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1 Introduction

In many applications, for instance the simulation of the behaviour of ferromagnetic materials, the demagnetization field, approximation of the Maxwell contribution, has to be computed (see [1, 4]). One of the applications aimed by this paper is the simulation of ferromagnetic periodic layers. This type of layers could be, for example, structured poly-cristals or textured thin layers of dispersed ferromagnetic dots (see for instance [2]). In these two cases, in order to compute the evolution of a periodic magnetization, it becomes necessary to carefully take into account the demagnetization contribution. Indeed, the extension of non-periodic computations to periodic computations is not straightforward in the case of demagnetization field. In the remainder of the paper, we will denote $H_d(m)$ the demagnetization contribution and m the magnetization. The magnetization m is a vector field from $\mathbb{R}^3$ to $\mathbb{R}^3$ which support is reduced to Ω, the open bounded set of $\mathbb{R}^3$ occupied by the ferromagnetic material. Then, the demagnetization field is deduced from the magnetization field by the formula

$$\left\{ \begin{array}{l} \mathbf{rot}(H_d(m)) = 0, \\ \mathrm{div}(H_d(m)) = \mathrm{div}(\tilde{m}), \end{array} \right. \tag{1}$$

in the sense of distribution on $\mathbb{R}^3$ where $\tilde{m}$ padding to zero of m on the whole space. The application so built $H_d(m)$ is non local, it is to say that in order to compute the contribution H_d in one point of the space, you need to know the magnetization in the whole space. This particularity can be easily seen using the the following representation formula

$$H_d(m) = \int_{\mathbb{R}^3} \mathbf{grad}\ \mathrm{div}\ \left(G(x-y)m(y)\right) dy,$$

where G is the Green kernel of the Laplacian in the whole space. In a previous work (see [5]) the fast computation of the demagnetization field has been treated for bounded domains of $\mathbb{R}^3$. The aim of this paper is to extend the result to periodic domain preserving the effectiveness of the method on bounded domains.

The main difficulty is that, in order to compute the demagnetization field on a periodic domain, one must know the magnetization on the whole infinite domain. To mesh this domain is obviously impossible, we have to build a good approximation of the far magnetization contributions necessary to compute the demagnetization at a given point of the domain.

The idea developped in this paper is to use a multi-grid mesh: the finer mesh will sharply discretize one period of the magnetic domain, afterward, we build a succession of grids whose superposition give a diadic mesh of the space. Then, two key points are used: the first is the fact that the magnetization is periodic and then it is possible to approximate it on each grid, the second is that the demagnetization field operator depends only on the shape of the domain and not on its size. This last property permits to preserve the same computation method in order to obtain the demagnetization field on each level of the multi-grid mesh.

In a first part, we will propose a mathematical framework to justify the computation of the demagnetization field on a periodic domain and to give a sense to the micromagnetism equations in this context. In a second part, we will expose the proposed multi-grid method. The last part will be dedicated to the numerical experimentations.

2 Problematic and equations

2.1 A mathematical framework for micromagnetism in a periodic domain

In a bounded domain, the dimensionless micromagnetism equations can be written as follows : find m in $H^1(\Omega\times]0,T[,\mathbb{R}^3)$ such that, for every m_0 in $H^1(\Omega,S^2)$ (S^2 is the unit sphere of $\mathbb{R}^3$) one has

$$\begin{cases} \frac{\partial m}{\partial t} = -m\wedge H(m) - \alpha m\wedge(m\wedge H(m)), \forall(x,t)\in\Omega\times]0,T[, \\ m(x,0)=m_0(x), \forall x\in\Omega, \\ H(m)=H_a(m)+A\triangle m+H_d(m), \end{cases}$$

where α is a strictly positive real number, H_a designate the anisotropy contribution (non differential and purely local in our case), A is the exchange constant and we set, in the sense of distributions on $\mathbb{R}^3$ valued in $\mathbb{R}^3$

$$\begin{cases} \mathbf{rot}(H_d(m))=0, \\ \mathrm{div}(H_d(m))=-\mathrm{div}(\tilde{m}). \end{cases} \tag{2}$$

We can define an energy for this system using the following formula: for every m in $H^1(\Omega\times]0,T[,\mathbb{R}^3)$, one has:

$$e(m) = -\int_\Omega H(m).m\ dx.$$

In the case of a periodic domain, the previous definitions can not be used directly. We must adapt the framwork as follows: given Ω_Π, a periodic domain of period $\widetilde{\Omega}$ and Ω a bounded domain included in $\widetilde{\Omega}$. We then set, for all separable Banach space W

$$H^1_\Pi(\Omega_\Pi,W)=\{u\in\Pi(\widetilde{\Omega},W)\mid u_{|\Omega}\in H^1(\Omega,W)\},$$

$$L^2_\Pi(\Omega_\Pi,W)=\{u\in\Pi(\widetilde{\Omega},W)\mid u_{|\Omega}\in L^2(\Omega,W)\},$$

where $\Pi(\widetilde{\Omega},W)$ designates the periodic functions, of period $\widetilde{\Omega}$ and valued in W. It is obvious that this space is not included neither in $L^2(\mathbb{R}^3)$ nor in $H^1(\mathbb{R}^3)$, then, most of the definitions given on a bounded domain become invalid. The Landau Lifshitz system is re-written as follows: find m in $H^1_\Pi(\Omega_\Pi\times]0,T[,\mathbb{R}^3)$ such that $supp(m_{|\widetilde{\Omega}})=\Omega$ and, for every m_0 in $H^1_\Pi(\Omega_\Pi,S^2)$ (S^2 is not a convex subset of $\mathbb{R}^3$, then $H^1_\Pi(\Omega,S^2)$ is not a space but a manifold) one has

$$\begin{cases} \frac{\partial m}{\partial t} = -m\wedge H_\Pi(m) - \alpha m\wedge(m\wedge H_\Pi(m)), \forall(x,t)\in\Omega_\Pi\times]0,T[, \\ m(x,0)=m_0(x), \forall x\in\Omega_\Pi, \\ H_\Pi(m)=H_{a,\Pi}(m)+A\triangle_\Pi m+H_{d,\Pi}(m), \end{cases}$$

$H_{a,\Pi}(m)$ is the field of $\Pi(\widetilde{\Omega},\mathbb{R}^3)$ such that, for every point x in Ω one has $H_{a,\Pi|\Omega}(m)(x) = H_a(m_{|\Omega})(x)$ and the vector vanishes on all other points. In the same way, we define the vector field $\triangle_\Pi(m)$ of $\Pi(\widetilde{\Omega},\mathbb{R}^3)$ by $\triangle_\Pi(m)_{|\Omega}=\triangle m$ which vanishes elsewhere in $\widetilde{\Omega}$. The demagnetization is then naturally given by the formula (2) applied to a periodic vector field m, but, this formula is not valid for all types of periodic domains as explained in the next subsection. For admissible domains, the demagnetization field obtained via formula (2) is an element of $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$ as proved in the next subsection.

