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Guided Quasicontinuous Atom Laser

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We report the first realization of a guided quasicontinuous atom laser by rf outcoupling a BEC, from a hybrid optomagnetic trap into a horizontal atomic waveguide. This configuration allows us to cancel the acceleration due to gravity and keep the de Broglie wavelength constant at $0.5 \mu\text{m}$ during 0.1 s of propagation. We also show that our configuration, equivalent to *pigtailing* an optical fiber to a (photon) semiconductor laser, ensures an intrinsically good transverse mode matching.

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The Bose-Einstein condensation of atoms in the lowest level of a trap represents the matter-wave analog to the accumulation of photons in a single mode of a laser cavity. In analogy to photonic lasers, atom lasers can be obtained by outcoupling from a trapped Bose-Einstein condensate (BEC) to free space [1, 2, 3]. However, since atoms are massive particles, gravity plays an important role in the laser properties: in the case of rf outcouplers, it lies at the very heart of the extraction process [4] and in general, the beam is strongly accelerated downwards, causing a rapid decrease of the de Broglie wavelength. With the growing interest in coherent atom sources for atom interferometry [5, 6, 7] and new studies of quantum transport phenomena [8, 9, 10, 11, 12, 13, 14] where large and well defined de Broglie wavelength are desirable, a better control of the atomic motion during its propagation is needed. One solution is to couple the atom laser into a horizontal waveguide, so that the effect of gravity is canceled, leading to the realization of a coherent matter wave with constant wavelength.

We report in this letter on the realization of such a guided quasicontinuous atom laser, where the coherent source, *i.e.* the trapped BEC, and the guide are *merged* together in a hybrid combination of a magnetic Ioffe-Pritchard trap and a horizontally elongated far off-resonance optical trap constituting an atomic waveguide (see Fig. 1). The BEC, in a state sensitive to both trapping potentials, is submitted to a rf outcoupler yielding atoms in a state sensitive only to the optical potential, resulting in an atom laser propagating along the weak confining axis of the optical trap. In addition to canceling the effect of gravity, this configuration has several advantages. Firstly, coupling into a guide from a BEC rather than from a thermal sample [15] allows us to couple a significant flux into a small number of transverse modes of the guide. Secondly, the weak longitudinal trapping potential of the guide can be compensated by the antitrapping potential due to the second order Zeeman effect acting onto the outcoupled atoms, resulting in an atom laser with a quasiconstant de Broglie wavelength. Thirdly, using an rf outcoupler rather than releasing a BEC into a guide [14, 16] results into quasi-

continuous operation, thus insuring sharp linewidth, and gives a better control on the beam parameters. Indeed, changing the frequency of the outcoupler allows one to *tune* the value of the de Broglie wavelength of the atom laser, and adjusting the rf coupler power allows one to independently vary the atom-laser density from the interacting regime to the noninteracting one [17]. In particular, those advantages opens new prospects for studying quantum transport phenomena, as, for instance, quantum reflection [18], where interactions dramatically suppress the reflection probability [19]. Finally, in spite of the lensing effect due to the interaction of the atom laser with the trapped BEC [3, 20], adiabatic transverse mode matching results into the excitation of only a small number of transverse modes, and we discuss the possibility of achieving single transverse mode operation.

Our setup [21] produces magnetically trapped cold clouds of ^{87}Rb in the $|F, m_F\rangle = |1, -1\rangle$ state. After evaporative cooling to $1 \mu\text{K}$, an optical guide produced by 120 mW of Nd:YAG laser ($\lambda = 1064 \text{ nm}$) focussed on a waist of $30 \mu\text{m}$ is superimposed along the z direction and after a final evaporation ramp of 6 s [22], Bose-Einstein condensation is directly obtained in the optomagnetic trap. We estimate the condensed fraction to 80% ($T \approx 0.4T_c \approx 150 \text{ nK}$) with 10^5 atoms in the

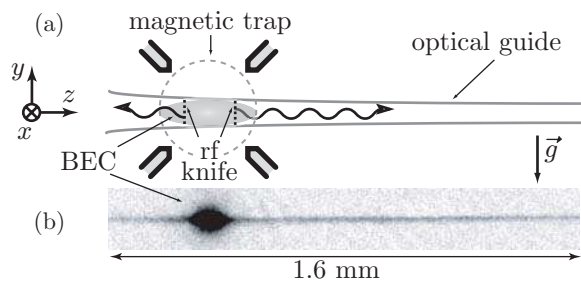


FIG. 1: (a) Setup. The BEC is produced at the intersection of a magnetic trap and a horizontal elongated optical trap acting as a waveguide for the atom laser. A “rf knife” provides outcoupling into the waveguide and an atom laser is emitted on both sides. (b) Absorption image (along x) of a guided atom laser after 100 ms of outcoupling.

