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String Dualities in the Presence of Anomalous $U(1)$ Symmetries

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Abstract

Anomalous $U(1)$ gauge symmetries in type II orientifold theories show some unexpected properties. In contrast to the heterotic case, the masses of the gauge bosons are in general of order of the string scale even in the absence of large Fayet-Iliopoulos terms. Despite this fact, the notion of heterotic-type II orientifold duality remains a useful concept, although this symmetry does not seem to hold in all cases considered. We analyse the status of this duality symmetry, clarify the properties of anomalous $U(1)$ gauge symmetry in the orientifold picture and comment on the consequences for phenomenological applications of such anomalous gauge symmetries.

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1 Introduction

Since its appearance in 4-dimensional ($d = 4$) heterotic string theories, anomalous $U(1)$ gauge symmetries have received considerable attention and have been extensively used in model building. One of the intriguing consequences of the presence of an anomalous $U(1)$ in the heterotic theory is the dynamical appearance of a Fayet-Iliopoulos term ξ in one-loop perturbation theory. This induces nontrivial vacuum expectation values of charged scalar fields that break the $U(1)$ and (potentially) other gauge symmetries spontaneously. This breakdown in connection with the Green-Schwarz mechanism [1] renders the gauge boson massive. In general the size of this mass and of ξ is set by the string scale, possibly suppressed by a constant factor $\epsilon \sim 10^{-2} - 10^{-1}$ that might have interesting consequences for model building. All in all the consequences of an anomalous $U(1)$ symmetry in the framework of the perturbative heterotic string are well understood.

The purpose of the present paper is to achieve a similar understanding of that situation in the framework of open string theories: the properties of anomalous $U(1)$ symmetries in type I and type II orientifolds. Here, in contrast to the situation in the perturbative heterotic theory, we might have to deal with several anomalous $U(1)$ gauge symmetries [2] and a generalized Green-Schwarz mechanism [3, 4]. Originally it was assumed that each of these symmetries comes with a one-loop Fayet-Iliopoulos term ξ_i and that the Green-Schwarz mechanism involves all the axions from the $d = 10$ antisymmetric tensor multiplets, including the “model independent axion” from the dilaton supermultiplet. With this multitude of anomalous $U(1)$ ’s one expected gauge boson masses and Fayet-Iliopoulos terms ξ_i of various sizes, including the possibility of anomalous gauge boson masses that are small compared to the string scale and might even tend to zero. In this paper we want to show that these expectations are not justified and try to clarify the situation. To do that, we rely on two new results concerning the mechanism of anomaly cancellation [5] and the generation of Fayet-Iliopoulos terms [6].

We shall argue that in the orientifold picture, the values of the ξ_i ’s are moduli that can take arbitrary values, but that the mass of the anomalous gauge bosons nonetheless is large and independent of the ξ_i ’s. Thus the gauge boson masses are of the size of the string scale even if some or all of the ξ_i ’s vanish. In that sense the role of the ξ ’s here is similar to those in a nonanomalous gauge symmetry where it can usually be adjusted to any desired value. This is in contrast to the heterotic theory, where we know that a nonvanishing value of ξ is induced. Of course, one should be aware of the fact, that there might be nonperturbative contributions to the ξ ’s in the orientifold picture. We shall come back to this question later in the paper.

Although the situation is so different in the heterotic and open string theories, there is the notion of heterotic-type I duality [7], that was assumed to hold in the orientifold picture as well [8, 9]. In the present paper we investigate this duality in view of the results [5, 6] mentioned earlier. At the moment we are not able to give a general answer, but we have to examine the situation on a model by

model basis. In some of the cases this duality seems to hold exactly, although the role played by the dilaton is different in heterotic and type I theories. Duality, where it holds, leads to interesting results: a blown up orbifold on the heterotic side can be dual to a type II model in the exact orientifold limit (not blown up). We also confirm the fact that anomalous gauge boson masses appear through the Green-Schwarz mechanism even in the presence of vanishing ξ 's, which are connected to the blowing up modes in the orientifold case.

But this duality symmetry should be taken with a grain of salt. In some of the cases (e.g. Z_7 and $Z_3 \times Z_3$ orientifold) problems appear at the level of maximally unbroken gauge group or massless spectrum³. At the moment we do not know how to interpret these discrepancies. It could be used as an argument against the validity of duality, but it could as well be that we are missing some nonperturbative mechanism that would restore it. Such a mechanism could be a nonperturbative induction of Fayet-Iliopoulos terms in type I theories. This could lead to a further breakdown of gauge groups and a picture consistent with duality, but at the moment this remains an open question.

The paper will be structured as follows. In the next chapter we shall explain in detail properties of models with anomalous $U(1)$ gauge theories. Chapter 3 will then give a discussion of the masses of anomalous gauge bosons. The question of heterotic-type I duality will be analyzed in four examples in chapter 4. The consequences for phenomenological application of anomalous $U(1)$ gauge symmetries will then be summarized in chapter 5.

2 The use of anomalous $U(1)$'s

In field theoretic models we were taught to discard anomalous gauge symmetries in order to avoid inconsistencies. This was even true for the condition on the trace of the charges $\sum_i Q_i = 0$ of a $U(1)$ gauge symmetry because of mixed gauge and gravitational anomalies [10]. Moreover a nonvanishing trace of the $U(1)$ charges would reintroduce quadratic divergencies in supersymmetric theories through a one-loop Fayet-Iliopoulos term [11]. In string theory we then learned that one can tolerate anomalous $U(1)$ gauge symmetries due to the appearance of the Green-Schwarz mechanism [1] that provides a mass for the anomalous gauge boson. In fact, anomalous $U(1)$ gauge symmetries are common in string theories and could be useful for various reasons. Before we discuss these applications in detail, let us first discuss the appearance of the anomalous symmetries in various string models.

³ If we assume of sufficient breakdown of gauge groups, as was done in previous investigations of duality [9, 22, 23], these problems disappear, in the sense that states that are inconsistent with duality are rendered heavy by the breakdown of gauge symmetries.

2.1 $U(1)_A$ in heterotic string theory

In this case one obtains models with at most one anomalous $U(1)$, and the Green-Schwarz mechanism involves the so-called model independent axion (the pseudoscalar of the dilaton superfield S). The number of potentially anomalous gauge bosons is in general limited by the number of antisymmetric tensor fields in the ten-dimensional ($d = 10$) string theory. This explains the appearance of only one such gauge boson in the perturbative heterotic string theory and leads to specific correlations between the various (mixed) anomalies [12]. This universal anomaly structure is tied to the coupling of the dilaton multiplet to the various gauge bosons.

The appearance of a nonvanishing trace of the $U(1)$ charges leads to the generation of a Fayet-Iliopoulos term ξ^2 at one loop. In the low energy effective field theory this would be quadratically divergent, but in string theory this divergence is cut off through the inherent regularization due to modular invariance. One obtains [13, 14]

$$\xi^2 \sim \frac{1}{(S + S^*)} M_{\text{Planck}}^2 \sim M_{\text{String}}^2 \quad (1)$$

where $(S + S^*) \sim 1/g^2$ with the string coupling constant g . The Fayet-Iliopoulos term of order of the string scale M_{String} is thus generated in perturbation theory. This could in principle lead to a breakdown of supersymmetry, but in all known cases there exists a supersymmetric minimum in which charged scalar fields receive nonvanishing vacuum expectation values (vevs), that break $U(1)_A$ (and even other gauge groups) spontaneously. This then leads to a mixing of the goldstone boson (as a member of a matter supermultiplet) of this spontaneous breakdown and the model-independent axion (as a member of the dilaton multiplet) of the Green-Schwarz mechanism. One of the linear combinations will provide a mass to the anomalous gauge boson. The other combination will obtain a mass via nonperturbative effects that might even be related to an axion-solution of the strong CP-problem [15].

