{22} = \frac{1}{2} \frac{c'(x)}{c(x)} - \frac{b(x)c(x)}{2}. \quad (25)$$

4. Scattering Operators

4.1. SCATTERING REPRESENTATION

To describe the scattering properties of the slab of porous material, the scattering operators are introduced [10]. For a specific incidence from $x < 0$, the incident wave $v^i(x, t)$ which propagates in the positive x direction, is of v^+ type. In the region $x < 0$, there is furthermore a reflected wave $v^r(x, t)$ which propagates in the negative x direction, i.e., which is of the v^- type. v^r may be interpreted as a response of the slab to the incident wave v^i . The Duhamel's principle establishes that there exist a linear relation between v^r and v^i namely $v^r(0, t) = \mathcal{R}(0, l, t)v^i(0, t)$ in the form [11].

$$v^r(0, t) = \int_{-\infty}^t R(0, l, t - t')v^i(0, t')dt'. \quad (26)$$

$R(0, l, t)$ is the reflection kernel which refers to the slab between $x = 0$ and $x = l$. In the same way we can introduce the transmission operator $\mathcal{T}(l, t)$ which relates the transmitted wave v^t and the incident wave v^i :

$$v^t(l, t) = v^i(0, t) + \int_{-\infty}^t T(0, l, t - t')v^i(0, t')dt', \quad (27)$$

where $T(0, l, t)$ is the kernel of \mathcal{T} and $v^t(l, t)$ is a v^+ type wave such that

$$v^t(l, t) = v^+(l, t + \tau(0, l)), \quad (28)$$

with $\tau(x_1, x_2)$ is the time it takes for the wave to travel from $x = x_1$ to $x = x_2$:

$$\tau(x_1, x_2) = \int_{x_1}^{x_2} \frac{dx'}{c(x')}. \quad (29)$$

The influence of the slab $0 < x < l$ is expressed through the arguments $(0, l \dots)$ of the kernels R and T . They indicate that these kernels depend of the whole slab between the two faces $x = 0$ and $x = l$. They are interpreted as the responses of the slab to a δ -function excitation:

$$v^i(x, t) = \delta(x - c_0 t)$$

4.2. IMBEDDING EQUATIONS FOR THE SCATTERING OPERATORS

When the slab is layered, we cannot consider just the global scattering kernels. By global kernels we understand functions which depend on the whole slab. We must also determine the variations of the scattering kernels through the slab. For that we define the local scattering kernels $R(x_1, x_2, t)$ and $T(x_1, x_2, t)$ relative to the subslab between $x = x_1$ and $x = x_2$. It is assumed that the subslab is embedded between two half-spaces specified by their velocities $c(x_1)$ and $c(x_2)$. The global kernels are obtained as the limits of the local kernels when x_1 and x_2 approach, respectively 0 and l . Stokes was the first to develop this approach [12] referred as invariant imbedding method.

4.2.1. Reflection Operator

The local reflection operator is defined by the equation

$$v^-(x, t) = R(x, t) * v^+(x, t). \quad (30)$$

Applying ∂_x to (30) leads to:

$$\partial_x v^-(x, t) = \partial_x R(x, t) * v^+(x, t) + R(x, t) * \partial_x v^+(x, t). \quad (31)$$

Thus, $\partial_x v^\pm$ are reported from the dynamic equation of the split fields to give:

$$\begin{aligned} 0 = & \frac{1}{2}a(x)c(x)\sqrt{\pi}\partial_t^{1/2}[v^+(x, t) + v^-(x, t)] + \frac{1}{c(x)}\partial_t v^-(x, t) \\ & + \delta_{21}v^+(x, t) + \delta_{22}v^-(x, t) - \partial_x R(x, t) * v^+(x, t) - \delta_{11}R(x, t) * v^+(x, t) \\ & - \delta_{12}R(x, t) * v^-(x, t) + \frac{1}{2}a(x)c(x)\sqrt{\pi}R(x, t) * \partial_t^{1/2}[v^+(x, t) + v^-(x, t)] \\ & + \frac{1}{c(x)}(R(x, t) * \partial_t v^+(x, t)). \end{aligned} \quad (32)$$