BEC. In this hybrid trap, the optical guide ensures a tight transverse confinement, with oscillation frequencies $\omega_{x,y}/2\pi = \omega_{\perp}/2\pi = 360$ Hz, large compared to those of the magnetic trap ($\omega_x^m/2\pi = 8$ Hz and $\omega_y^m/2\pi = 35$ Hz). In contrast, the confinement along the z axis is due to the shallow magnetic trap with an oscillation frequency $\omega_z^m/2\pi = 35$ Hz. The chemical potential is then $\mu_{\text{BEC}}/h \simeq 3.2$ kHz and the Thomas-Fermi radii are $R_z = 25$ μm and $R_{\perp} = 2.4$ μm . The guided atom laser is obtained by rf-induced magnetic transition [2] between the $|1, -1\rangle$ state and the $|1, 0\rangle$ state, which is submitted to the same transverse confinement due to the optical guide, but is not sensitive (at first order) to the magnetic trapping. We thus obtain a guided coherent matter wave propagating along the optical guide [Fig. 1(b)]. This configuration, where the optical guide dominates the transverse trapping of both the source BEC and the atom laser, enables to collect the outcoupled atoms into the guide with 100% efficiency.

As explained below, the propagation of the guided atom laser, after leaving the region of interaction with the remaining BEC, is dominated by a potential $V_{\text{guide}}(z)$ resulting from the repulsive second order Zeeman effect $V_{\text{ZQ}}(z) = -m\omega_{\text{ZQ}}^2(z - z_m)^2/2$ and the weakly trapping optical potential $V_{\text{op}}(z) = m\omega_{\text{op}}^2(z - z_0)^2/2$, where z_m and z_0 are respectively the magnetic and optical traps centers relative to the BEC center [23]. For our parameters the curvatures of $V_{\text{ZQ}}(z)$ and $V_{\text{op}}(z)$ cancel each other ($\omega_{\text{op}}/2\pi \simeq \omega_{\text{ZQ}}/2\pi = 2$ Hz), so that $V_{\text{guide}}(z)$ is nearly linear, with a slope corresponding to an acceleration $a_{\text{guide}} = \omega_{\text{op}}^2 z_0$, several orders of magnitude smaller than gravity [Fig. 2]. Then the atom-laser velocity remains almost constant at $v = 9$ mm.s^{-1} , corresponding to a de Broglie wavelength $\lambda_{\text{db}} = h/mv$ of 0.5 μm .

Besides its de Broglie wavelength, an atom laser is characterized by its flux. In quasicontinuous rf outcoupling and in the weak coupling regime [4, 24], this flux can be controlled by adjusting the rf power. We work at a flux $\mathcal{F} = 5 \times 10^5$ at.s^{-1} which is appropriate for efficient absorption imaging of the atom laser. The dimensionless parameter $n_{1\text{D}}a_s$ characterizing the interactions [25] is about 0.25. In this expression, $a_s = 5.3$ nm is the (3D) atomic scattering length and $n_{1\text{D}}$ is the linear density ($n_{1\text{D}} = \mathcal{F}/v \simeq 45$ $\text{at.}\mu\text{m}^{-1}$ at $v = 9$ mm.s^{-1}). For $n_{1\text{D}}a_s < 1$ we are in the “1D mean field” regime [26], where the mean-field intralaser interaction may influence the longitudinal dynamics but not the transverse one.

