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Λ effect in the cosmological expansion of voids

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Abstract

The dynamical effect of the cosmological constant Λ on a single spherical void evolving in a the universe is investigated within a non linear perturbation of Newton-Friedmann models. The void expands with a huge initial burst which freezes asymptotically with time up to matching Hubble flow. For $\Omega_o \sim 0.3$, Λ -effect on the kinematics intervenes significantly by amplifying the expansion rate at redshift $z \sim 1.7$. As a result, the size increases with the background density and with Λ , what interprets by the gravitational attraction of borders from outside regions and the gravitational repulsion of borders. The velocity flow within the void region depends solely on Λ , it reads $\vec{v} = \sqrt{\Lambda/3}\vec{r}$. Hence, the empty regions are swept out for spatially closed Friedmann models what provides us with a stability criterion.

Keywords: Cosmology : Theory, Cosmological Constant, Voids, Large Scale Structures.

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1 Introduction

The understanding of the foam like patterns with large empty regions in the distribution of galaxies within scale up to 100 Mpc, which have been ob-

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served since three decades (see *e.g.*, [27, 16, 17, 7, 25, 5, 15, 4]), has become an important challenge for the formation theory of cosmological structures at large scale. Investigations on their statistical properties have been performed by improving identification techniques (see *e.g.*, [24]), by exploring their formation process in a Λ CDM model through N-body simulations (see *e.g.*, [13, 26, 12, 20, 3, 29, 23]), by probing their origins (see *e.g.*, [22]), on the kinematics of giant voids (see *e.g.*, [18]) and on the dynamics by testing models of void formation (see *e.g.*, [1, 11, 2]). Herein, we contribute to such a goal by providing an exact solution of spherical void dynamics. Thanks to their stability with respect to linear perturbations [19], Friedmann-Lemaître models provide us with a suitable description of the universe at large scales. These models can be described with a Newtonian approach (see *e.g.* [21]) by means of Euler-Poisson equations system solutions for whom kinematics satisfy Hubble (cosmological) law. Except the description of the gravitational waves, Newtonian treatment of perturbations leads indeed to similar results as those obtained in general relativity [9]. Herein, we define a model describing the dynamics of an empty spherical shell evolving in an expanding distribution of dust with the aim of understanding the effect of the cosmological constant on the evolution of the void.

2 Euler-Poisson equations system

The motion of a pressureless medium (dust) is described by its specific density $\rho = \rho(\vec{r}, t)$ and velocity $\vec{v} = \vec{v}(\vec{r}, t)$ at position \vec{r} and time t . These fields are constraint by Euler equations system

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{v}) = 0 \quad (1)$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial \vec{r}} \vec{v} = \vec{g} \quad (2)$$

where $\vec{g} = \vec{g}(\vec{r}, t)$ stands for the gravitational field, and satisfy the modified Poisson-Newton equations

$$\overrightarrow{\text{rot}} \vec{g} = \vec{0} \quad (3)$$

$$\text{div} \vec{g} = -4\pi G \rho + \Lambda \quad (4)$$

where G is Newton constant of gravitation and Λ the cosmological constant.

For the purpose of our investigation, it is convenient to write these equations with new coordinates, herein named *reference coordinates*, defined by

$$(t, \quad \vec{x} = \frac{\vec{r}}{a}), \quad a > 0 \quad (5)$$

where $a = a(t)$ is a monotonic function which verifies Friedmann equation

$$\dot{H} + H^2 - \frac{\Lambda}{3} + \frac{4\pi G}{3}\rho_0 a^{-3} = 0, \quad H = \frac{\dot{a}}{a}, \quad a_0 = a(t_0) = 1 \quad (6)$$

where ρ_0 is an arbitrary constant, the dotted variables stand for time derivatives. This differential equation admits an integration constant

$$K_0 = \frac{8\pi G}{3}\rho_0 + \frac{\Lambda}{3} - H_0^2, \quad H_0 = H(a_0) \quad (7)$$

and hence it integrates for providing us with

$$H^2 = \frac{\Lambda}{3} - \frac{K_0}{a^2} + \frac{8\pi G}{3}\frac{\rho_0}{a^3} \geq 0 \quad (8)$$

The function $H(a)$, which shows an asymptotical value

$$H_\infty = \lim_{a \rightarrow \infty} H = \sqrt{\frac{\Lambda}{3}} \quad (9)$$

provides us with a chronology $t \mapsto a$ as reciprocal mapping of a quadrature. Thus the function $a(t)$ depends on three parameters chosen among Λ , ρ_0 , H_0 and K_0 , see Eq. (7).