Let suppose that the domain Ω_Π is an admissible periodic domain for the computation of the demagnetization field, the demagnetization energy can be written as follows:

$$e_\Pi = -\int_\Omega H_\Pi(m).m\ dx.$$

For the anisotropic and exchange contributions, there is no major modifcation of the bounded case. But, as shown in the preceeding formula, the demagnetization energy contribution involves the demagnetizing field due to one period but also the radiation of the field from all the periods of the infinite domain. Then, in order to compute the demagnetization field on a periodic domain, one has to compute the limit of a serie. The admissible domains will be those such that this serie converges.

2.2 Demagnetization field: the admissible domains

The first question is: given the micromagnetism model, what are admissible periodic domains, it is to say domains on whom one can define an energy. In this part we will then give a definition of the energy for periodic domain and prove that admissible periodic domains have no more than two periodic directions. The first property we will need is the decay of the magnetic field given in the following lemma

Lemma 1. *Given m in $L^2(\mathbb{R}^3)$ such that $supp(m) = \omega$ where ω is a compact set of $\mathbb{R}^3$, then, the demagnetization field radiate by m outside of ω is smooth. Furthermore, there exists C_1 and C_2, strictly positive real numbers, such that for every x in $(\mathbb{R}^3\backslash\bar{\omega})^2$ and $\lambda > 1$*

$$C_1 \left(\frac{d(\omega, x)}{d(\omega, \lambda x)}\right)^3 \leq \frac{|H_d(m)(x)|}{|H_d(m)(\lambda x)|} \leq C_2 \left(\frac{d(\omega, x)}{d(\omega, \lambda x)}\right)^3,$$

where $d(\omega, x) = sup_{z\in\omega}|x - z|$.

Proof. As proven in [5], for all m in $L^2(\Omega)$, $P_{0,h}(m)$ converges to m when h vanishes where $P_{0,h}$ is the constant piecewise projection of m and moreover, $H_d(P_{0,h}(m))$ tends to $H_d(m)$. Then, let consider m, a constant vector field on a spheric domain of radius η, the generalization to a an indifferent smooth domain is straightforward.
Let m a constant vector filed on the ball $B(0, \eta)$ of center 0 and radius η and x a given point of $\mathbb{R}^3$ not in $\overline{B(0, \eta)}$, the demagnetizing field of m is given by:

$$\begin{aligned} H_d(m)(x) &= \frac{-1}{4\pi} \int_{B(0,\eta)} \mathbf{grad}_x \mathrm{div}_x \frac{m}{|x-y|} dy, \\ &= \frac{-1}{4\pi} \left(\int_{B(0,\eta)} \frac{1}{|x-y|^3} K(x-y) dy \right) m, \end{aligned}$$

where K is the matrix is the application from $\mathbb{R}^3$ into the space of real matrices of order 3 defined by, for all $x = (x_1, x_2, x_3)^t \in \mathbb{R}^3$:

$$K(x) = \begin{pmatrix} -1 + \frac{3x_1^2}{|x|^2} & \frac{3x_1x_2}{2|x|^2} & \frac{3x_1x_3}{2|x|^2} \\ \frac{3x_1x_2}{2|x|^2} & -1 + \frac{3x_2^2}{|x|^2} & \frac{3x_2x_3}{2|x|^2} \\ \frac{3x_1x_3}{2|x|^2} & \frac{3x_2x_3}{2|x|^2} & -1 + \frac{3x_3^2}{|x|^2} \end{pmatrix}.$$

Then, let now compute the demagnetizing field of λx where λ is a real number strictly superior to one. We have

$$\begin{aligned}
|H_d(m)(\lambda x)| &= |\frac{-1}{4\pi}\left(\int_{B(0,\eta)} \frac{1}{|\lambda x - y|^3} K(\lambda x - y) dy\right) m|, \\
&= |\frac{-\lambda^3}{4\pi}\left(\int_{B(0,\frac{\eta}{\lambda})} \frac{1}{|\lambda x - \lambda y|^3} K(\lambda x - \lambda y) dy\right) m|, \\
&= |\frac{-\lambda^3}{4\lambda^3\pi}\left(\int_{B(0,\frac{\eta}{\lambda})} \frac{1}{|x - y|^3} K(x - y) dy\right) m|, \\
&= |\frac{-1}{4\lambda^3\pi}\left(\int_{B(0,\eta)} \frac{1}{|x - \frac{y}{\lambda}|^3} K(x - \frac{y}{\lambda}) dy\right) m|, \\
&\leq \frac{C}{\lambda^3}|H_d(m)(x)|.
\end{aligned}$$

The other part of the inequality is obtain identically using underestimations in the Riemann integral obtained after the projection phase of the limit process.

□

Then, using the decay properties of the demagnetization field and a summation over concentric spheres we prove that only bi-periodic magnetic domains are admissible:

Definition 1. *A periodic domain Ω_Π is said to be admissible for the demagnetization energy if for every m in $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$ one has*

$$0 \leq -\int_\Omega H_{\Pi,d}(m).m \; dx < \infty.$$

In the remainder of the paper, we will focus the study on rectangular domains Ω_Π. These domains are such that their period $\widetilde{\Omega}$ has the following form

$$\widetilde{\Omega} =]-L_x, L_x[\times]-L_y, L_y[\times]-L_z, L_z[,$$

where L_x, L_y and L_z are strictly positive reals, eventually infinite. for example, a domain such that L_x, L_y and L_z are infinite is nothing more than a non periodic domain.

Proposition 1. *Rectangular periodic domains such that at least one of the three parameters L_x, L_y or L_z is infinite are admissible domains.*

Proof. As shown in Lemma 1, the demagnetizing field has a strong decreasing property. We will use this property in order to prove the theorem. Let Ω_Π be a periodic domain of period $\tilde{\Omega}$ and I a set of indices who designates the periods as follows: for every i in I, Ω_i is a transformation of $\tilde{\Omega}$. Then, we can write the periodic demagnetizing energy as follows, for every m in $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$:

$$\begin{aligned}
e_{d,\Pi}(m) &= -(H_{d,\Pi}(m), m)_{\tilde{\Omega}} \\
&= -\sum_{i\in I}(H_d(\chi_i \; m), m)_{\tilde{\Omega}},
\end{aligned}$$

where χ_i is the characteristic function the domain Ω_i. Then, setting for every Ω_1 and Ω_2, open bounded sets of $\mathbb{R}^3$

$$d(\Omega_1, \Omega_2) = \inf_{x\in\Omega_1} \sup_{y\in\Omega_2} |x - y|,$$

we can apply the Lemma 1 and obtain the following estimation

$$C_1 \sum_{i\in I} \left(\frac{d(\tilde{\Omega})}{d(\tilde{\Omega}, \Omega_i)} \right)^3 \leq e_{d,\Pi}(m) \leq C_2 \sum_{i\in I} \left(\frac{d(\tilde{\Omega})}{d(\tilde{\Omega}, \Omega_i)} \right)^3,$$

and, organizing grouping of cells Ω_j whose $d(\tilde{\Omega}, \Omega_i)$ are close, we obtain the following estimate

$$\sum_{i=0}^{\infty} A_i \frac{C_1}{i^3} \leq e_{d,\Pi}(m) \leq \sum_{i=0}^{\infty} A_i \frac{C_2}{i^3},$$