Our modeling of the dynamics of the guided atom laser is based on the formalism used in [25]. The strong transverse confinement allows us to assume that the quantized transverse dynamics adiabatically follows the slowly varying transverse potential as the laser propagates along the z axis. In this “quasi-1D regime”, the laser wave function takes the form:

$$\Psi(\vec{r}, t) = \phi(z, t)\psi_{\perp}(\vec{r}_{\perp}, z) \quad (1)$$

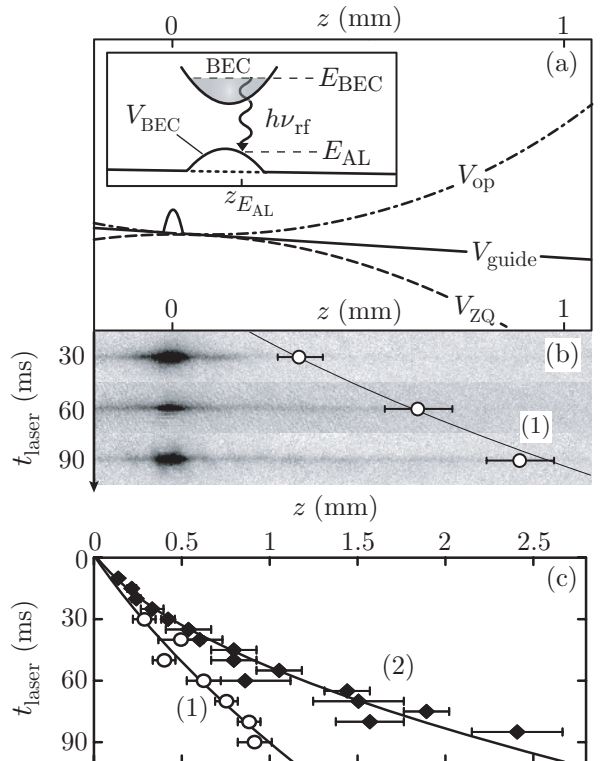


FIG. 2: Longitudinal dynamics of the guided atom laser. (a) Longitudinal potential $V_{\text{guide}} + V_{\text{BEC}}$, sum of the quadratic Zeeman (dashed), optical (dash-dot) and BEC mean-field (inset) potentials. (b) Guided atom laser after different lasing times t_{laser} . These images allow us to determine the wavefront position (estimated error bars are shown). (c) Wavefront position versus t_{laser} for two different adjustments of the optical potential. Each set of data is fitted by a second degree polynomial, yielding the same initial velocity $v_0 = 9 \pm 2$ mm.s^{-1} , and different accelerations $a_1 = 0.07 \pm 0.06$ m.s^{-2} (1) and $a_2 = 0.36 \pm 0.04$ m.s^{-2} (2).

with the normalization $\int |\psi_{\perp}|^2 d\vec{r}_{\perp} = 1$ so that the linear density is $n_{1\text{D}} = \int |\Psi|^2 d\vec{r}_{\perp} = |\phi(z, t)|^2$. In the following we will assume that $\psi_{\perp}(\vec{r}_{\perp}, z)$ is the ground state of the local transverse potential including the mean-field interaction due to the BEC, so that it matches perfectly the BEC transverse shape in the overlap region and evolves smoothly to a gaussian afterwards. The longitudinal dynamics can then be described in terms of hydrodynamical equations, bearing on $n_{1\text{D}}$ and the phase velocity $v = \hbar \nabla \mathcal{S} / m$ such that $\phi = \sqrt{n_{1\text{D}}} e^{i\mathcal{S}}$. In the stationary regime, for an atom laser of energy E_{AL} , these equations reduce to the atomic flux and energy conservations:

$$n_{1\text{D}}(z) v(z) = \mathcal{F}, \quad (2)$$

$$\frac{1}{2} m v(z)^2 + V_{\text{guide}}(z) + \mu(z) = E_{\text{AL}}. \quad (3)$$

The quantity $\mu(z)$ is an effective local chemical potential which takes into account both intralaser interaction and transverse confinement [25]. Inside the BEC, $\mu(z)$

is dominated by the interaction with the trapped BEC and we can rewrite $\mu(z) = V_{\text{BEC}}(z) = \mu_{\text{BEC}}(1 - z^2/R_z^2)$. Outside the BEC and in the “1D mean field” regime, one has $\mu(z) = \hbar\omega_{\perp}(1 + 2a_s n_{1\text{D}}(z))$.