Using the following relations:

$$R(x, t) * \partial_t v^+(x, t) = R(x, 0)v^+(x, t) + \partial_t R(x, t) * v^+(x, t) \quad (33)$$

$$\partial_t v^-(x, t) = 2R(x, t) * v^+(x, t) + R(x, 0)v^+(x, t) \quad (34)$$

Equation (32) becomes:

$$\begin{aligned} 0 = & \frac{1}{2}a(x)c(x)\sqrt{\pi}\partial_t^{1/2}[v^+(x, t) + R(x, t) * v^+(x, t)] \\ & + \frac{2}{c(x)}\partial_t R(x, t) * v^-(x, t) + \frac{1}{c(x)}R(x, 0)v^+(x, t) \\ & + \delta_{21}v^+(x, t) + \delta_{22}v^-(x, t) - \partial_x R(x, t) * v^+(x, t) \\ & + \frac{1}{c(x)}R(x, 0) * v^+(x, t) + \frac{1}{c(x)}\partial_t R(x, t) * v^+(x, t) \\ & + \frac{1}{2}a(x)c(x)\sqrt{\pi}R(x, t) * \partial_t^{1/2}[v^+(x, t) + R(x, t) * v^+(x, t)] \\ & - \delta_{11}R(x, t) * v^+(x, t) - \delta_{12}R(x, t) * R(x, t) * v^+(x, t). \end{aligned} \quad (35)$$

In these equations the fractional derivatives $\partial_t^{1/2} v^\pm(x, t)$ can be written as the following convolutions:

$$\partial_t^{1/2} v^+(x, t) = -\frac{1}{2\sqrt{\pi}} \frac{1}{t^{3/2}} * v^+(x, t) \quad (36)$$

$$\partial_t^{1/2} v^-(x, t) = -\frac{1}{2\sqrt{\pi}} \frac{1}{t^{3/2}} * R(x, t) * v^+(x, t). \quad (37)$$

A term by term identification of (35) gives:

– from the constant terms:

$$R(x, 0) = -\frac{1}{4}(c'(x) + b(x)c^2(x)) \quad (38)$$

– from the convolution terms

$$\begin{aligned} \partial_x R(x, t) - \frac{3}{c(x)} \partial_t R(x, t) = & -\frac{1}{2} \left[\frac{c'(x)}{c(x)} - a(x)c(x) \right] R(x, t) * R(x, t) \\ & - \frac{1}{4} a(x)c(x) \left[\frac{1}{t^{3/2}} + \frac{2}{t^{3/2}} * R(x, t) + \frac{1}{t^{3/2}} * R(x, t) * R(x, t) \right] \\ & + b(x)c(x)R(x, t), \end{aligned} \quad (39)$$

with the conditions:

$$R(x, 0) = -\frac{1}{a}[c'(x) + b(x)c^2(x)] \quad \text{and} \quad R(l, t) = 0. \quad (40)$$

4.2.2. Transmission Operator

The transmission operator is defined by the equation

$$v^+(x, t + \tau(L, x)) = a(l, x)v^+(x, t) + T(x, t) * v^+(x, t), \quad (41)$$

where $a(l, x)$ is the attenuation factor. The differential equation for transmission operator is determined in the same way that the one of the reflection operator. After a straightforward calculation and the term by term identification one finds the following equations:

– from the constant terms

$$\partial_x a(l, x) = -\delta_{11} a(l, x), \quad (42)$$

or

$$a(l, x) = \sqrt{\frac{c(x)}{c(l)}} \exp\left(-\frac{1}{2} \int_x^l b(x')c(x')dx'\right); \quad (43)$$

– from the coefficient of $\partial_t v^+(x, t)$

$$\partial_x \tau(l, x) = -\frac{1}{c(x)}, \quad (44)$$

leading to

$$\tau(l, x) = \int_x^l \frac{dx'}{c(x')}; \quad (45)$$

– from the convolution terms

$$\begin{aligned} \partial_x T(x, t) + \frac{1}{c(x)} \partial_t T(x, t) = & -\delta_{11} T(x, t) - \delta_{12} \{a(l, x) R(x, t) + T(x, t) * R(x, t)\} \\ & - \frac{1}{4} \left\{ a(l, x) \frac{1}{t^{3/2}} + a(l, x) \frac{1}{t^{3/2}} * R(x, t) \right. \\ & \left. + \frac{1}{t^{3/2}} * R(x, t) + \frac{1}{t^{3/2}} * R(x, t) * T(x, t) \right\} \end{aligned} \quad (46)$$

with the condition $T(0, t) = 0$.