As we can see from (1), both the mass of the $U(1)_A$ gauge boson and the value of the Fayet-Iliopoulos term ξ are of the order of the string scale. Nonetheless, models with an anomalous $U(1)$ have been considered under various circumstances and lead to a number of desirable consequences. Among those are

- (i) the breakdown of some additional nonanomalous gauge groups [16],
- (ii) a mechanism to parametrize the fermion mass spectrum in an economical way [17],
- (iii) the possibility to induce a breakdown of supersymmetry [18],
- (iv) a satisfactory incorporation of D-term inflation [19],
- (v) the possibility for an axion solution of the strong CP-problem [15].

The nice property of the perturbative heterotic string theory in the presence of an anomalous $U(1)$ is the fact that both ξ and the mass of the anomalous gauge boson are induced dynamically and not just put in by hand. Both of them, though, are of order of the string scale M_{String} , which might be too high for some of the applications, notably (iv) and (v). We will now compare this for the case of

2.2 $U(1)_A$ in type I and type II orientifolds.

These are in general $d = 4$ string models of both open and closed strings that are derived from either type I or type II string theories in $d = 10$ by appropriate orbifold or orientifold projections [20]. As a first surprise it was noticed, that in these cases more than a single anomalous $U(1)$ symmetry could be obtained [2]. This led to the belief that here we can deal with a new playground of various sizes of ξ 's and gauge boson masses in the phenomenological applications.

The appearance of several anomalous $U(1)$'s is a consequence of the fact that these models contain various antisymmetric tensor fields in the higher dimensional theory and the presence of a generalized Green-Schwarz mechanism [3, 4] involving axion fields in new supermultiplets M . In the type II orientifolds under consideration these new axion fields correspond to twisted fields in the Ramond-Ramond sector of the theory.

From experience with the heterotic case it was then assumed [9] that for each anomalous $U(1)$ a Fayet-Iliopoulos term was induced dynamically. With a mixing of the superfields M and the dilaton superfield S one hoped for $U(1)_A$ gauge boson masses of various sizes in connection with various sizes of the ξ 's.

The picture of duality between heterotic orbifolds and type II orientifolds as postulated in [8] seemed to work even in the presence of several anomalous $U(1)$ gauge bosons assuming the presence of Fayet-Iliopoulos terms in perturbation theory and the presence of the generalized Green-Schwarz mechanism.

Meanwhile we became aware of two decisive new results that initiated our renewed interest in these questions and forces us to reanalyse this situation. The first one concerns the inspection of the anomaly cancellation mechanism in various type II orientifolds. As was observed by Ibáñez et al. [5], in this class of models there is no mixing between the dilaton multiplet and the M -fields. It is solely the latter that contribute to the anomaly cancellation. Thus the dilaton that is at the origin of the Green-Schwarz mechanism in the heterotic theory does not participate in that mechanism in the dual orientifold picture. The second new result concerns the appearance of the Fayet-Iliopoulos terms. As was shown by Poppitz [6] in a specific model, there were no ξ 's generated in one-loop perturbation theory. The one loop contribution vanishes because of tadpole cancellation in the given theory. This result seems to be of more general validity and could have been anticipated from more general arguments, since in type I theory a (one-loop) contribution to a Fayet-Iliopoulos term either vanishes or is quadratically divergent, and the latter divergence is avoided by the requirement of tadpole cancellation. Of course, there is a possibility to have tree level contri-

butions to the ξ 's, but they are undetermined, in contrast to the heterotic case where ξ is necessarily nonzero because of the one loop contribution. In type II theory such a contribution would have to be of nonperturbative origin.

In the heterotic theory the mass of the anomalous gauge boson was proportional to the value of ξ . If a similar result would hold in the orientifold picture, this would mean that some of the $U(1)$ gauge bosons could become arbitrary light or even massless, a situation somewhat unexpected from our experience in quantum field theory. In any case a careful reevaluation of several questions is necessary in the light of this new situation. Among those are:

- the size of the ξ 's,
- the size of the masses of anomalous $U(1)$ gauge bosons
- relation of ξ and gauge boson mass,
- the fate of heterotic - type IIB orientifold duality,

which we will discuss in the remainder of this paper.

3 Anomalous gauge boson masses

3.1 $D = 4, N = 1$ heterotic compactifications

Let us first recall some facts about anomalous $U(1)$'s in $D = 4, N = 1$ compactifications of the heterotic string. The gauge group of such vacua often possesses several abelian factors, one of which may be anomalous. Its anomalies are harmless, however, since they are compensated for by a four-dimensional version of the Green-Schwarz mechanism [1] which ensures the consistency of the underlying $D = 10$ string theory. A Fayet-Iliopoulos term ξ^2 is generated in the $D = 4$ vacuum at the one string loop level, as well as a gauge boson mass at two loops. A string computation gives [14]

$$\xi^2 = \frac{\text{Tr } X}{192\pi^2} M_{Str}^2 \quad (2)$$

where $\text{Tr } X$ denotes the trace of the anomalous charge, called X in the following, over all massless states of the theory.

As shown by Dine, Seiberg and Witten [13], much information about the anomalous $U(1)$ can be obtained from a four-dimensional supersymmetric formulation of the Green-Schwarz mechanism. Before going through this formulation, let us recall how the Green-Schwarz mechanism works in ten dimensions. $D = 10, N = 1$ supergravity coupled to supersymmetric Yang-Mills theory has in general gauge and gravitational anomalies which are generated by hexagon diagrams with six external gauge bosons and/or gravitons. When the gauge group is $SO(32)$ or $E_8 \times E_8$, all anomalies can be cancelled by the addition of counterterms such as $B \text{tr } F^4$, where B and F are the two-forms corresponding

to the antisymmetric tensor B_{MN} from the supergravity multiplet and to the Yang-Mills field strength F_{MN} , respectively ($M, N = 0 \dots 9$). This term, together with the coupling $\partial^M B^{NP} \omega_{MNP}^{YM}$ present in the supergravity action (where $\omega^{YM} = \text{tr}(AF - \frac{1}{3}A^3)$ is the Yang-Mills Chern-Simons three-form), generates an anomalous tree diagram with a B propagator and six external gauge fields. Such diagrams compensate for the hexagon diagrams. It has been shown that the type I and heterotic string theories automatically contain the counterterms required for anomaly cancellation.

In $D = 4, N = 1$ heterotic vacua, abelian anomalies are compensated for by a remnant of this mechanism. The role of the ten-dimensional B_{MN} is played by the four-dimensional antisymmetric tensor $B_{\mu\nu}$ coming from the components of B_{MN} that are tangent to the non-compact dimensions. This field couples to the four-dimensional Chern-Simons form:

$$\partial^\mu B^{\nu\rho} \omega_{\mu\nu\rho}^{YM} \quad (3)$$

and to the field strength of the anomalous $U(1)_X$:

$$\epsilon_{\mu\nu\rho\sigma} B^{\mu\nu} F_X^{\rho\sigma} . \quad (4)$$

This last coupling is nothing but the four-dimensional remnant of the ten-dimensional Green-Schwarz counterterms (it can be obtained, for example, by giving background expectation values to the field strengths with compact indices in the term $B \text{tr} F^4$).