From now, let $\overrightarrow{\text{div}}$ and $\overrightarrow{\text{rot}}$ denote the differential operators defined with respect to \vec{x} , Euler-Poisson-Newton equations system reads in term of reference coordinates defined in Eq. (5) as follows

$$\frac{\partial \rho_c}{\partial t} + \text{div}(\rho_c \vec{v}_c) = 0 \quad (10)$$

$$\frac{\partial \vec{v}_c}{\partial t} + \frac{\partial \vec{v}_c}{\partial \vec{x}} \vec{v}_c + 2H \vec{v}_c = \vec{g}_c \quad (11)$$

$$\overrightarrow{\text{rot}} \vec{g}_c = \vec{0} \quad (12)$$

$$\text{div} \vec{g}_c = -\frac{4\pi G}{a^3}(\rho_c - \rho_0) \quad (13)$$

where

$$\rho_c = \rho a^3, \quad \vec{v}_c = \frac{d\vec{x}}{dt} \quad (14)$$

act respectively as the density and the velocity fields of medium in the *reference frame*, and

$$\vec{g}_c = \frac{\vec{g}}{a} + \left(\frac{4\pi G}{3a^3}\rho_0 - \frac{\Lambda}{3} \right) \vec{x} \quad (15)$$

as the acceleration field.

2.1 Newton-Friedmann and Vacuum models

Equations (10, 11) have two obvious solutions :

1. herein named Newton-Friedmann model (NF), it is defined by

$$\rho_{\mathbf{c}} = \rho_{\circ}, \quad \vec{v}_{\mathbf{c}} = \vec{0}, \quad \vec{g}_{\mathbf{c}} = \vec{0} \quad (16)$$

According to Eq.(8,14), a and H recover now their usual interpretations in cosmology as *expansion parameter* and *Hubble parameter* respectively, $\rho_{\circ} = \rho a^3$ identifies to the density of sources in the co-moving space and K_{\circ} (interprets in General Relativity as) its scalar curvature. We limit our investigation to motions which do not correspond to bouncing solutions. Accordingly, an analysis on roots of third degree polynomials tell us that the constraint

$$K_{\circ}^3 < (4\pi G \rho_{\circ})^2 \Lambda \quad (17)$$

must be satisfied. Hence, the kinematics shows two distinct behaviours characterised by the sign of K_{\circ} . Namely, H decreases with time by reaching its asymptotic value H_{∞} either upward ($K_{\circ} \leq 0$) or downward ($K_{\circ} > 0$) from a minimum defined by

$$H_m = H_{\infty} \sqrt{1 - \frac{K_{\circ}^3}{\Lambda (4\pi G \rho_{\circ})^2}} < H_{\infty} \quad (18)$$

at $a = 4\pi G \rho_{\circ} K_{\circ}^{-1}$ (named *loitering period*).

2. herein named Vacuum model (V), it is defined by

$$\rho_{\mathbf{c}} = 0, \quad \vec{v}_{\mathbf{c}} = (H_{\infty} - H) \vec{x}, \quad \vec{g}_{\mathbf{c}} = \frac{4\pi G}{a^3} \rho_{\circ} \vec{x} \quad (19)$$

3 Spherical voids in Newton-Friedman universe

For modelling the dynamics of a spherical void in a uniform dust distribution we use a covariant formulation of Euler-Poisson equations system (see Appendix A). The model is obtained by sticking together the local solutions V and NF of Euler-Poisson equations system. It accounts for the dynamics of their common border (*i.e.* boundaries conditions), which is a *material shell*, see Sec. 3.1. For convenience in writing, each symbol S, V and NF denotes both the medium and the related dynamical model. A qualitative analysis of solutions is performed in Sec. 3.2, a general discussion is given in Sec. 3.3.