To write Eq. (3), we have neglected the longitudinal quantum pressure since the density $n_{1\text{D}}$ varies smoothly along z . With this simplification, Eqs. (2) and (3) are equivalent to the standard WKB approximation. The amplitude of $\phi(z, t)$ is determined by the flux \mathcal{F} [Eq. (2)] and its phase $\mathcal{S}(z)$ can be derived from the classical motion of an atom of energy E_{AL} submitted to the 1D potential $V_{\text{AL}}(z) = V_{\text{guide}}(z) + \mu(z)$. The parameters E_{AL} and \mathcal{F} , determining the atom-laser wave function, are fixed by the frequency and power of the output coupler.

In the weak coupling regime, the coupling between the trapped BEC and the continuum of propagating atom-laser wave functions can be described by the Fermi Golden Rule (see [4] and references *therein*). The atom-laser energy is thus given by the resonance condition

$$E_{\text{AL}} = E_{\text{BEC}} - \hbar\nu_{\text{rf}}, \quad (4)$$

where E_{BEC} is the BEC energy, and the coupling rate, which determines \mathcal{F} , depends on the overlap integral between the BEC and the atom-laser wave functions. For a uniformly accelerated atom laser, the longitudinal wave function $\phi(z, t)$ is an Airy function with a narrow lobe around the classical turning point $z_{E_{\text{AL}}}$, defined by $v(z_{E_{\text{AL}}}) = 0$ in Eq. (3), and the overlap integral is proportional to the BEC wave function at $z_{E_{\text{AL}}}$ [4]. This can be interpreted by the so-called Franck-Condon principle, which states that the rf coupler selects, via the resonance condition, the atom laser extraction position $z_{E_{\text{AL}}}$ [27]. In contrast to the case where the atom laser is extracted by gravity, here the acceleration due to $V_{\text{guide}}(z)$ is small enough that the potential $V_{\text{AL}}(z)$ is dominated by the bump $V_{\text{BEC}}(z)$ [Fig. 2(a)], so that there are two outcoupling points corresponding to two atom lasers emitted on both sides of the trapped condensate [Fig. 2(b)]. If the slope of the potential $ma(z_{E_{\text{AL}}})$ varies slowly around the outcoupling point at the scale of the first lobe of the corresponding Airy function, the atom-laser wave function can be locally approximated by the Airy function and we can use the result of [4] where gravity acceleration is replaced by $a(z_{E_{\text{AL}}})$:

$$\mathcal{F} = \frac{\pi\hbar\Omega_{\text{rf}}^2}{2} \frac{n_{1\text{D}}^{\text{BEC}}(z_{E_{\text{AL}}})}{ma(z_{E_{\text{AL}}})}. \quad (5)$$

Here Ω_{rf} is the Rabi frequency characterizing the rf coupling between the different atomic internal states, and $n_{1\text{D}}^{\text{BEC}}(z) = \int d\vec{r}_{\perp} |\psi_{\text{BEC}}(\vec{r}_{\perp}, z)|^2$ is the condensate linear density. More rigorously, one can solve the Schrödinger equation in a parabolic antitrapping potential [28]. We checked that the two calculations give the same result when the local slope approximation is valid, and the second approach is necessary only when the coupling is close

to the maximum of the potential bump. As expected, the flux is then predicted to reach its maximum value.

The modeling above allows us to analyze our experimental data. Firstly, for a Rabi frequency of $\Omega_{\text{rf}}/2\pi = 40$ Hz, a BEC of $N_{\text{BEC}} \simeq 10^5$ atoms and assuming a coupling at about $5 \mu\text{m}$ from the center of the BEC, Eq. (5) gives $\mathcal{F} = 5 \times 10^5 \text{ at.s}^{-1}$, in agreement with the observed decay of the atom number in the BEC. Secondly, this modeling shows that with our parameters, the axial dynamics of the atom laser associated to Eqs. (2) and (3) is revealed by the propagation of the wavefront of the atom laser [Fig. 2(b)]. Indeed, out of the region of overlap with the trapped BEC, and for a coupling close to the potential maximum, the atoms have a kinetic energy of the order of the BEC chemical potential ($\mu_{\text{BEC}}/h \simeq 3.2$ kHz), large compared to $\mu(z)$ ($\mu(z)/h \sim \omega_{\perp}/2\pi = 360$ Hz). We can thus neglect $\mu(z)$ in Eq. (3), and out of the BEC the wavefront acceleration is dominated by $V_{\text{guide}}(z)$, while the atomic velocity just leaving the BEC is determined by $V_{\text{BEC}}(z_{E_{\text{AL}}})$. For an outcoupling at the center of the BEC, the expected value is $v_0 \simeq 5.4 \text{ mm.s}^{-1}$, somewhat less than the observed value $v_0 = 9 \pm 2 \text{ mm.s}^{-1}$. The discrepancy will be discussed below.