Let us now recall the standard supersymmetric formulation of this mechanism [13]. In four dimensions, an antisymmetric tensor $B_{\mu\nu}$ describes only one degree of freedom and is related through a duality transformation to a pseudo-scalar field: $\partial_\mu B_{\nu\rho} \sim \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a$. After a duality transformation, the coupling (3) may be rewritten (using the identity $d\omega^{YM} = \text{tr} F^2$) as $F^{\mu\nu} \tilde{F}_{\mu\nu}$. Since the tree-level gauge kinetic function of heterotic compactifications is simply the dilaton supermultiplet (in the weakly coupled regime), this tells us that the axion a has to lie in this multiplet. Indeed, writing $S|_{\theta=\bar{\theta}=0} = s + ia$ and developing the gauge kinetic terms in components, we obtain, omitting the fermionic terms:

$$\begin{aligned} \mathcal{L}_{GK} &= \frac{1}{4} \sum_A \int d^2\theta S W^A W^A + \text{h.c.} \\ &= -\frac{1}{4} s F^{A\mu\nu} F_{\mu\nu}^A + \frac{1}{4} a F^{A\mu\nu} \tilde{F}_{\mu\nu}^A + \frac{1}{2} s D^A D^A \end{aligned} \quad (5)$$

where the index $A = (a, i)$ runs over the factors of the four-dimensional gauge group, $G = \otimes_a G_a \otimes_i U(1)_i$ (the G_a are simple groups, and $i = X$ corresponds to the anomalous $U(1)$), and we have omitted the Kac-Moody levels for simplicity. In these notations, the string coupling constant is given by $\langle s \rangle = \frac{1}{g^2}$.

The Green-Schwarz counterterm (4), which after a duality transformation becomes $\partial_\mu a A_X^\mu$, is described in a supersymmetric manner by modifying the kinetic term of the dilaton, $K(S, \bar{S}) = -\ln(S + \bar{S})$, to

$$\mathcal{L}_K = -\int d^4\theta \ln(S + \bar{S} - \delta V_X) \quad (6)$$

where the Green-Schwarz parameter δ characterizes the coupling of the anomalous gauge boson to the axion. Gauge invariance then requires that, under a $U(1)_X$ transformation with parameter Λ_X :

$$S \rightarrow S + \frac{i}{2} \delta \Lambda_X \quad (7)$$

This results in a shift of the axion field $a \rightarrow a + \frac{\delta}{2} \theta_X$ (where $\theta_X = \text{Re } \Lambda_X |_{\theta=\bar{\theta}=0}$), which through the variation of (5) compensates for the anomaly⁴

$$\delta \mathcal{L} |_{1\text{-loop}} = -\frac{\theta_X}{32\pi^2} \sum_A C_A F^A \tilde{F}^A \quad (8)$$

For this mechanism to work, the anomalies of the charge X must satisfy the relations:

$$C_A = 4\pi^2 \delta \quad (9)$$

In heterotic compactifications, this is automatically the case. Thus all anomalies (including the mixed gravitational anomaly $C_g = \text{Tr } X$, which is also compensated for by the axion shift) are proportional to the Green-Schwarz parameter δ . This property, which ensures that heterotic string vacua contain at most one anomalous $U(1)$, is a consequence of the universal coupling of the dilaton superfield to the gauge fields. We shall for this reason refer to the above anomaly compensation mechanism as the *universal Green-Schwarz mechanism*.

Let us now develop the Kähler potential (6) in component fields (again omitting fermionic terms):

$$-\frac{1}{4s^2} (\partial^\mu s \partial_\mu s + \partial^\mu a \partial_\mu a) + \frac{\delta}{4s^2} \partial_\mu a A_X^\mu - \frac{\delta^2}{16s^2} A_X^\mu A_{X\mu} + \frac{\delta}{4s} D_X \quad (10)$$

where, since we are working in the Einstein frame, $M_{Pl} = 1$ (here M_{Pl} refers to the reduced Planck mass, $M_{Pl} = 2.4 \times 10^{18} \text{GeV}$). The supersymmetrization of the Green-Schwarz counterterm has introduced a mass term for the anomalous gauge boson and a Fayet-Iliopoulos term. These arise in string perturbation theory at the level of two loops and one loop, respectively. Their expressions are given by:

$$M_X^2 = \frac{1}{8} g^4 \delta^2 M_{Str}^2 \quad \xi^2 = \frac{\delta}{4} M_{Str}^2 \quad (11)$$

where we have restored the string scale $M_{Str} = g M_{Pl}$, and rescaled the vector multiplet $V_X \rightarrow g V_X$ in order to have canonical kinetic terms for A_μ^X . The Green-Schwarz parameter δ is determined by the string computation of ξ^2 (2):

$$\delta = \frac{\text{Tr } X}{48\pi^2} \quad (12)$$

⁴In (8), the anomaly coefficient C_A is defined by $C_a = \sum_{R_a} 2T(R_a)X_{R_a}$ for a non-abelian group G_a (where $T(R_a)$ and X_{R_a} are respectively the index and the X -charge (14) of the representation R_a), and by $C_i = 2 \text{Tr } (Y_i^2 X)$ for an abelian group $U(1)_{Y_i}$ (there is an additional symmetry factor for the cubic anomaly of $U(1)_X$, $C_X = \frac{2}{3} \text{Tr } (X^3)$).

This fixes the proportionality coefficient between the mixed gravitational anomaly and the gauge anomalies in (9).

In the original vacuum, supersymmetry is broken by the Fayet-Iliopoulos term, and the axion becomes the longitudinal component of the anomalous gauge boson. In many compactifications, however, there exist shifted vacua in which supersymmetry is preserved upon some scalar fields charged under $U(1)_X$ acquiring a vev: $\langle D_X \rangle = \sum_\alpha X_\alpha |\langle \Phi_\alpha \rangle|^2 + \xi^2 = 0$. For this to happen, some of the X_α must have the opposite sign to ξ^2 . A combination of the S and the Φ_α chiral superfields is then absorbed by the anomalous vector multiplet and disappears from the massless spectrum, while the orthogonal combination yields a low-energy axion. The mass of the anomalous gauge boson is now (assuming that ξ^2 is compensated for by a single vev with charge X) $M_X^2 = \frac{1}{8} g^2 \delta (g^2 \delta - 4 X) M_{Str}^2$. Note that, since the Φ_α often carry other charges, it is likely that other gauge groups, either abelian or non-abelian, are broken together with the anomalous $U(1)$ [16].

The universal Green-Schwarz mechanism then leads to the following picture: once the dilaton assumes its vacuum expectation value, the anomalous $U(1)_X$ is broken and a Fayet-Iliopoulos term is generated. Comparing the two scales

$$M_X^2 = \frac{1}{2} g^2 (g^2 \delta - 4 X) \xi^2 \quad (13)$$

we see that while ξ^2 is tied to the string scale, one could in principle make M_X light with respect to M_{Str} by lowering the string coupling constant. This possibility would conflict, however, with the successful gauge coupling unification of the MSSM.

3.2 $D = 4, N = 1$ type IIB orientifolds

As stressed in the introduction, $D = 4, N = 1$ type IIB orientifolds show a very different pattern of anomaly cancellation. The gauge group of such vacua may contain more than one anomalous $U(1)$. Their anomalies are not universal in the sense of Eq. (9), and they are compensated by a generalized version of the Green-Schwarz mechanism, which involves several antisymmetric tensors. Also, a string computation [6] has shown that no Fayet-Iliopoulos term is generated at the one-loop level. While this result has been obtained in a specific vacuum (the Z_3 orientifold of Ref. [8]), it is believed to hold in a larger class of models, since it is related to tadpole cancellation.