3.1 Dynamical model

We consider three distinct media : a material shell with null thickness (S), an empty inside (V) and outside a uniform dust distribution (NF). These media behave such that S makes the juncture of V with NF as given in Eq. (16,19). The tension-stress on S is assumed to be negligible, what is characterised by a (symmetric contravariant) mass-momentum tensor defined as follows

$$T_S^{00} = (\rho_S)_c, \quad T_S^{0j} = (\rho_S)_c v_c^j, \quad T_S^{jk} = (\rho_S)_c v_c^j v_c^k \quad (20)$$

The background is described by the following mass-momentum tensor

$$T_{NF}^{00} = \rho_c, \quad T_{NF}^{0j} = 0, \quad T_{NF}^{jk} = 0 \quad (21)$$

According to Appenix A, since the eulerian function(al)

$$\mathcal{T}(x \mapsto \gamma) = \int T_S^{\mu\nu} \gamma_{\mu\nu} dt dS + \int T_{NF}^{\mu\nu} \gamma_{\mu\nu} dt dV \quad (22)$$

vanishes when γ reads in the form $\gamma_{\mu\nu} = \frac{1}{2} (\hat{\partial}_\mu \xi_\nu + \hat{\partial}_\nu \xi_\mu)$, one has

$$\begin{aligned} & \int_S \left((\partial_0 \xi_0 + g_c^j \xi_j) + (\partial_j \xi_0 + \partial_0 \xi_j - 2H \xi_j) v_c^j + v_c^j v_c^k \partial_j \xi_k \right) (\rho_S)_c dt x^2 d\Omega \\ &= - \int_{NF} \rho_c \partial_0 \xi_0 dt x^2 dx d\Omega \end{aligned} \quad (23)$$

where $d\Omega$ stands for the solid angle element. The radial symmetry of solutions enables us to write the *reduced* peculiar velocity and acceleration of a test particle located on the shell as follows

$$\vec{v}_c = \alpha \vec{x}, \quad \vec{g}_c = \beta \vec{x} \quad (24)$$

where the functions $\alpha = \alpha(t)$ and $\beta = \beta(t)$ have to be determined. A by part integration of Eq. (23) provides us with

$$\begin{aligned} & \int_{t_1}^{t_2} (\partial_0 (\rho_S)_c + 3(\rho_S)_c \alpha - \rho_c \alpha x) x^2 \xi_0 dt \\ &= \int_{t_1}^{t_2} (\partial_0 ((\rho_S)_c \alpha) + 4(\rho_S)_c \alpha^2 + 2H(\rho_S)_c \alpha - (\rho_S)_c \beta x^{-1}) x^3 \tilde{\xi} dt \end{aligned} \quad (25)$$

where $x = \|\vec{x}\|$ stands for the radius of S and $\tilde{\xi} = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}$. This equality must be fulfilled for all bounded time interval and compact support 1-form. Hence, we easily derive the conservation equations for the mass

$$\partial_0 (\rho_S)_c + (3(\rho_S)_c - \rho_c x) \alpha = 0 \quad (26)$$

and for the momentum

$$\frac{d\alpha}{dt} + \left(1 + \frac{\rho_c}{(\rho_S)_c} x\right) \alpha^2 + 2H\alpha + \frac{\beta}{x} = 0 \quad (27)$$

With Eq. (5), the calculation of the gravitational force from the entire shell acting on a particular point¹ provides us with

$$\beta = \frac{4\pi G}{a^3} \left(\frac{\rho_c}{3} - \frac{(\rho_S)_c}{2x} \right) \quad (28)$$

About mass conservation, it is noticeable that

$$(\rho_S)_c = \frac{1}{3} \rho_c x \quad (29)$$

is solution of Eq. (26), what ensures that the amount of matter which forms the shell comes from its interior. Hence, Eq. (27) transforms

$$\frac{d\alpha}{dt} + 4\alpha^2 + 2H\alpha - \frac{2\pi G}{3} \frac{\rho_c}{a^3} = 0 \quad (30)$$