A_i is the number of cells such that $\frac{d(\tilde{\Omega})}{d(\tilde{\Omega},\Omega_i)}$ is close to i. For instance, when the domain is periodic in one direction, $A_i = K$, when the domain is periodic in two dimension, $A_i = Ki^2$ and in three direction $A_i = i^3$. Hence, to ensure that the energy is finite, one the number of periodic direction has to be equal to 1 or 2. $\square$

In the remainder of the paper we will focus on admissible rectangular periodic domains, it is to say domains which period is at least infinite in one direction of the space. For those domains, we set

Definition 2. *Given Ω_Π an admissible rectangular periodic domain whose non infinite parameters are L_x or L_x and L_y (up to a rotation, it is always possible to be in this case), then, for every (i, j) in $\mathbb{Z}^2$, one has*

$\Omega_i = \{x \in \mathbb{R}^3 | x - (2(i-1)L_x)e_x \in \widetilde{\Omega}\}$ *if Ω has two infinite directions,*

$\Omega_{i,j} = \{x \in \mathbb{R}^3 | x - [(2(i-1)L_x)e_x + (2(j-1)L_y)e_y] \in \widetilde{\Omega}\}$ *if Ω has only one infinite direction.*

2.3 Some properties of the demagnetization field on periodic rectangular domains

On a non periodic domain, the demagnetization operator is a projector in sense of L^2 norm on divergence free space functions; it means that the kernel of this operator is the set of divergence free functions in L^2. This point is quite important and will have to be carefully treated by the discretization.

Then, one can prove the following result

Proposition 2. *For every m in $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$, one has*

$$H_{d,\Pi}(H_{d,\Pi}(m)) = H_{d,\Pi}(m).$$

Moreover,

$$Ker(H_{d,\Pi}) = \{u \in L^2_\Pi(\Omega_\Pi, \mathbb{R}^3) | div(u) = 0 \text{ in } \mathbb{R}^3\}.$$

Proof. This property is directly induced by the system (2) as shown in [3] for bounded domains. $\square$

Then, it is possible to filter divergence free part of a field in $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$ using the demagnetization field operator.

One of the properties of the demagnetization field which is used to build the algorithm in the scale independance of the demagnetization operator

Proposition 3. *For every n in $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$, for every λ strictly positve real, one set*

$$n \circ \Lambda(\lambda) = m,$$

then m is in $L^2_\Pi(\Lambda(\frac{1}{\lambda})(\Omega_\Pi), \mathbb{R}^3)$ (where $\Lambda(\lambda)$ is the application who associates $\lambda\, x$ to each x in $\mathbb{R}^3$) and

$$H_{d,\Pi}(n) \circ \Lambda(\lambda) = H_{d,\Pi}(m).$$

Proof. This property is deduced form the fact that the operator H_d is a zero order operator. $\square$

2.4 Algorithm constraints

The algorithm we build has to obey to several constraints:

- to preserve the more accurately the kernel of the demagnetization operator,
- to preserve the positivity of the demagnetization energy,
- to have a complexity comparable to the complexity of the fast algorithm on non periodic domains.

3 A multilevel algorithm for the computation of demagnetization field on rectangular periodic domains

3.1 Un quick recall of the algorithm on non periodic domains

Given Ω a bounded domain. We set $(\omega_i)_{i\in\mathcal{I}}$ a family of conne open sets of non empty interior (see [5]):

$$\forall (i,j) \in \mathcal{I}^2, i \neq j : \omega_i \cup \omega_j = \emptyset,$$

where $\mathcal{I}$ is a finite set

$$\bar{\Omega} = \bigcap_{i\in\mathcal{I}} \bar{\omega}_i.$$

Then, for every $U = (u_i)_{i\in\mathcal{I}}$ in $\mathbb{R}^3$, we set $m = \sum_{i\in\mathcal{I}} u_i \chi_{\omega_i}$. The approximation $H_{d,h}$ of the demagnetization field associted to a vector U is given by

$$(H_{d,h}(U))_j = \sum_{(i,j)\in\mathcal{I}^2} K_{i,j} u_j,$$

with

$$\forall (i,j) \in \mathcal{I}^2, \forall u \in \mathbb{R}^3,\ K_{i,j} u = \frac{-1}{4\pi} \int_{\omega_j} \int_{\omega_i} \mathrm{grad}_x \mathrm{div}_x \left(\frac{u}{|y-x|} \right) \, dy \, dx.$$

For a regular cubic mesh of a rectangular domain Ω, the matrix H_d associated to the linear operator $h_{d,h}$ is block-Toeplitz. This structure allows fast computations of complexity $O(n \log(n))$ (where n is the number of elements in the set $\mathcal{I}$).

3.2 Mesh of the infinite domain: the diadic mesh

Given Ω_Π an admissible periodic rectangular domain, in order to compute an approximation of the demagnetization field of an element of $L^2_\Pi(\Omega_\Pi, \mathbb{R}^3)$ we have to mesh Ω_Π. This mesh must, obviously, contains a finite number of elements; then we use the decay property of the demagnetization field in the computation of the interaction between two elements of the mesh.

The discrete diadic domain we build is an assembly of crowns. In the center, the fine mesh of the domain $\widetilde{\Omega}$ is put. Then, on each new crown, the domain is meshed with elements four times greater than the elements of the preceeding crown (see Fig. 1).

Moreover, to improve the method, we add a zone of fine cells as it is shown for example in Figure 2 : for the cells marked by $\mathbf{x}$, the corresponding fine zone added is drawing with dotted lines. In consequence, each cell located in $\widetilde{\Omega}$ has its associated fine zone. Every non-overlapping fine zones have the same size that the fine mesh discretizing $\widetilde{\Omega}$ and for a three dimensional domain, the number of fine zones is equal to eight (four in the case of a bidimensional domain). These fine zones allow a better approximation of the demagnetization field due to the fact that we compute more precisely the closest contributions which are the most important.