The cancellation of $U(1)$ anomalies in toroidal Z_N type IIB orientifolds has been studied in Ref. [5]. Let us summarize here their results. In addition to the antisymmetric tensor $B_{\mu\nu}$ from the untwisted sector, which is also present in heterotic compactifications, there are several RR antisymmetric tensors $B_{k\mu\nu}$, $k = 1 \dots M$ from the twisted sector, which are associated to the fixed points of the underlying orbifold. Similarly to the heterotic $B_{\mu\nu}$, those twisted antisymmetric tensors couple to the Yang-Mills Chern-Simons forms:

$$\partial^\mu B_k^{\nu\rho} \omega_{\mu\nu\rho}^{AYM} \quad (14)$$

and to the field strength of the N abelian factors $U(1)_i$ present in the gauge group:

$$\epsilon_{\mu\nu\rho\sigma} B_k^{\mu\nu} F_i^{\rho\sigma} \quad (15)$$

But, contrary to the Green-Schwarz counterterm of heterotic compactifications, the couplings (15) are present at tree-level. Another striking difference with the heterotic case is that there is no such coupling for $B_{\mu\nu}$ - implying that the dilaton superfield does not play any role in anomaly cancellation.

The pseudoscalar duals a_k of the twisted antisymmetric tensors lie in the same chiral multiplets as the NS-NS twisted moduli m_k corresponding to the blowing-up modes associated with the singularities of the orbifold:

$$M_k |_{\theta=\bar{\theta}=0} = m_k + i a_k \quad (16)$$

Performing a duality transformation on the couplings (14), we obtain the following expression for the gauge kinetic function:

$$f_A = f_p + \sum_k c_A^k M_k \quad (17)$$

where f_p is a function of the untwisted moduli⁵ and the c_A^k are model-dependent coefficients. Similarly, the couplings (15) can be rewritten:

$$\sum_{i,k} \delta_i^k \partial_\mu a_k A_i^\mu \quad (18)$$

where the Green-Schwarz parameters δ_i^k are model-dependent coefficients as well⁶. As stressed before, however, the δ_i corresponding to the model-independent axion always vanishes. Thus only the twisted moduli, and not the dilaton, participate in the generalized Green-Schwarz mechanism. This tells us that the Kähler potential for the M_k fields takes the generic form⁷:

$$K = K \left(\left\{ M_k + \bar{M}_k - 2 \sum_{i=1}^N \delta_i^k V_i \right\}_{k=1\dots M} \right) \quad (19)$$

and that, under a $U(1)_i$ transformation with gauge parameter Λ_i , the M_k undergo a shift:

$$M_k \rightarrow M_k + i \delta_i^k \Lambda_i \quad (20)$$

⁵In type IIB orientifolds, the dependence on the untwisted moduli of the gauge kinetic function associated with a gauge group depends on the D-brane sector this gauge group comes from. For example, gauge groups coming from 9-branes have $f_p = S$ [2].

⁶More precisely, the c_A^k and δ_i^k are given by $c_A^k = \text{Tr}(\gamma_k^{-1} \lambda_A^2)$ ($A = a, i$) and $\delta_i^k = \text{Tr}(\gamma_k \lambda_i)$ respectively, where γ_k represents the action of the k^{th} orbifold twist on the Chan-Paton factors, and λ_i (λ_a) is the Chan-Paton matrix associated with the gauge group $U(1)_i$ (G_a) [5].

⁷For the sake of simplicity, we are working in the basis where the kinetic terms for the twisted moduli are canonical in the orbifold limit, namely $\frac{\partial^2 K}{\partial M_k \partial \bar{M}_l} |_{M_k=0} = \frac{1}{2} \delta_{kl}$.

while the dilaton, as well as the other untwisted moduli, remains unshifted. Anomaly cancellation then requires the non-universal relations:

$$C_A^i = 8\pi^2 \sum_k c_A^k \delta_i^k \quad (21)$$

As mentioned above, a string computation [6] has shown that no Fayet-Iliopoulos term is generated at one-loop level in such orientifolds. However, a tree-level ξ^2 can appear upon the m_k assuming a vacuum expectation value. As stressed in [5], this statement does not rely on any particular assumption regarding the Kähler potential. In the presence of the Green-Schwarz counterterms $\partial_\mu a_k A_i^\mu$, supersymmetry requires couplings

$$- \sum_{i,k} \delta_i^k m_k D_i \quad (22)$$

which, if the orbifold singularities are blown up, generate Fayet-Iliopoulos terms $\xi_i^2 = -\sum_k \delta_i^k \langle m_k \rangle$ (for generic Kähler potentials, the ξ_i^2 receive other contributions than (22), which also depend on the $\langle m_k \rangle$). It should be stressed that, since the m_k are moduli, the ξ_i^2 are arbitrary. This is to be contrasted with the heterotic case, in which ξ^2 is tied to the string scale by a model-dependent coefficient: in the orientifold case, the Fayet-Iliopoulos terms are just moduli.

Since the values of the Fayet-Iliopoulos terms are arbitrary, one may wonder whether this is also the case for the anomalous gauge boson masses - recall that in the heterotic case, M_X^2 is proportional to ξ^2 . In [6], it was noticed that M_X^2 has a string-scale value if the Kähler potential for the twisted moduli (19) is a square. Our goal here is to show that *this statement is of general validity and does not depend strongly on the particular choice of the Kähler potential*. We shall first consider the orbifold limit $\langle m_k \rangle = 0$, in which the couplings (22) are computed. In this case, one can identify the scalar components of the massive vector multiplets as combinations of the m_k fields and compute their masses (which by supersymmetry are the same as the gauge boson masses) without knowing the Kähler potential explicitly. Indeed, the couplings (22) induce a mass matrix for the m_k fields

$$\mu_{kl}^2 = \sum_i g_i^2 \delta_i^k \delta_i^l \quad (23)$$

(we have performed the canonical rescaling on the abelian vector multiplets: $V_i \rightarrow g_i V_i$, where $g_i = \langle \text{Re } f_i \rangle^{-1/2}$). This matrix is diagonalized by some rotation R :

$$\mu_p^2 \delta_{pq} = \sum_i g_i^2 \bar{\delta}_i^p \bar{\delta}_i^q \quad \bar{\delta}_i^p = \sum_k R_{pk} \delta_i^k \quad (24)$$

One can always choose R such that the first r eigenvalues are nonzero. This defines in an unambiguous way r massive combinations of the m_k :

$$m'_p = \sum_k R_{pk} m_k \quad p = 1 \dots r \quad (25)$$

the $M-r$ remaining combinations, which can always be redefined, being massless. Performing the same rotation R on the NS-NS partners of the scalars m_k , one finds that only the a'_p , $p = 1 \dots r$ couple to the abelian gauge bosons:

$$\sum_{p=1}^r \mu_p \partial_\mu a'_p A_p'^\mu \quad (26)$$

where we have redefined the A_i^μ to

$$A_p'^\mu = \sum_i \frac{g_i \bar{\delta}_i^p}{\mu_p} A_i^\mu \quad p = 1 \dots r \quad (27)$$

and orthogonal combinations for the remaining $N-r$ gauge bosons $A_p'^\mu$, $p = r+1 \dots N$. Equations (24) and (26) tell us that the chiral superfields $M'_p = \sum_k R_{pk} M_k$, $p = 1 \dots r$ are absorbed by the r vector superfields $V'_p = \sum_i g_i \bar{\delta}_i^p V_i / \mu_p$ to form r massive vector multiplets. Then supersymmetry allows us to conclude that the abelian gauge bosons $A_p'^\mu$ have the same mass as the scalars m'_p .