It is convenient to use the dimensionless variable

$$\chi = 4 \frac{\alpha}{H_\circ} a^2 \quad (31)$$

where the ratio αH_\circ^{-1} stands for the expansion rate of S in the reference frame. Hence, Eq. (30) transforms into a Riccati equation

$$\frac{d\chi}{da} = \left(\Omega_\circ - \frac{\chi^2}{a} \right) \frac{1}{\sqrt{P(a)}} \quad (32)$$

where

$$P(a) = \lambda_\circ a^4 - k_\circ a^2 + \Omega_\circ a, \quad P(1) = 1 \quad (33)$$

with the following dimensionless parameters²

$$\lambda_\circ = \frac{\Lambda}{3H_\circ^2}, \quad \Omega_\circ = \frac{8\pi G \rho_\circ}{3H_\circ^2}, \quad k_\circ = \frac{K_\circ}{H_\circ^2} \quad (34)$$

According to Eq. (14,31), the evolution of the radius of S is given by

$$x = x_i \exp \left(\int_{a_i}^a \frac{\chi da}{4a\sqrt{P(a)}} \right) \quad (35)$$

where x_i and a_i stands for the initial values at time t_i .

¹The modified newtonian gravitation field reads $\vec{g} = \left(\frac{\Lambda}{3} - \frac{Gm}{r^3} \right) \vec{r}$

²These notations are preferred to the usual $\Omega_\Lambda = \lambda_\circ$ and $\Omega_K = -k_\circ$ for avoiding ambiguities on the interpretation of cosmological parameters, see *e.g.*, [8],[10].

3.2 Qualitative analysis

For the analysis of the expansion of the shell S, we use the following dimensionless quantities : the *magnification* X and the *expansion rate* Y . They are defined as follows

$$X = \frac{x}{x_i}, \quad Y = \frac{\alpha}{H_o} \quad (36)$$

Their corresponding diagrams versus the expansion parameter a characterise the dynamics of S, they are obtained from Eq. (32, 35) by numerical integration. Since the mapping $t \mapsto a(t) = 1/(1+z)$ is a monotonic function of t in the present investigation, the evolution of X and Y with cosmic time t or with redshift z can be straightforwardly derived. Instead of having an exhaustive analysis, we limit our investigation around the generally accepted values $\lambda_o = 0.7$ and $\Omega_o = 0.3$. The initial conditions lie on the expansion rate Y_i and the formation date t_i , as expressed by means of $a_i = a(t_i)$. The values $a_i = 0.003$ and $Y_i = 0$ (void initially expanding with Hubble flow) are used as standard in our discussion. The kinematics of the expansion is described in Sec. 3.2.1, the dependence on cosmological parameters is analysed in Sec. 3.2.2, and on initial conditions in Sec. 3.2.3, the results are synthesised in Sec. 3.2.4.

3.2.1 Kinematics

The expansion velocity of the shell S with respect to its centre reads

$$\vec{v} = yH\vec{r}, \quad y = 1 + \frac{Y}{h}, \quad h = \frac{H}{H_o} \quad (37)$$

where y stands for the corrective factor to Hubble expansion. The general trend of the kinematics can be derived from the diagram y versus a in Fig. 1. It results from the initial conditions $a_i = 0.003$, $Y_i = 0$, and the cosmological parameters $\Omega_o = 0.3$. and $\lambda_o = 0$, $\lambda_o = 0.7$, $\lambda_o = 1.4$. The shell S expands faster than Hubble expansion ($y > 1$) at early stages of its evolution with no significant dependence on Λ . Λ -effect appears later by preserving the curve from an earlier decreasing. If $\lambda_o > 0.7$ (*i.e.*, $k_o > 0$) then it is characterised by a significant protuberance at $z \sim 1.7$, the larger the λ_o (*i.e.*, k_o) the higher the bump. It is due to the existence of a minimum value H_m of Hubble parameter H which is reached during the cosmological expansion, see Eq. (18). After this period the expansion of S reaches asymptotically Hubble behaviour. It is interesting to note that the present epoch ($a = 1$)