The magnetization field, thanks to the periodicity, is known on the whole space. It is the possible to project it, in sense of the means, on the diadic mesh. Where the elements are the finer, values will not be changed, on the coarse part, values will be averaged. It is the magnetization field of the piece-wize magnetization that we will compute. The internal crown which meshes $\widetilde{\Omega}$, will be designate by the index 0. Then, each following crown is indexed by an increasing index. Each of the p levels will have an index between 0 and $p-1$.

For example, we considere an admissible domain periodic in the two directions Ox and Oy. For a magnetization field $m = (0, 0, 1)$, we show on the Figures 3, 4, 5 and 6 the values of the z component of m for each level of the diadic mesh. The fine zone considered in this example is that one which is associated to the cells marked by $\mathbf{x}$ in Figure 2. The magnetization field on the other fine zones is computed by the same way.

3.3 Computation of the demagnetization field on each level: taking into account the order of the operator

The goal is to compute the demagnetizing field on the fine mesh at the center of the diadic mesh, this computation will have to take into account the contributions from all the meshed levels. In order to fulfil this purpose, we use that for every $(U_{i,j})_{(i,j)\in\{0,..,p-1\}\times\{1,...,n(i)\}}$ computed with the mean values of m on the fine mesh, we have:

$$\forall j \in \mathcal{I},\ (H_{d,h,\Pi}(m))_j = \int_{\Omega_j} \left(\sum_{k=0}^{p} \sum_{i=1}^{n(k)} H_d(U_{k,i}\chi_{\omega_{k,i}}) \right) dx.$$

Then, using the scale independance of the demagnetizing field (induced by the fact that the operator is a 0 zero order operator), it is possible to shift the estimation of the demagnetizing field on the coarse mesh to the estimate on the finer mesh. It comes from the fact that the interaction between two elements is the same than the one between this elements expanded by an arbitrary factor.

If we consider the example presented in Section 3.2 (see Figure 2) then we have to compute the demagnetization field for the levels 0, 1 and 2 (see Figures 3, 5 and 6) and for the associated fine zone (see Figure 4). From the point of view of computational time, for the levels 0 and 2 we have 2 computations of the demagnetization field and for the level 1 we have 8×2 computations in a three dimensional domain (8 zones). Hence, for p levels ($p > 1$), we have $(p-1)+16$ computations of the demagnetization field at each iteration. However, these computations are fully independant (the projections of the magnetization field on each levels are also fully independant) so they can be easily computed in parallel. The optimal number of processors needed is clearly $(p-1)+16$.

4 Numerical results

4.1 Benchmarks : the demagnetization field of constant fields

In order to validate the method, we present three benchmarks. First, we consider the constant field $m = (0, 0, 1)$ in a periodic domain in the directions Ox and Oy. The computational domain (i.e. $\widetilde{\Omega}$) is a cube. In this case, we know the demagnetization field solution : $H_d(m) = (0, 0, -1)$. On Figure 7 we compare the periodic solution with the non-periodic solution. In the periodic case, we see that the demagnetization field is well computed.

Figure 8 shows that the order of our method is 1 like in the non periodic case. It means that when the space step decreases the accuracy increases as usual for finite volume schemes. To obtain this accuracy, we have defined the number of levels p function of the number of control volumes : for a computational domain meshed by $2^{p_x} \times 2^{p_y} \times 2^{p_z}$ control volumes, the number of levels is $p = min(p_x, p_y, p_z)$.

Then, the second benchmark consists in taking a magnetization field equal to $m = (0, 1/\sqrt{2}, 1/\sqrt{2})$ on a computational domain of the size $16 \times 64 \times 1$. The periodicity is still in the Ox and Oy directions. The expected result is $H_d(m) = (0, 0, -1/\sqrt{2})$ and the Figure 9 shows that the result obtained with the method is correct.

Finally, we consider again the previous configuration but we change the value of the magnetization field in taking $m = (1, 0, 0)$. So, the expected result is $H_d(m) = (0, 0, 0)$ which is coherent with the result presented in Figure 10.

Notice that at eight points on Figure 10, the magnitude of the demagnetization field is about 10^{-2} instead of 10^{-4} like in the other points. Inspite of that this drawback does not modify the order of the method, a perspective is to reduce this particularity which comes from the fine zones.

4.2 Computation of equilibrium states

In the micromagnetism context, we seek for equilibrium states whose characterization is : find the magnetization field m which minimize the energy :

$$E(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |H_d(m)|^2 dx.$$

In the following computations, the exchange constante is $A = 1.3e^{-11}$ and the space step is $d = 2.5e^{-9}$. Moreover, we use a random initialization for m at the initial time step.

First, we compute the equilibium state in a cube $8 \times 8 \times 8$ with different periodic directions : see Figures 11, 12 and 13.

Then, we compute the equilibrium state for a $16 \times 16 \times 4$ domain with two vertical bars. In Figure 14 we show the results when the periodicity is in different directions.

5 Conclusion

In order to compute a good approximation of the demagnetization field on a periodic domain, we have to apply $(p-1)+16$ times the fast computation algorithm of this field on a bounded domain restraint to one periodicity cell, where p is the number of levels. To preserve the complexity of the bounded domain algorithm, $(p-1)+16$ processors are needed to compute all these computations independantly. Moreover, using the same multilevel ideas, it is also possible to only apply p times the bounded domain algorithm (i.e. the fast block-Toeplitz vector-matrix multiplication) if we assemble a specific discrete operator from the core demagnetizing operator to take into account the fine contributions (corresponding to the fine zones).
In practice, good results are obtained from p equal to 2.

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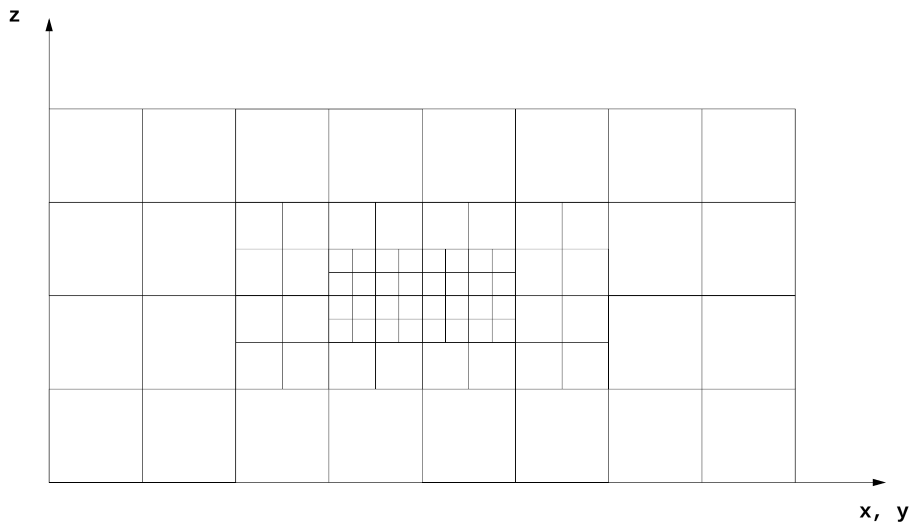

Figure 1: Diadic mesh for a core mesh of 128 cells.