We can ask the question how the above formulae are modified when one blows up the orientifold. For a generic Kähler potential, both the normalization of the kinetic terms of the twisted moduli and their couplings to the abelian gauge bosons are corrected by the non-vanishing of the blowing-up modes. These corrections can be taken into account by moving to the basis where the kinetic terms of the twisted moduli are canonical, $\widetilde{M}_k = \sqrt{2} \sum_l K_{kl}^{1/2} M_l$ (where $K_{kl} = \partial^2 K / \partial M_k \partial M_l$ is the Kähler metric), before applying the procedure of the previous paragraph. All formulae then remain valid, with the δ_i^k replaced by moduli-dependent coefficients $\tilde{\delta}_i^k = \sqrt{2} \sum_l K_{kl}^{1/2} \delta_i^l$. Therefore, in a blown-up orientifold, the gauge boson masses μ_p depend on the values of the blowing-up modes both through the gauge couplings g_i , with $g_i^{-2} = \langle \text{Re} f_i \rangle = \langle \text{Re} f_p \rangle + \sum_k c_i^k \langle m_k \rangle$, and through the Kähler metric $K_{kl} = \sum_{N=0}^\infty \frac{2^N}{N!} K_{kl n_1 \dots n_N}(0) \langle m_{n_1} \rangle \dots \langle m_{n_N} \rangle = \frac{1}{2} \delta_{kl} + 2 \sum_n K_{kln}(0) \langle m_n \rangle + \dots$.

It is not difficult to identify the combinations of the $U(1)$'s that are anomaly-free. Eq. (26) tells us that there are no couplings between the a'_p and the $N-r$ massless gauge bosons, so the corresponding $U(1)$'s must be anomaly-free. This can be checked explicitly by redefining the charges accordingly to the gauge bosons, $Y'_p = \sum_i R_{pi}^V g_i Y_i / \sqrt{\sum_i g_i^2}$, where $R_{pi}^V = g_i \bar{\delta}_i^p / \mu_p$ for $p = 1 \dots r$, and by computing the anomalies in the new basis. One finds that the charges $Y'_{p=r+1 \dots N}$ associated with the massless gauge bosons have no anomalies, except mixed anomalies with the anomalous charges $Y'_{p=1 \dots r}$ which are compensated for by the generalized Green-Schwarz mechanism. After integrating out the massive vector multiplets, we end up with $N-r$ anomaly-free $U(1)$'s and $M-r$ twisted moduli $M'_{p=r+1 \dots M}$. Note that contrary to the heterotic case, a vacuum shift is not required to maintain supersymmetry, unless the orientifold is blown up and Fayet-Iliopoulos terms are generated. Eq. (26) implies that this can happen only for an anomalous vector multiplet V'_p , and only if the scalar partner of the massive

gauge boson m'_p has a nonzero vev. Indeed, the anomalous D -terms read:

$$D'_p = -\sqrt{\sum_i g_i^2} \left(\sum_\alpha Y'_p{}^\alpha |\Phi_\alpha|^2 - \frac{\mu_p}{\sqrt{\sum_i g_i^2}} m'_p \right) \quad p = 1 \dots r \quad (28)$$

At the level of unbroken supersymmetry, and in the absence of any nonperturbative mechanism that would stabilize them, the vevs of the m'_p are only restricted by the vanishing of the anomalous D -terms (together with the vevs of the matter fields Φ_α , which are constrained by the other D -terms as well). Thus nothing forces them to be nonzero, and the Fayet-Iliopoulos terms in type IIB orientifolds are just moduli. In particular, there is an obvious vacuum $\langle \Phi_\alpha \rangle = 0$, $\langle m_k \rangle = 0$, corresponding to the orbifold limit, in which all ξ_p^2 vanish. In this vacuum, only the anomalous $U(1)$'s are broken, and their associated gauge bosons become heavy and decouple, leaving r residual global symmetries. On the other hand, any nonzero $\langle m'_p \rangle$, $p = 1 \dots r$ would force some of the matter fields Φ_α to acquire a vev, possibly leading to further breakdown of the gauge group.

To summarize, in $D = 4, N = 1$ type IIB orientifolds, the vector supermultiplets V'_p associated with the anomalous $U(1)$'s become massive by absorbing a twisted modulus chiral supermultiplet M'_p . The masses of the massive multiplets can be computed from the diagonalization of the twisted moduli mass matrix (23):

$$\mu_p^2 = \sum_i g_i^2 \bar{\delta}_i^p \bar{\delta}_i^p M_{Pl}^2 \quad p = 1 \dots r \quad (29)$$

(where we have restored the Planck mass). To each of these massive multiplets is associated a moduli-dependent Fayet-Iliopoulos term

$$\xi_p^2 = -\frac{\mu_p}{\sqrt{\sum_i g_i^2}} \langle m'_p \rangle M_{Pl} \quad p = 1 \dots r \quad (30)$$

which is proportional to the vev of the scalar partner of the gauge boson. From the relation (30) we conclude that the anomalous gauge boson masses and their Fayet-Iliopoulos terms are essentially decoupled. Indeed, the latter can be made arbitrarily small by tuning the $\langle m_k \rangle$, while the former have a Planck-scale value in the orbifold limit, from which they can possibly depart only for large values of the blowing-up modes. This is to be contrasted with the heterotic case, in which the relation between the Fayet-Iliopoulos term and the anomalous gauge boson mass is controlled by the gauge coupling.

One may still ask whether it is possible to make the anomalous gauge bosons light for large values of the blowing-up modes. Let us consider for simplicity the case where the gauge group contains a single abelian factor $U(1)_X$ (this is the case in the Z_3 orbifold [8]), with gauge coupling g_X . The gauge boson mass and the Fayet-Iliopoulos term are, respectively:

$$M_X^2 = 2g_X^2 \sum_{k,l} K_{kl} \delta_X^k \delta_X^l M_{Pl}^2 \quad \xi^2 = -2 \sum_{k,l} K_{kl} \delta_X^k \langle m_l \rangle M_{Pl}^2 \quad (31)$$

Since $g_X^{-2} = \langle \text{Ref}_X \rangle = \langle \text{Ref}_p \rangle + \sum_k c_X^k \langle m_k \rangle$, one could try to make M_X^2 small by giving very large values to the blowing-up modes. However, if the Kähler potential differs from a square, the large values of $\langle m_k \rangle$ also contribute to the Kähler metric, $K_{kl} = \frac{1}{2} \delta_{kl} + 2 \sum_n K_{kln}(0) \langle m_n \rangle + \dots$, making it less natural to envisage a light anomalous gauge boson. Even if the Kähler potential were quadratic, a nonzero value of the m_k would induce a vev of some field with anomalous charge X , resulting in:

$$M_X^2 = \frac{\sum_k \delta_X^k \delta_X^k + 2X \sum_k \delta_X^k \langle m_k \rangle}{\langle \text{Ref}_p \rangle + \sum_k c_X^k \langle m_k \rangle} M_{Pl}^2 \quad (32)$$

Thus it would be very difficult, even in this case, to obtain light gauge bosons. To conclude, one does not expect the blowing-up of the orientifold to lower the masses of the anomalous gauge bosons. The safest possibility to make them light would be to tune the values of the untwisted moduli so as to make the gauge coupling small, much like in the heterotic case.