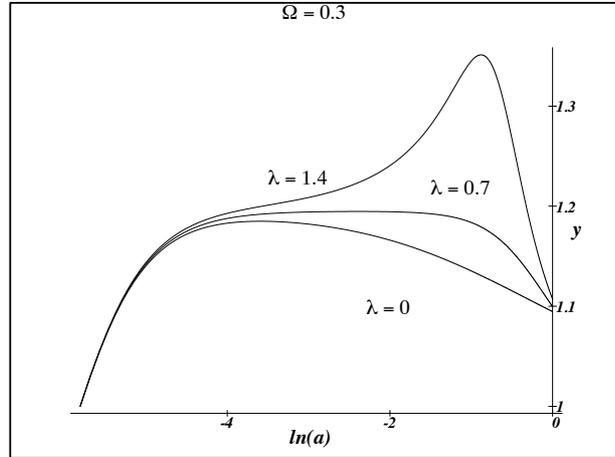


Figure 1: The corrective factor to Hubble expansion.

appears quite peculiar because of the relative proximity of curves, but it is solely an artefact³.

3.2.2 Dichotomy between cosmological parameters

The growth of spherical voids is investigated by means of X and Y diagrams with respect to cosmological parameters Ω_o and λ_o , with the aim of disentangling their related effects, herein named Ω -effect and Λ -effect respectively.

1. The dependence on the outside density. — The comparison of diagrams with a vanishing cosmological constant $\lambda_o = 0$ corresponding to parameter values $\Omega_o = 0.15, 0.3, 0.45$ on Fig. 2 shows that the larger the Ω_o the larger the magnification. Such an effect results from the attraction of shell particles toward denser regions. The growth looks like a huge burst which freezes asymptotically up to matching Hubble expansion ($Y = 0$), the larger the Ω_o the larger the expansion rate Y , see Fig. 3. This trend is not significantly modified by other acceptable values of λ_o .
2. The dependence on the cosmological constant. — With a constant $\Omega_o = 0.3$ the comparison of diagrams corresponding to parameter val-

³Indeed, the three curves cross at $a > 1$ but not all in only one point.

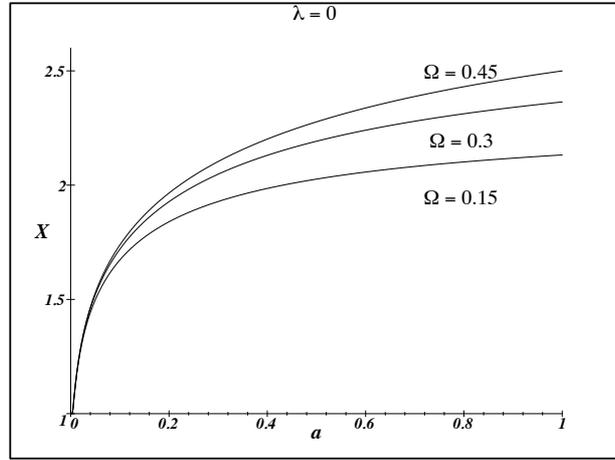


Figure 2: The magnification X — dependence on density.

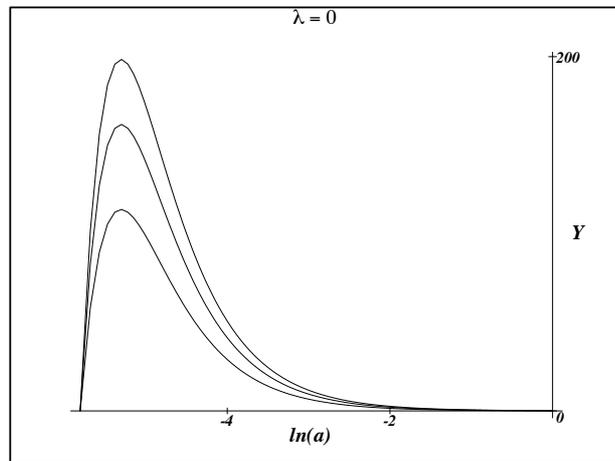


Figure 3: The expansion rate Y — dependence on density.