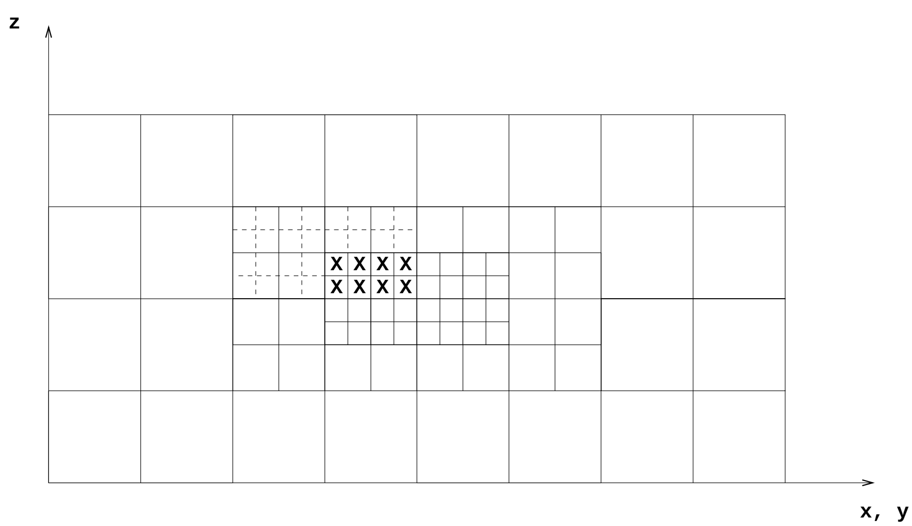

Figure 2: Diadic mesh and the fine zone associated to the cells marked by **x**.

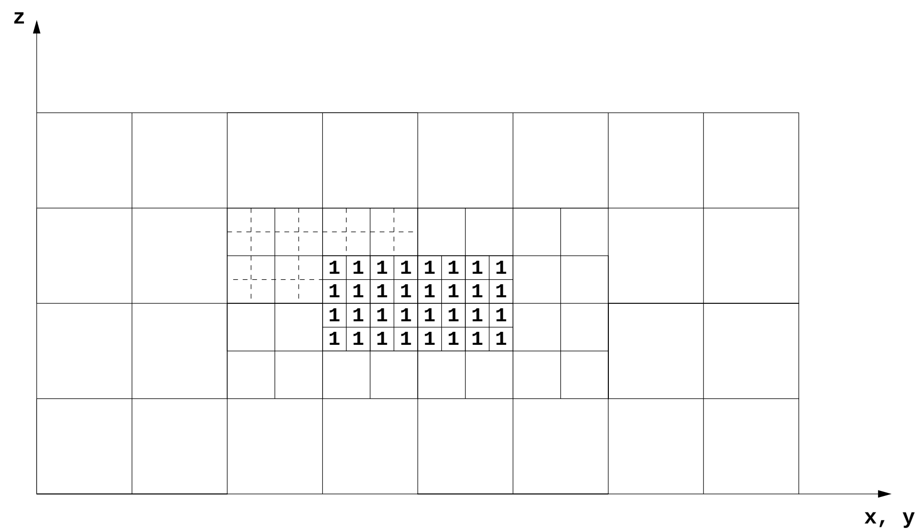

Figure 3: Values of the z component of the magnetization field $m = (0, 0, 1)$ at level 0.

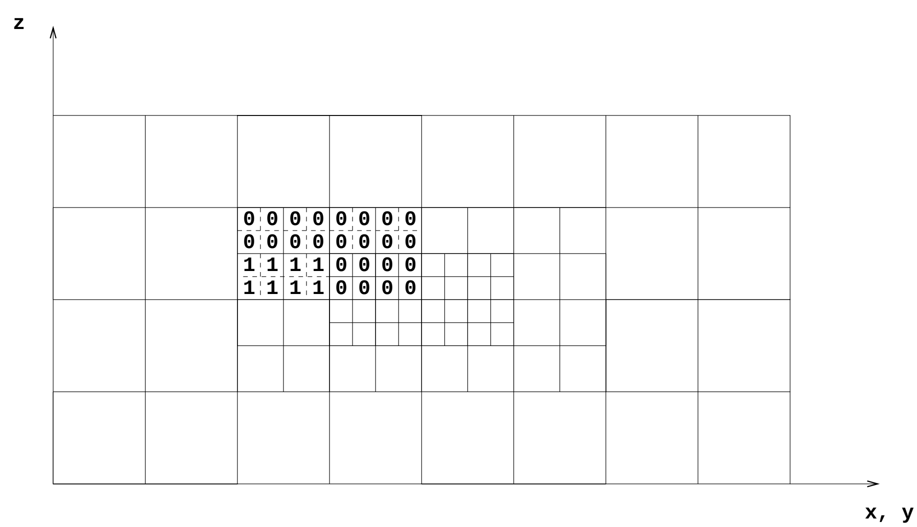

Figure 4: Values of the z component of the magnetization field $m = (0, 0, 1)$ on a fine zone at level 1.

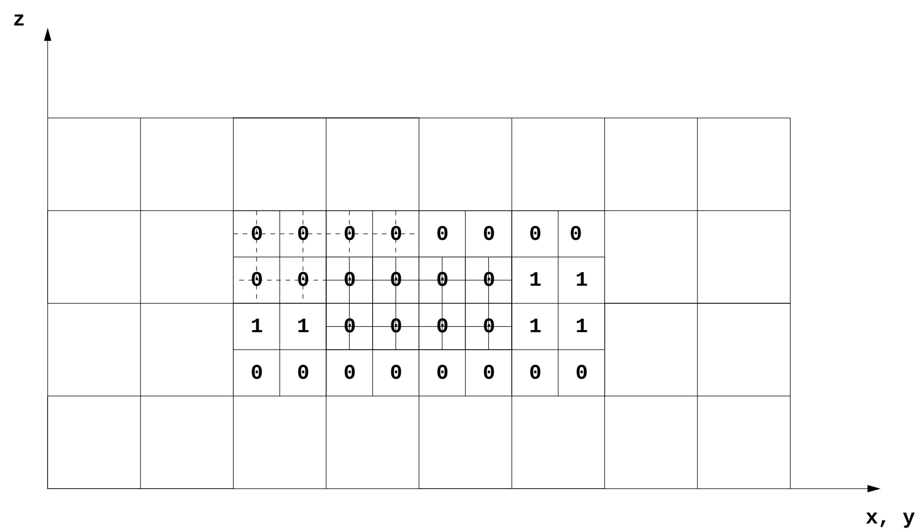

Figure 5: Values of the z component of the magnetization field $m = (0, 0, 1)$ at level 1.