4 Heterotic-Type I Duality

The class of models containing anomalous $U(1)$ factors offers a playground for studying details of Type I - Heterotic duality in four dimensions. As pointed out in [8] this duality, which is of the weak coupling - strong coupling type in ten dimensions, upon compactification to lower dimensions gives rise to weak coupling - weak coupling dualities in certain portions of the moduli space. In four dimensions the relation between the heterotic and type I dilatons is

$$\phi_H = \frac{1}{2} \phi_I - \frac{1}{8} \log(G_I) \quad (33)$$

where G_I is the determinant of the metric of the compact 6d space, which depends on some of the moduli fields. As the string coupling is $e^{\phi_{I,H}}$, and the volume of the compact space is at least unity in string units for phenomenologically relevant models, then it is obvious that dual models on both sides can be simultaneously weakly coupled. One may question whether this requirement of weak string coupling on both sides constrains in any way the values of the moduli which we need to solve D- and F-flatness conditions as well as to give masses to unwanted particle states. The answer is no and does not depend on the model. The basic observation is that the generalized Green-Schwarz terms which we find on type I side do not depend on the dilaton (i.e. on the 4d universal S modulus), and the compact space volume does not depend on the twisted moduli M which enter the generalized Green-Schwarz terms and, consequently, four dimensional anomalous D-terms. The independence of the anomalous $U(1)$ D-terms of the dilaton does not hold on the heterotic side, where the Fayet-Iliopoulos term ξ_H depends on the dilaton only, and not on any other modulus. But we know already from earlier sections, and shall see in more detail here, that thanks to the existence of certain states charged under anomalous $U(1)$, these models also fulfill all the consistency

checks, like the requirement of the unification of gauge couplings at the proper scale and value.

Before we proceed to analyze specific examples, let us specify the criteria for two models to be called dual to each other. First of all, we remind the reader that already in the heterotic models the anomalous $U(1)$ appears in the low energy lagrangian only to cause a shift of the vacuum which restores supersymmetry, and to be immediately decoupled in a supersymmetric manner. What is left behind, is the supersymmetric model with a global $U(1)$ symmetry realized on the matter supermultiplets in a linear way (the moduli which remain massless do not transform under the global $U(1)$). Thus, already there, the perturbative couplings between would-be ‘light’ states play an important role in finding the correct supersymmetric vacuum. The same phenomenon is found in the present case on both sides, and, moreover, to establish the duality equivalence of two models, we shall need perturbative superpotential couplings between light fields on the heterotic side. Hence, we shall be working at the level of the effective lagrangian valid just below the respective string scale on each side, heterotic and type I ⁸. For the duality to hold between two models we require that they have supersymmetric families of vacua, and that the spectrum of the massless excitations around these vacua is equivalent. This means, in particular, that the unbroken gauge groups and their massless representations should be the same. In the sector of gauge singlet fields, we require that the number of truly massless states be the same on both, heterotic and type I, sides. In addition, we require that the masses of the states which become massive upon the choice of the vacua of the field theoretical lagrangians we are analyzing be of the same order of magnitude. This requirement means in particular that we expect the masses of the gauge bosons of the anomalous $U(1)$ s to be very close to each other among the dual pairs of models. In practice, we solve the D-flatness conditions for all anomalous and nonanomalous $U(1)$ groups on both sides and then for the F-flatness conditions on the heterotic side. It is legitimate in the string context to assume that our unbroken supersymmetry minimum corresponds to flat space and, hence, that the use of the globally supersymmetric lagrangian instead of the locally supersymmetric one is justified.

The pairs of models which we study are type IIB orientifolds models in 4d and their candidate heterotic duals which can be found in the existing literature [8, 9, 22, 23, 24, 2, 25, 26].

4.1 Z_3 models without Wilson lines

The first two examples are Z_3 orientifolds/orbifolds without and with Wilson lines. The model without Wilson lines is actually the original example proposed for the conjectured type I - heterotic duality in four dimensions in [8]. The type IIB orientifold model has the gauge group $G = SU(12) \times SO(8) \times U(1)_A$ where the $U(1)_A$ factor is anomalous. The anomalies are non-universal and get cancelled

⁸The scales and couplings in orientifold models were recently discussed in [21].

by means of the generalized GS mechanism. This mechanism involves twenty-seven twisted singlets $M_{\alpha\beta\gamma}$, a particular combination of which combines with the anomalous vector superfield to form a massive multiplet. After the decoupling of this heavy vector multiplet we obtain the nonanomalous model with the gauge group $G' = SU(12) \times SO(8)$.

On the heterotic side, which is the heterotic $SO(32)$ superstring compactified on T^6/Z_3 , the gauge group is $G = SU(12) \times SO(8) \times U(1)_A$ and the $U(1)_A$ is again anomalous. Its anomalies, however, are universal in this case, and the universal, only dilaton-dependent, Fayet-Iliopoulos parameter is generated. In this case there are also fields which are charged only under the anomalous $U(1)$ and that can compensate for the Fayet-Iliopoulos term by assuming an expectation value, without breaking the gauge group any further; a combination of these fields and of the dilaton supermultiplet is absorbed by the anomalous vector multiplet. These nonabelian singlets are the counterparts of the $M_{\alpha\beta\gamma}$ moduli of the orientifold model. However, on the heterotic side we have additional states charged under $U(1)_A$ (and also under $SO(8)$) the counterparts of which are not present in the orientifold model. These unwanted states become heavy in a supersymmetric manner through the superpotential couplings [9]

$$W_H = \Lambda_{\alpha\beta\gamma} \alpha' \beta' \gamma' \alpha'' \beta'' \gamma'' Tr(M_{\alpha\beta\gamma} V_{\alpha' \beta' \gamma'} V_{\alpha'' \beta'' \gamma''}) . \quad (34)$$

Upon giving expectation values to the M 's, the supermultiplets V obtain supersymmetric mass terms of the order of ξ . Below the scale of the heavy gauge boson mass we have a pair of models which exactly fulfill our duality criteria.

One should note that on the heterotic side we have a blown-up orbifold, since the scalars that assume a vacuum expectation value correspond to the blowing-up modes. Thus, in this case, a Type IIB orientifold is found to be dual to a blown-up heterotic orbifold⁹. The next point to be stressed is that the duality works even though no Fayet-Iliopoulos term is present on the orientifold side. In Ref. [9] where, according to the general belief, the generation of a 1-loop Fayet-Iliopoulos term in the orientifold model had been assumed, duality held only in a region of the moduli space where the nonabelian gauge groups are broken. If such a term were generated on the Type IIB side, perhaps by a nonperturbative mechanism, the duality would still hold, but one would have to blow up the orientifold to achieve D-flatness on the Type IIB side.

4.2 Z_3 models with a discrete Wilson line

One can add a discrete Wilson line to the Z_3 orientifold construction [24]. In this case the gauge group of the orientifold model is $G = SU(4)^4 \times U(1)^4$ where three of the four $U(1)$ s are anomalous and decouple, as they become massive upon mixing with three combinations of the orientifold blowing-up modes. The combination of the four $U(1)$ generators which is orthogonal to the three anomalous generators defines a nonanomalous $U(1)$ under which all fields in the massless spectrum

⁹The blowing-up of the Z_3 orientifold has been recently discussed in Ref. [27].

are neutral. Hence, this last abelian factor cannot be spontaneously broken, and the associated abelian vector boson remains in the massless spectrum of the orientifold model. On the heterotic side there is, as always, only one anomalous $U(1)_A$ which gets massive through the universal GS mechanism. The fields which participate in forming the massive vector multiplet are the dilaton and 27 fields which are neutral under the nonabelian factors but have nonzero abelian charges: $9(1, 1, 1, 1)_{\frac{4}{3}, \frac{8}{3}, 0, 0} \oplus 9(1, 1, 1, 1)_{\frac{4}{3}, -\frac{4}{3}, -4, 0} \oplus 9(1, 1, 1, 1)_{\frac{4}{3}, -\frac{4}{3}, 4, 0}$, where the subscript numbers are the abelian charges (the anomalous charge is the first one). Giving vevs to three combination of these fields allows to make all abelian D-terms vanish, while breaking spontaneously the first two nonanomalous $U(1)$ factors together with the anomalous $U(1)$. The last $U(1)$ is not broken, as all the fields given above are neutral with respect to it. Below the scale of breaking of the three $U(1)$'s, which lies slightly below the string scale, the corresponding gauge bosons decouple, and both models have the same gauge groups $G' = SU(4)^4 \times U(1)$ and the same massless spectra at the end. It should be noted that this example contains on the Type IIB side three independent anomalous $U(1)$ factors, which is a real novelty when compared to the heterotic models where one gets always a single anomalous $U(1)$.