Note that Equations (26) and (46) which contains R and T are coupled nonlinear differential equations. Equation (26) contains only the unknown $R(x, t)$. They both contain fractional derivatives which model the memory effects which develop in the porous medium when the waves propagate in it.

These equations may be solved analytically when the profiles of parameters variations are tractable [13] or by numerical methods [14] as for example finite difference method or the method of characteristics [15].

5. Conclusion

In this paper the fractional differential equations for the scattering operators have been established. However, they are deduced from the fractional telegraph equation which describes the transient acoustics signals propagation in a layered porous medium. This model is well adapted to investigate the propagation of ultrasonic pulses in acoustics porous materials. The model is restricted to a one-dimensional problem to reduce the inverse scattering problem but more general situations like obliquely incident wave may be tackled in the same way.

Acknowledgements

We would like to thank the anonymous referees for their invaluable suggestions.

References

1. Fellah, Z. E. A. and Depollier, C., 'Transient acoustic wave propagation in rigid porous media: A time-domain approach', *Journal of the Acoustical Society of America* **107**, 2000, 683–688.
2. Johnson, D. L., 'Recent developments in the acoustic properties of porous media', in *Proceedings of the International School of Physics "Enrico Fermi" Course XCIII Frontiers in Physical Acoustics*, D. Sette (ed.), North Holland, 1986, pp. 255–290.
3. Allard, J. F., *Propagation of Sound in Porous Media: Modeling Sound Absorbing Materials*, Chapman & Hall, London, 1993, pp. 255–290.
4. Lafarge, D., Lemarnier, P., Allard, J. F., and Tarnow, V., 'Dynamic compressibility of air in porous structures at audible frequencies', *Journal of the Acoustical Society of America* **102**, 1997, 1995–2006.
5. Johnson, D. L., Koplik, J., and Dashen, R., 'Theory of dynamic permeability and tortuosity in fluid-saturated porous media', *Journal of Fluid Mechanics* **176**, 1987, 379–402.
6. Champoux, Y. and Allard, J. F., 'Dynamic tortuosity and bulk modulus in air-saturated porous media', *Journal of Applied Physics* **70**, 1991, 1975–1979.
7. Samko, S. G., Kilbas, A. A., and Marichev, O. I., *Fractional Integrals and Derivatives*, Gordon and Breach, Yverdon, Switzerland, 1993.
8. Hanyga, A. and Rok, V., 'Wave propagation in micro-heterogeneous porous media: A model based on an integro-differential wave equation', *Journal of the Acoustical Society of America* **107**, 2000, 2965–2972.
9. Cascaval, R. C., Eckstein, E. C., Frota, C. L., Goldstein, J. A., 'Fractional telegraph equations', *Journal of Mathematic Analysis and Application* **276**, 2002, 145–159.

10. Kristensson, G. and Krueger, R. J., 'Direct and inverse scattering in the time domain for a dissipative wave equation: I. Scattering operators', *Journal of Mathematical Physics* **27**, 1986, 1667–1682; Kristensson, G. and Krueger, R. J., 'Direct and inverse scattering in the time domain for a dissipative wave equation: II. Simultaneous reconstruction of dissipation and phase velocity profiles', *Journal of Mathematical Physics* **27**, 1986, 1683–1693.
11. Courant, R. and Hilbert, D., *Methods of Mathematical Physics*, Vol. 2, Interscience, New York, 1962.
12. Stokes, G. C., 'On the intensity of the light reflected and transmitted through a pile of plates', in *Mathematical and Physical Papers*, Vol. 2, Cambridge University Press, Cambridge, 1883.
13. Fella, Z. E. A., Depollier, C., and Fella, M., in preparation.
14. Kirsh, A. and Kress, R., 'Two methods for solving the inverse acoustic scattering problem', *Inverse Problems* **4**, 1988, 749–770.
15. Weng, C. C., *Waves and Fields in Inhomogeneous Media*, IEEE Press, New York, 1995.