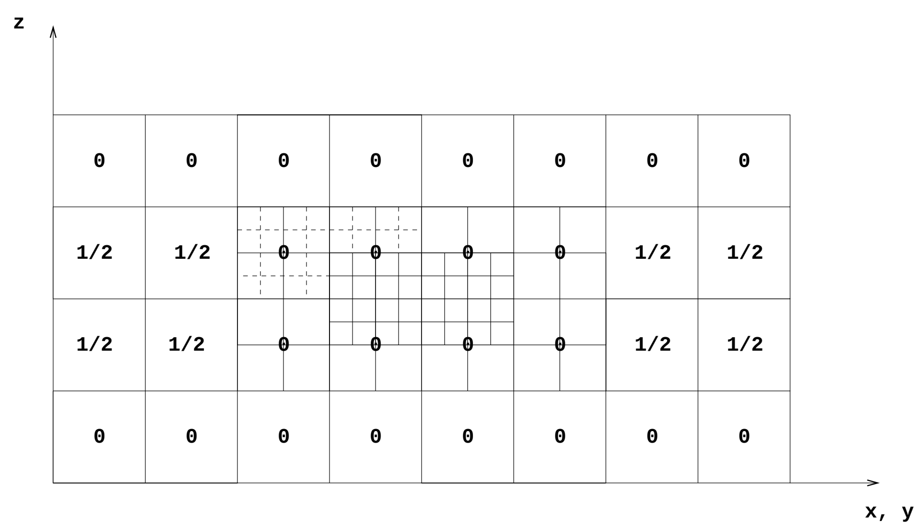

Figure 6: Values of the z component of the magnetization field $m = (0, 0, 1)$ at level 2.

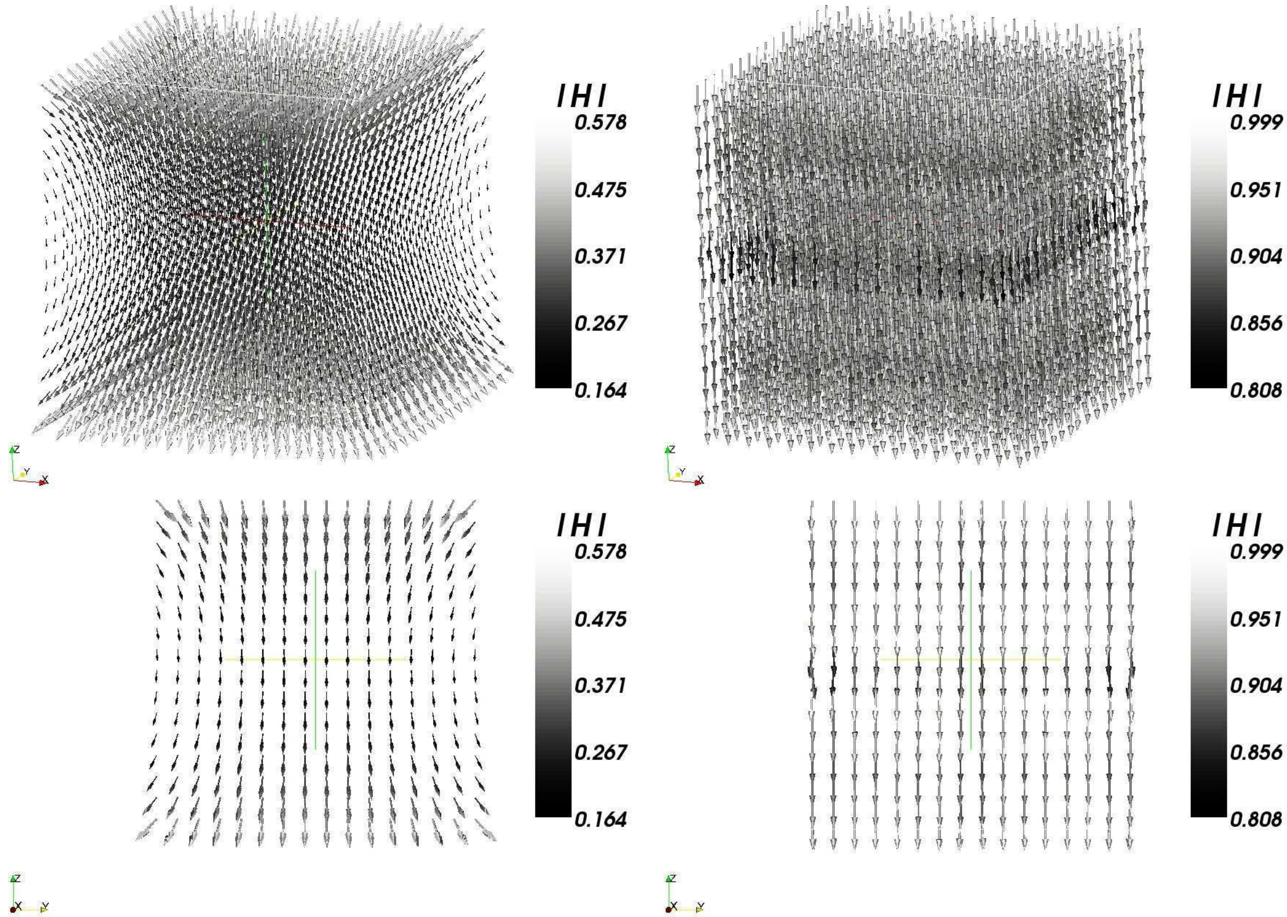

Figure 7: Demagnetization field in a non periodic domain (on the left) and in a periodic domain (on the right) in the directions Ox and Oy.

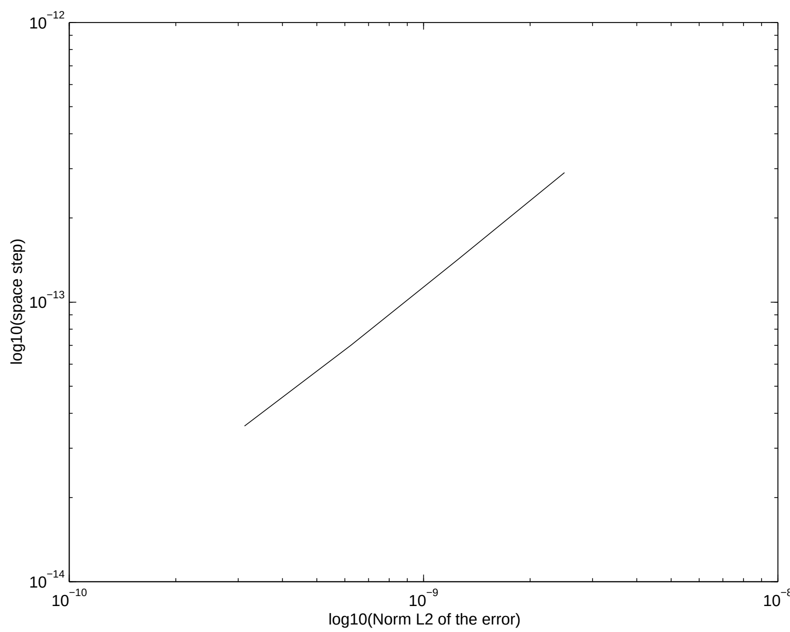

Figure 8: Accuracy of the multilevel algorithm.