4.3 Z_7 models

There exist, however, examples where exact duality (in the sense specified at the beginning of this section) cannot be achieved. The first of the examples we present here is the Z_7 orientifold/orbifold model given in [22]. The orientifold model has the gauge group $G = SU(4)^3 \times SO(8) \times U(1)^3$. All three $U(1)$ factors are anomalous and their gauge bosons decouple upon getting masses by the nonuniversal GS mechanism. These gauge bosons mix with combinations of the chiral superfields M_α^a which transform nonlinearly under the $U(1)$'s. In this case the unbroken gauge group is large, $G' = SU(4)^3 \times SO(8)$, since the inspection of the D- and F-flatness conditions shows that the charged fields are not forced to assume vevs breaking the nonabelian subgroups. The situation is very different on the heterotic orbifold side. Here we have a unique anomalous $U(1)$ and a Fayet-Iliopoulos term $\xi^2 \propto TrQ > 0$. The only fields at hand which can cancel the anomalous D-term and participate in giving a mass to the gauge boson are the Q 's from the table given below (where the indices give $U(1)$ charges and the first abelian factor is anomalous).

untwisted (partial)	Q_1	$(1, 4, 4, 1)_{-2,1,1}$
	Q_2	$(\bar{4}, 1, 4, 1)_{-2,1,-1}$
	Q_3	$(\bar{4}, \bar{4}, 1, 1)_{-3,-3,0}$
twisted $\alpha = 1 \dots 7$	M_α^1	$7(1, 1, 1, 1)_{0, \frac{36}{7}, -\frac{6}{7}}$
	M_α^2	$7(1, 1, 1, 1)_{0, -\frac{12}{7}, \frac{20}{7}}$
	M_α^3	$7(1, 1, 1, 1)_{0, -\frac{24}{7}, -\frac{16}{7}}$
twisted $\alpha = 1 \dots 7$	V_α^1	$7(1, 1, 6, 1)_{2, \frac{8}{7}, -\frac{4}{7}}$
	V_α^2	$7(1, 6, 1, 1)_{2, \frac{2}{7}, \frac{6}{7}}$
	V_α^3	$7(6, 1, 1, 1)_{0, -\frac{10}{7}, -\frac{2}{7}}$

Since those fields are charged under the $SU(4)^3$ nonabelian factor, this group is spontaneously broken together with the nonanomalous $U(1)$ at the string scale, and the low-energy gauge group is different from that on the Type IIB side. The second problematic aspect is that the fields M_α^a of the heterotic model must acquire vevs in order to make massive in a supersymmetric way the unwanted states V_α^a , which are not present in the orientifold model. However, on the orientifold side the corresponding M_α^a states are gauge singlets, and nothing forces them to assume nonzero vacuum expectation values.

Thus, in the Z_7 example neither the low energy gauge groups nor the massless spectra match in the supposedly dual pair, at least at the level of the perturbative effective lagrangian we rely on here. The question is whether a nonperturbative contribution to the superpotential or, perhaps a nontrivial Kähler potential dependence on the fields M_α^a would change the picture. The second type of corrections, although somewhat exotic in details, could achieve duality. This comes from the fact that certain additional contributions to the Kähler potential would enforce nonzero vevs for the M_α^a states on the Type IIB side (through the D-flatness conditions) and then the two models could appear as a dual pair. The same effect would be achieved if nonzero Fayet-Iliopoulos terms were generated, perhaps by nonperturbative effects.

4.4 $Z_3 \times Z_3$ models

A second example which sheds doubts on the exact weak-weak coupling 4d duality conjecture is the $Z_3 \times Z_3$ orbifold/orientifold constructions of Ref. [23]. In this case after giving masses to the anomalous gauge bosons (there is just one on each side) the gauge group is the same in both models in the pair, namely $G' = SU(4)^3 \times SO(8) \times U(1)_1 \times U(1)_2$, but the spectra cannot be matched. There are massless states in the heterotic model which are charged under the nonanomalous $U(1)_1 \times U(1)_2$, which is not the case on the orientifold side - the corresponding massless states are neutral under the $U(1)_1 \times U(1)_2$ factor.

The direct inspection shows that in this particular case no obvious modification of the Kähler potential, or nonperturbative superpotential, can help restoring duality.

4.5 Global anomalous $U(1)$ symmetries

As pointed out long ago by Witten [28], when the ‘anomalous’ gauge boson decouples, there remain many chiral superfields in the massless spectrum which were charged under that $U(1)$, and their interactions, in particular the perturbative superpotential, still respect the global version of that symmetry. Under this global $U(1)'$ chiral fields transform linearly with inherited charges, hence this global symmetry is anomalous. The low energy dilaton, S' , does not transform under this symmetry. The reason is that in terms of the original string variables the chiral multiplet which is absorbed by the gauge boson to form the massive vector multiplet is a linear combination of the original dilaton, S , and the charged chiral multiplets which obtain the vacuum expectation values. The composition of the combination which is eaten depends on the vacuum configuration, but it always contains an admixture of S . The orthogonal combinations are massless, and one of them, S' , enters the gauge fields kinetic functions the way the original dilaton did: $(S' W^\alpha W_\alpha)_F + h.c.$. This combination does not have any other couplings, hence it supports another anomalous global $U(1)''$ symmetry under which $S' \rightarrow S' + i\gamma$ and other fields are inert¹⁰. There is a unique combination of the $U(1)'$ and $U(1)''$ which is anomaly free, and the orthogonal combination which is anomalous. This, somewhat simplified but physically accurate, reasoning convinces us that there is a global anomalous $U(1)$ symmetry left behind the original anomalous local $U(1)$. This is the statement in the context of the heterotic string compactifications, where there is always only one local anomalous $U(1)$. Hence, on the heterotic side we expect naturally one anomalous global $U(1)$ symmetry. Then, the question is what happens in the Type IIB orientifold models, where one has more anomalous $U(1)$ factors, like for example in the Z_3 with Wilson lines, or in Z_7 models discussed earlier. The answer is that the phenomenon described above occurs separately for each anomalous factor in exactly the same way as described above. The role of the dilaton is played this time by the nonuniversal fields M which are numerous in these models and whose combinations help to form massive vector supermultiplets on the Type IIB side. Among the orthogonal combinations are the superfields M' which are completely analogous to the field S' . At this point we can illustrate this mechanism in the Type II case with a simple example. Let us take for simplicity a single modulus M and a single charged chiral field Y . The Kähler potential with the two fields is $K = \frac{1}{2}(M + \bar{M} - 2\delta V)^2 + Y\bar{Y}$, and the relevant kinetic function $f = S + M$, where $g^2 = \text{Re}(f)^{-1}$. The Lagrangian is

$$\mathcal{L} = \partial M \partial \bar{M} + \partial Y \partial \bar{Y} - \frac{1}{2} g^2 (M + \bar{M} - Y \bar{Y})^2 \quad (35)$$

Let us take the vacuum expectation values along the real directions of the fields: $m = \text{Re}(M)$, $y = \text{Re}(Y)$. Then one can find the eigenvalues of the mass matrix of fluctuations around the vacuum given by $\langle m \rangle$, $\langle y \rangle$, and corresponding

¹⁰This acts like the global shift of the model independent axion in original string variables.