ues $\lambda_{\circ} = 0, 0.7, 1.4$ on Fig 4 shows that the larger the λ_{\circ} the larger the magnification. As for Ω -effect, Λ -effect amplifies the magnification

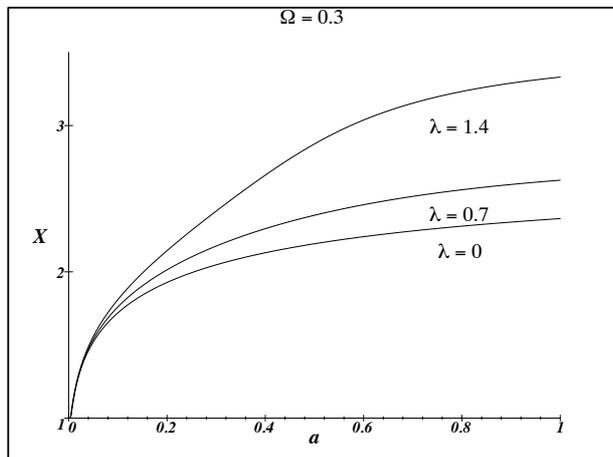


Figure 4: The magnification X — dependence on the cosmological constant.

which increases nonlinearly with λ_{\circ} . With $\Lambda > 0$ this phenomenon interprets as a repulsive effect of gravity in empty regions (also named gravitational repulsion of vacuum). The expansion rate Y does not characterises Λ since the related curves do not disentangle, see Fig 5. Figure 1 shows that Λ -effect is late and weak on Y . However, it has a cumulative effect which is reflected on X .

3. Interpretation of parameter k_{\circ} . — According to Eq. (19), the velocity of a test-particle within S reads $\vec{v} = H_{\infty} \vec{r}$. In the reference frame this motion translates as follows : in general, the test-particle moves toward the centre of S but if $k_{\circ} > 0$ then it starts moving toward its border at date $a(t^*) = \frac{8}{3} \pi G \rho_{\circ} K_{\circ}^{-1}$. Such a property of sweeping out the void region (due to $H < H_{\infty}$) interprets as a stability criterion for void regions. A dimensional analysis of Eq. (7) shows that the Newtonian interpretation of $k_{\circ} = \lambda_{\circ} + \Omega_{\circ} - 1$ corresponds to a dimensionless binding energy for the universe⁴. Because K_{\circ} is an integration constant, this provides us with a meaningful procedure for comparing

⁴The lower k_{\circ} the faster the cosmological expansion. Note that it works with an opposite direction for the expansion of spherical voids.

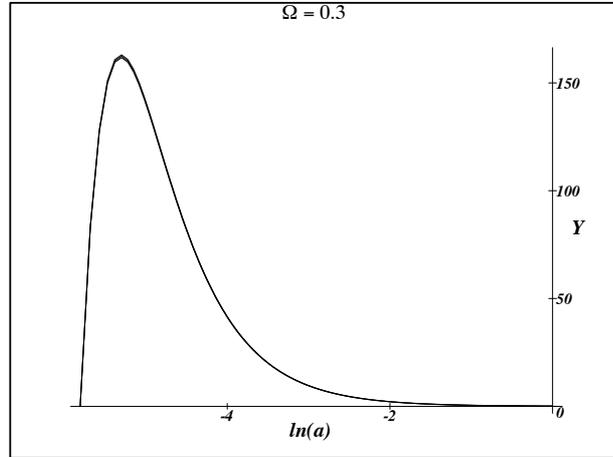


Figure 5: The expansion rate Y — dependence on the cosmological constant.

the magnitudes of Λ -effect or Ω -effect between different world models at constant k_o . For Friedmann model, k_o stands for the dimensionless spatial curvature (of the comoving space), $k_o > 0$ world models correspond to spatially closed universes.