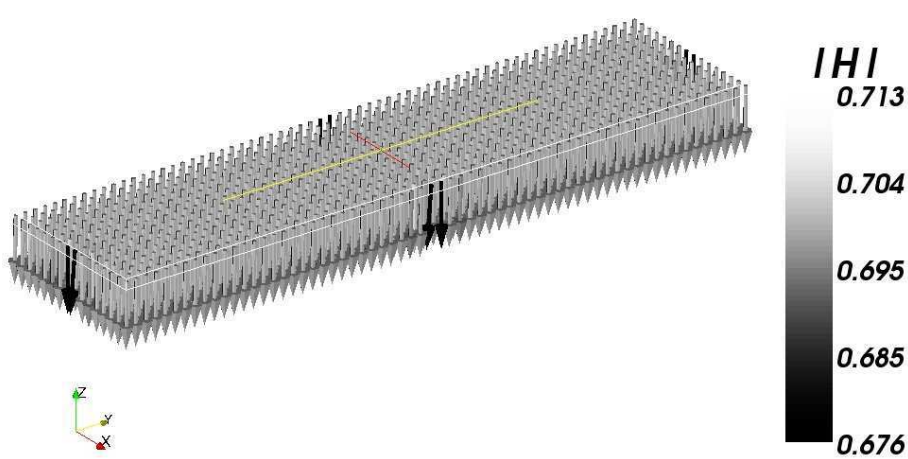

Figure 9: Demagnetization field.

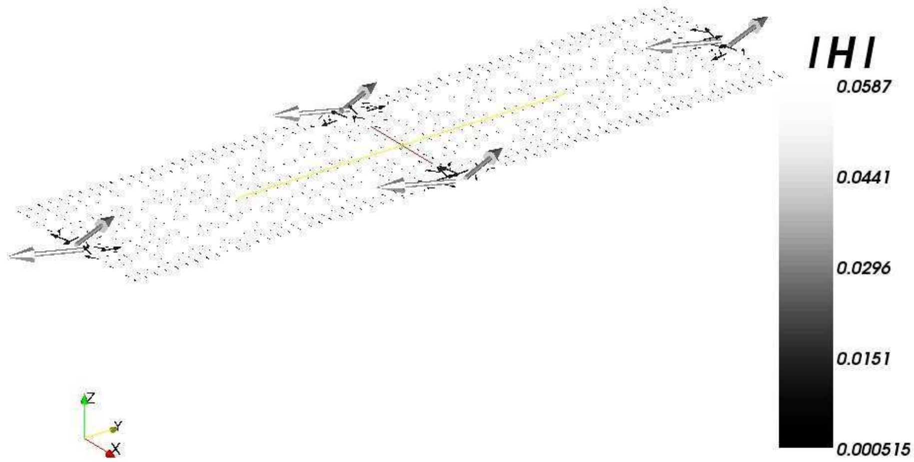

Figure 10: Demagnetization field.

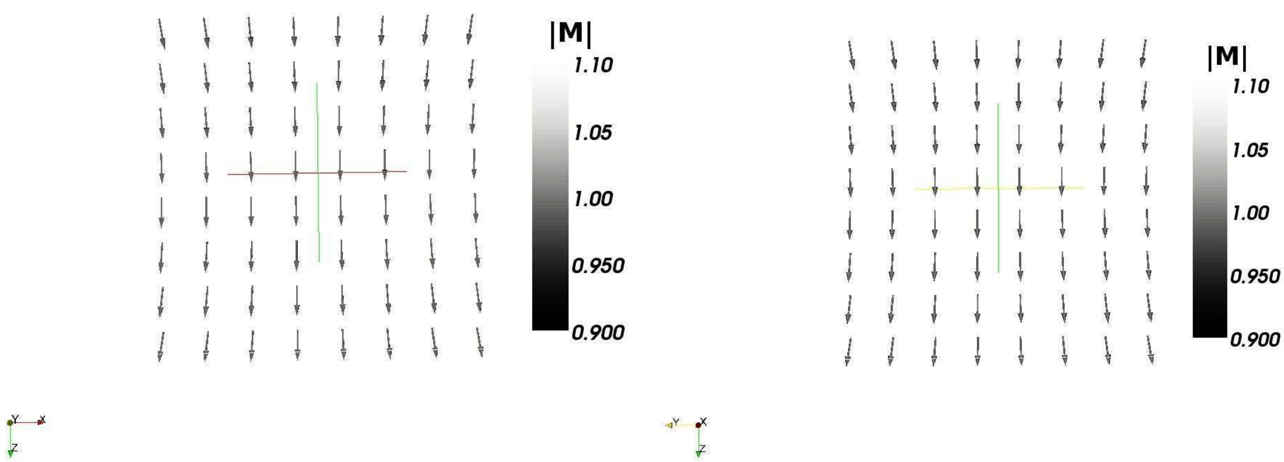

Figure 11: Equilibrium state on a non periodic domain.

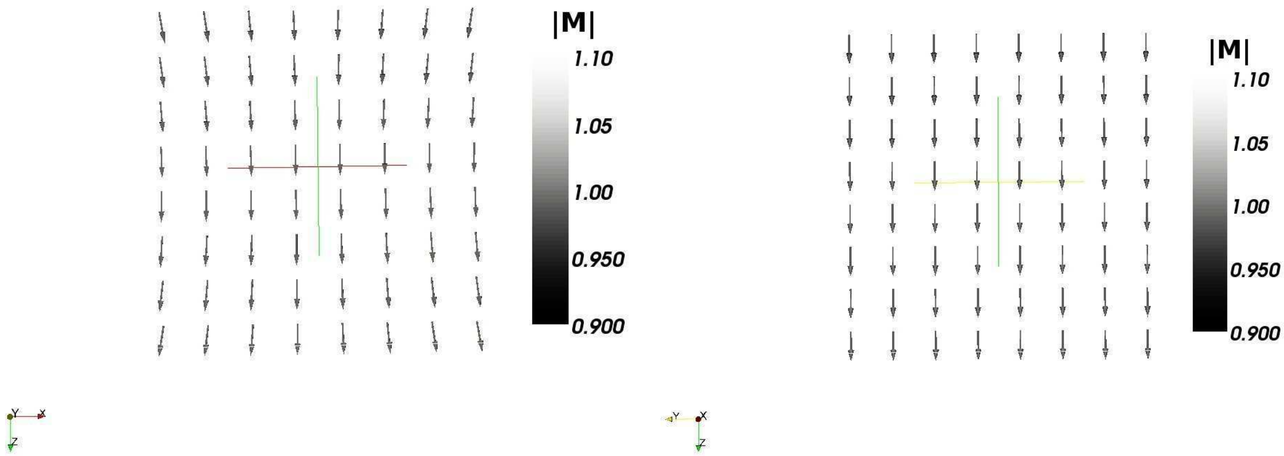

Figure 12: Equilibrium state on a periodic domain in the direction Oy.