eigenmodes. The zero eigenmode is

$$\phi_0 = \frac{1}{\sqrt{1 + \langle y \rangle^2}} (\langle y \rangle \delta m + \delta y) \quad (36)$$

and the orthogonal mode with the mass $m^2 = 4g^2(1 + \langle y \rangle^2)$ is

$$\phi_m = \frac{1}{\sqrt{1 + \langle y \rangle^2}} (\langle y \rangle \delta y - \delta m) \quad (37)$$

One can express the gauge kinetic function in terms of the eigenmodes which we have found

$$Re(f) = Re(S) + \langle m \rangle + \frac{1}{\sqrt{1 + \langle y \rangle^2}} (\langle y \rangle \phi_0 - \phi_m) \quad (38)$$

The heavy mode ϕ_m becomes part of the heavy gauge boson multiplet and decouples from the massless fields. The field which is a flat direction of the potential and enters the gauge kinetic function is ϕ_0 . To make the expression for the effective gauge kinetic function more transparent, it is convenient to define the field M' through $Re(M') = \langle m \rangle + \frac{1}{\sqrt{1 + \langle y \rangle^2}} \langle y \rangle \phi_0$. Then the gauge kinetic function is simply $f_{eff} = S + M'$. We have assumed that the heavy mode ϕ_m , together with its whole supermultiplet decouples completely from the massless fields (which is precisely the case for the models discussed here). Then the anomalous global symmetry acting on massless fields is unbroken at the renormalizable level. However, one should bear in mind that when we consider the full set of fields, light and heavy, then the anomalous global $U(1)$ is spontaneously broken. This means, strictly speaking, that even in the sector of massless fields this symmetry shall be broken through suppressed interactions with the heavy fields. We are working here with the renormalizable interactions only, hence we can justifiably treat the global symmetries as exact ones.

The same mechanism works for each anomalous $U(1)$ factor. This, however, leads to the conclusion that in Type IIB models we have several global anomalous $U(1)$ symmetries. This conclusion is correct, as can easily be verified for example in the Z_3 model with Wilson lines described here. However, the dual heterotic model has exactly the same superpotential, and the same light fields, hence there are three, not just one, anomalous global $U(1)$ symmetries also on the heterotic side. How could they appear here? The answer is straightforward, although somewhat unexpected. Recall, that on the heterotic side we have also additional local but nonanomalous $U(1)$'s. These additional factors are spontaneously broken and their gauge bosons also decouple. However, as discussed in specific examples, to match the spectra we have to make certain chiral multiplets heavy on the heterotic side, through superpotential couplings. It turns out that this process 'knocks-out' from the massless spectrum some states charged under nonanomalous local $U(1)$'s in such a way, that what is left are anomalous global $U(1)$'s whose anomalies and charges are exactly the ones needed to match anomalous global factors borne in the dual Type IIB model. This is a somewhat

unexpected observation, which might lead to interesting phenomenological consequences. From the point of view of the present discussion this gives further consistency check for heterotic - type I duality in four dimensions.

5 Conclusion

In this paper, we have studied the properties of anomalous $U(1)$'s in a large class of $D = 4, N = 1$ type IIB orientifolds, and reconsidered some candidate evidence for heterotic-type I duality in 4 dimensions, in the light of the recent results of Ref. [5] and [6]. We have shown that the masses of the anomalous gauge bosons are proportional to the Planck scale, and can be made light only at the expense of small gauge couplings, very much like in the heterotic case. They appear to be decoupled from the Fayet-Iliopoulos terms, whose scales are set by the values of the blowing-up modes of the underlying orbifold, and are therefore undetermined at the perturbative level. This is a noticeable difference with the heterotic anomalous $U(1)$, whose Fayet-Iliopoulos term has a nonzero value, of the order of the string scale.

The absence of such Fayet-Iliopoulos terms in type IIB orientifolds seems at first sight to contradict the generally admitted duality between type IIB orientifolds and heterotic orbifolds, which has been considered as a $D = 4, N = 1$ manifestation of the postulated heterotic-type I duality in ten dimensions. However, this conclusion is too crude, since one should compare the supersymmetric low-energy theories rather than the original vacua. From this point of view, the vacuum shift induced by the heterotic Fayet-Iliopoulos term appears to be a necessary, but not always sufficient ingredient to match the gauge groups and massless spectra of both low-energy theories. If duality holds, one expects indeed the heterotic counterparts of the twisted moduli that participate in the generalized Green-Schwarz mechanism on the orientifold side to assume a vacuum expectation value in order to cancel the heterotic Fayet-Iliopoulos term, resulting in the breaking and decoupling at a high scale of the same number of $U(1)$'s on both sides. At the same time, those vevs give large supersymmetric masses to states that have no perturbative orientifold counterpart [9], making it possible for both massless spectra to match.

This duality picture works perfectly well for the Z_3 models of Ref. [8] and [24]. However, it fails in at least two known candidate dual examples. In the Z_7 model [22] the heterotic vacuum shift triggers the breaking of nonabelian gauge factors, which is not required on the orientifold side. In the $Z_3 \times Z_3$ model [23] some of the remaining charged states of the heterotic model have singlet counterparts in the orientifold model. While duality could be restored in the Z_7 case if Fayet-Iliopoulos terms were generated in the orientifold model, presumably by some nonperturbative mechanism, this would not be sufficient in the $Z_3 \times Z_3$ case if one insists on the requirement that the matching be enforced by the vacuum shifts. It could be that type IIB orientifolds do not always have a heterotic dual model. This would not necessarily contradict the conventional heterotic-type I

duality [7], since type IIB orientifolds represent a generalization of genuine type I compactifications. Still, it would be interesting to see how nonperturbative effects could possibly influence the notion of heterotic-type II orientifold duality. We reserve this question for future investigation.

Finally, let us say a few words about the possible consequences of the orientifold anomalous $U(1)$'s. In heterotic string compactifications, the presence of an anomalous $U(1)$ has been shown to have numerous implications of great relevance for phenomenology. Among them is the possibility of explaining the origin and hierarchies of the small dimensionless parameters present in the low-energy lagrangian, such as the Yukawa couplings [17], in terms of the ratio $\sqrt{|\xi^2|}/M_{Pl}$. Particularly encouraging is the fact that, in explicit string models, ξ^2 is found to be of the order of magnitude necessary to account for the value of the Cabibbo angle. Furthermore, the universality of the mixed gauge anomalies implies a successful relation between the value of the Weinberg angle at unification and the observed fermion mass hierarchies [29]. The anomalous $U(1)$ also plays an important role in supersymmetry breaking: not only it takes part in its mediation from the hidden sector to the observable sector (as implied by the universal Green-Schwarz relation among mixed gauge anomalies), but also it can trigger the breaking of supersymmetry itself, due to an interplay between the anomalous D -term and gaugino condensation [18]. Also, the heterotic anomalous $U(1)$ is likely to have outstanding implications in cosmology, in particular its Fayet-Iliopoulos term can dominate the vacuum energy of the early Universe, leading to inflation [19]. Finally, it may provide a solution of the strong CP problem [15].

One may now ask whether the anomalous $U(1)$'s present in type IIB orientifolds are likely to have similar consequences - or even have the potential to solve some of the problems encountered in the heterotic case. In order to answer this question, it is important to note that all the phenomenological implications of the heterotic $U(1)_X$ rely on the appearance of a Fayet-Iliopoulos term whose value, a few orders of magnitude below the Planck mass, is fixed by the anomaly. The situation is very different in orientifolds, where the Fayet-Iliopoulos terms are moduli-dependent: the freedom that is gained (and allows for example to cure the problems of D -term inflation in heterotic models [30]) is paid for by a loss of predictivity. In that respect, one may conclude that the orientifold anomalous $U(1)$'s are not very different from anomaly-free $U(1)$'s, whose Fayet-Iliopoulos terms are unconstrained and can be chosen at will.

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