3.2.3 Dependence on initial conditions

The dependence of S on the formation date is investigated with $Y_i = 0$ and $a_i = 0.003$, $a_i = 0.03$, and $a_i = 0.3$ respectively, and by assuming $\Omega_o = 0.3$ and $\lambda_o = 0.7$. According to Fig. 6, the earlier the date of birth the larger the magnification X . Figure 7 suggests the existence of a limiting curve $a \mapsto Y$ defined by $a_i = 0$, which characterises Ω_o . All expansion rate curves related to other formation dates are located in lower part and reach it asymptotically. One can expect that the effects on the dynamics of growth resulting from reasonable initial expansion rates $Y_i \neq 0$, related to physical processes (*e.g.*, supernovae explosions), are negligible at primordial epoch⁵, what legitimises the initial condition $Y_i = 0$ at $a_i = 0.003$. Evolutions with other initial conditions on Y_i can be deduced from that since any given point (a_i, Y_i) in the diagram belongs to a single evolution curve.

⁵Since Hubble expansion is all the more important towards the past, the earlier the formation date the weaker this effect, according to Eq. (37).

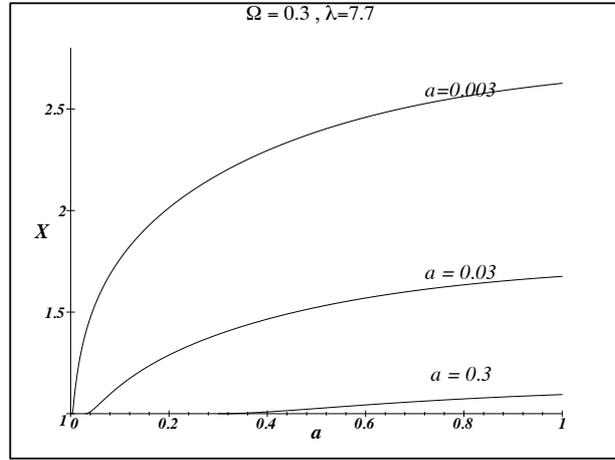


Figure 6: The magnification X — dependence on the formation date.

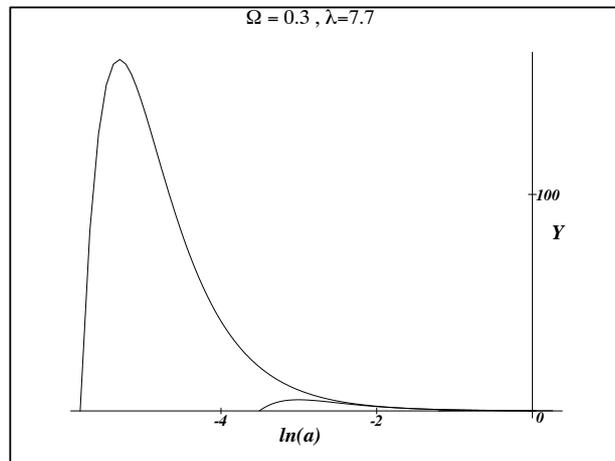


Figure 7: The expansion rate Y — dependence on the formation date.

3.2.4 Synthesis

As a result, the expansion of a spherical void does not show a *linear regime*, but a huge initial burst which freezes asymptotically up to matching Hubble expansion. Its radius (in the reference frame) x increases from an initial size x_i with the cosmological parameters Ω_o and λ_o . The related individual effects interpret respectively by the gravitational attraction from the outer parts and by the gravitational repulsion of its borders (or the vacuum from the inner parts). The larger these parameters the higher the magnification $X = x/x_i$. The dynamics is sensitive to Ω -effect at primordial epochs and to Λ -effect later on. The evolution of its expansion velocity (in the reference frame) $\vec{v}_c = YH_o\vec{x}$ with time does not characterise λ_o but Ω_o . On the other hand, the cosmological constant intervenes significantly on the kinematics by means of the corrective factor y to Hubble expansion by preserving the expansion from an earlier decreasing. Moreover, spatially closed world models are characterised by a significant expansion of voids which reaches its maximum at redshift $z \sim 1.7$ (with $\Omega_o \sim 0.3$), the larger the λ_o (*i.e.*, k_o) the higher the expansion rate. Notwithstanding, the perturbation on redshift of sources located on the shell of expanding voids does not exceed