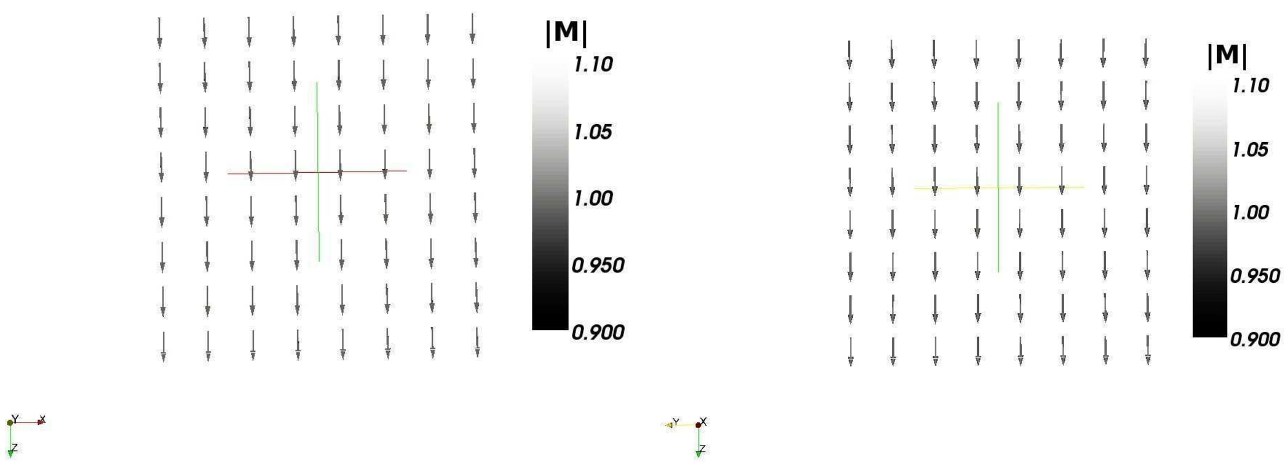

Figure 13: Equilibrium state on a periodic domain in the directions Ox and Oy.

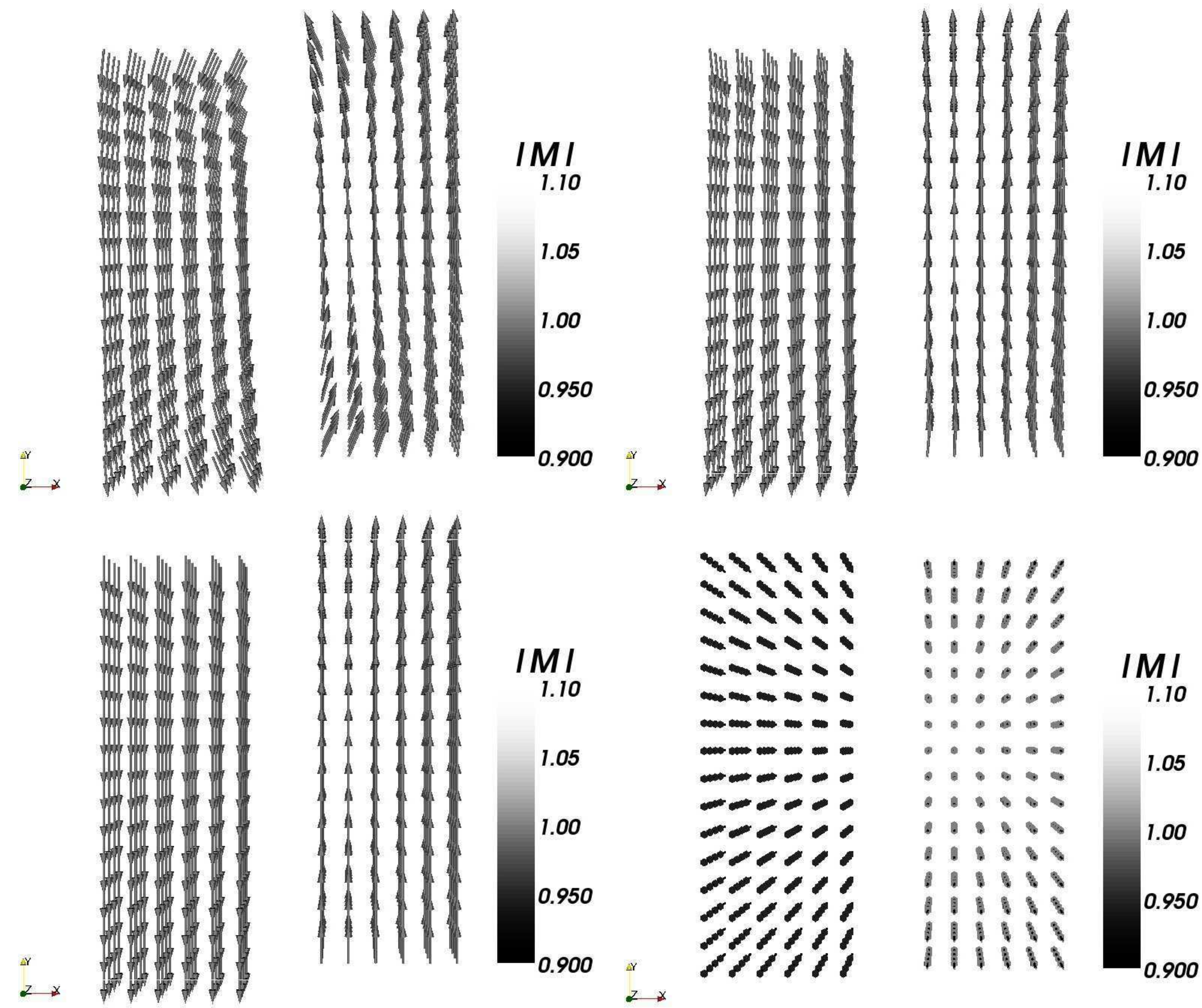

Figure 14: Equilibrium state : on a non periodic domain (at the top, on the left), on a periodic domain in the direction Ox (at the top, on the right), Oy (at the bottom, on the left) or Oz (at the bottom, on the right).