We now turn to the transverse mode of the guided atom laser. To characterize it, we measure the transverse energy using a time-of-flight: after 60 ms of propagation, the optical guide is suddenly switched off and we measure the expansion along the y axis. The evolution of the rms size is directly related to the transverse kinetic energy according to $\sigma(t)^2 = \sigma_0^2 + \langle v_y^2 \rangle t^2$, where σ_0 is the resolution of the imaging system ($7.5 \mu\text{m}$) which dominates the initial transverse size ($0.6 \mu\text{m}$). A fit gives $\langle v_y^2 \rangle = 4.5 \pm 0.2 \text{ mm}^2/\text{s}^2$. Assuming cylindrical symmetry, this corresponds to a total transverse energy $E_{\perp} = (5.5 \pm 0.8)\hbar\omega_{\perp}$, *i.e.* an average excitation quantum number of 2 along each transverse direction. This shows that only a few transverse modes are excited, and we may wonder whether single transverse mode operation is achievable.

Actually, we expect the atom laser to be outcoupled in its lowest transverse mode. Indeed, the transverse potential experienced by an atom in the atom laser has the same shape as the one experienced by an atom of the BEC, *i.e.*, in the Thomas-Fermi approximation, quadratic trapping edges and a flat bottom of width $2R_{\perp}(z)$. As z increases, this width decreases monotonically to 0 until the point where the atom laser leaves the BEC and experiences a pure harmonic potential. A numerical simulation shows that this evolution is smooth enough to enable the transverse atom-laser wave function $\psi_{\perp}(r_{\perp}, z)$ to adiabatically adjust to the local ground state, resulting in the prediction of almost single-mode emission. The observed multimode behavior may be attributed to different experimental imperfections, which can be fixed in future experiments. Firstly, if the magnetic trap is not exactly centered on the optical guide,

transverse mode matching between the BEC and the guide is not perfect. Secondly, excitation of higher transverse modes can be provoked by the position noise of the guide (we observe a heating rate of 100 nK/s). Finally, a numerical resolution of the coupled Gross-Pitaevskii equations suggests that at our value of the atomic flux, the BEC decay is not adiabatic enough [4] so that the outcoupling could induce excitations inside the BEC and thus increase the energy transferred to the atom laser. This might also explain why the observed values of atom-laser velocity correspond to an energy somewhat larger than μ_{BEC} .

In conclusion, we have demonstrated a scheme for efficiently coupling a BEC into a waveguide. We have obtained a guided atom laser with an almost constant de Broglie wavelength, at a value of $0.5\ \mu\text{m}$, and by coupling near the boundary of the BEC it should be possible to obtain even larger de Broglie wavelengths. Such values are of interest for experiments in atom interferometry as, for instance, the coherent splitting at the crossing of two matterwave guides [29, 30], which could be implemented in miniaturized components [31]. Furthermore, as the atomic wavelength reaches values similar to visible light wavelength, transport properties through wells, barriers or disordered structures engineered with light should enter the quantum regime [8, 9, 10, 11, 12, 13, 14]. Also the control of the atom-laser flux offers the possibility to tune the amount of interaction inside the guided atom-laser beam. For instance, the possibility of combining a large and well defined de Broglie wavelength together with a density small enough to suppress interactions, should provide the conditions to observe Anderson-like localization [13]. On the other hand, the interacting regime should allow investigation of effects such as the breakdown of superfluidity through obstacles [9, 10], or nonlinear resonant transport [11, 12]. We thus believe that our scheme constitutes a very promising tool for further development of coherent guided atom optics.

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