$$\Delta z = \frac{XY}{1+z} \frac{x_i H_o}{c} \quad (38)$$

which is a tiny value ($\sim 10^{-3}$) because of counterbalancing behaviours of X and Y . The interior of voids shows a de Sitter expansion $\vec{v} = \sqrt{\Lambda/3}\vec{r}$ which sweeps out it if $k_o > 0$, what interprets as a stability criterion. The expansion rate evolutions related to formation dates show a common envelope curve which characterises the density parameter Ω_o .

3.3 Discussion

It is clear that such a simple model should be applied with caution to observed distribution of galaxies since the effects related to foamlike patterns are not considered. Moreover, because Newtonian approach is used instead of general relativity, it applies solely to reasonable sized voids⁶ so that the main features should remain qualitatively reliable. On the other hand, based on the connection of two exact solutions of Euler-Poisson equations system,

⁶At first glance, one might argue that the related scale should be larger than the size of background structures in order to make the uniform density hypothesis an acceptable approximation, what looks like a dilemma. However, such a hypothesis, which applies to the entire universe, is ensured by the isotropy of CMB.

which is a huge advantage for investigating properly the behaviour of a single void, it provides us with an appreciable hint to the dynamics with respect to cosmological parameters.

4 Conclusion

We investigate the dynamics of an isotropic universe constituted by a spherical void surrounded by a uniform distribution of dust by means of Euler-Poisson equations system with cosmological constant. It turns out that the behaviours of both regions (inside and outside the shell) coincide with Newton-Friedmann (NF) solutions. The connection conditions between these two regions are investigated in Classical Mechanics, what provides us with the dynamics of the shell. Such a schema does not correspond to the usual embedding of a void solution into a cosmological background solution, but interprets as a non linear perturbation of NF models.

The general behaviour of the void expansion shows a huge initial burst, which freezes asymptotically up to match Hubble expansion. While the corrective factor to Hubble law on the shell depends weakly on cosmological constant at early stages, it enables us to disentangle significantly cosmological models around redshift $z \sim 1.7$. The magnification of spherical voids increases with the density parameter and with the cosmological constant. An interesting feature is that for spatially closed Friedmann models, the empty regions are swept out, what provides us with a stability criterion.

A Appendix : Covariant formulation of Euler-Poisson equations system

Souriau's covariant formulation of Euler-Poisson equations[28] can be summarized as follows : The geometrical interpretation of Newton dynamics[6] shows that the component of gravitational field \vec{g} identify to the only non null Christoffel components of newtonian connexion

$$\Gamma_{00}^j = -g^j$$

Hence, one obtains their expression in the new coordinates system defined in Eq. (5), and the only non null components read

$$(\Gamma_c)^j_{k0} = H\delta_k^j, \quad (\Gamma_c)^j_{00} = -g_c^j$$

Let \mathcal{T} be a function(al) defined on the set of symmetric density tensors $x \mapsto \gamma$ on newtonian spacetime \mathbb{R}^4

$$\mathcal{T}(x \mapsto \gamma) = \int_{\mathbb{R}^4} T^{\mu\nu} \gamma_{\mu\nu} dt dV$$

where $dt dV$ stands for the volume element and T for a symmetric contravariant tensors which accounts for the media. The measure density \mathcal{T} is eulerian if and only if it vanishes for all covariant tensor fields which reads

$$\gamma_{\mu\nu} = \frac{1}{2} \left(\hat{\partial}_\mu \xi_\nu + \hat{\partial}_\nu \xi_\mu \right)$$

where $\hat{\partial}$ stands for the covariant derivative and $x \mapsto \xi$ for a compact support 1-form. In such a case, it is obvious to show that

$$\hat{\partial}_\mu T^{\mu\nu} = 0$$

interprets as Euler equations, in any coordinates systems.

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