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A Guided Quasicontinuous Atom Laser

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We report the first realization of a guided quasicontinuous atom laser by rf outcoupling a BEC from a hybrid optomagnetic trap into a horizontal atomic waveguide. This configuration allows us to cancel the acceleration due to gravity and thus to keep the de Broglie wavelength constant at a value of about $0.5 \mu\text{m}$ during 0.1 s of propagation. We also show that our configuration, equivalent to *pigtailing* an optical fiber to a (photon) semiconductor laser, ensures an intrinsically good transverse mode matching.

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The Bose-Einstein condensation of atoms in the lowest level of a trap represents the matter-wave analog to the accumulation of photons in a single mode of a laser cavity. In analogy to photonic lasers, atom lasers can be obtained by outcoupling from a trapped Bose-Einstein condensate (BEC) to free space [1, 2, 3]. However, since atoms are massive particles, gravity plays an important role in the laser properties: in the case of rf outcouplers, it lies at the very heart of the extraction process [4] and in general, the beam is strongly accelerated downwards, causing a rapid decrease of the de Broglie wavelength. With the growing interest in coherent atom sources for atom interferometry [5, 6, 7] and new studies of quantum transport phenomena [8, 9, 10, 11, 12, 13, 14] where large de Broglie wavelength are desirable, a better control of the atomic motion during its propagation is needed. One solution is to couple the atom laser into a horizontal waveguide, so that the effect of gravity is canceled, leading to the realization of a coherent matter wave with constant wavelength.

We report in this letter on the realization of such a guided quasicontinuous atom laser, where the coherent source, *i.e.* the trapped BEC, and the guide are *merged* together in a hybrid combination of a magnetic Ioffe-Pritchard trap and an horizontally elongated far off-resonance optical trap constituting an atomic waveguide (see Fig. 1). The BEC, in a state sensitive to both trapping potentials, is submitted to a rf outcoupler yielding atoms in a state sensitive only to the optical potential, resulting in an atom laser propagating along the weak confining axis of the optical trap. In addition to canceling the effect of gravity, this configuration has several advantages. Firstly, coupling into a guide from a BEC rather than from a thermal sample [15] allows us to couple a significant flux into a small number of transverse modes of the guide. Secondly, using a rf outcoupler rather than releasing a BEC into a guide [14, 16] results into quasicontinuous operation. Thirdly, it is possible to compensate the weak longitudinal trapping potential of the guide by the antitrapping potential due to the second order Zeeman effect acting onto the outcoupled atoms, re-

sulting in an atom laser with a quasiconstant de Broglie wavelength (about $0.5 \mu\text{m}$ over 0.1 s of propagation in this work). Fourthly, changing the frequency of the outcoupler allows one to *tune* the value of the de Broglie wavelength of the atom laser. Fifthly, by adjusting the rf coupler power, one can vary the atom-laser density from the interacting regime to the noninteracting regime [17], offering the prospect to study linear as well as nonlinear atom optics phenomena. Finally, in spite of the lensing effect due to the interaction of the atom laser with the trapped BEC [3, 18], adiabatic transverse mode matching results into the excitation of only a small number of transverse modes, and we discuss the possibility of achieving single transverse mode operation of the atom laser.

Our setup [19] produces magnetically trapped cold clouds of ^{87}Rb in the $|F, m_F\rangle = |1, -1\rangle$ state. During the evaporative cooling, an optical guide produced by 120 mW of Nd:YAG laser ($\lambda = 1064 \text{ nm}$) focussed on a waist of $30 \mu\text{m}$ is superimposed along the z direction and Bose-Einstein condensation is directly obtained in the optomagnetic trap. In this hybrid trap, the optical guide ensures a tight transverse confinement, with oscillation

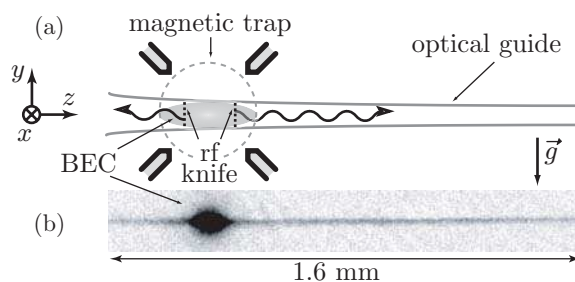


FIG. 1: (a) Schematic view of the setup. The BEC is produced at the intersection of a magnetic trap and a horizontal elongated optical trap acting as an atomic waveguide for the outcoupled atom laser. An “rf knife” provides outcoupling into the waveguide and an atom laser is emitted on both sides. (b) Absorption image (along x) of a guided atom laser after 100 ms of outcoupling.

frequencies $\omega_{x,y}/2\pi = \omega_{\perp}/2\pi = 360$ Hz, large compared to the frequencies characterizing the magnetic confinement along the same axes ($\omega_x^m/2\pi = 8$ Hz and $\omega_y^m/2\pi = 35$ Hz). In contrast, the longitudinal confinement of the BEC along the z axis is due to the shallow magnetic trap with an oscillation frequency $\omega_z^m/2\pi = \omega_m/2\pi = 35$ Hz. A BEC of 10^5 atoms has then a chemical potential $\mu_{\text{BEC}}/h \simeq 3.2$ kHz and Thomas-Fermi radii $R_z = 25$ μm and $R_{\perp} = 2.4$ μm . The guided atom laser is obtained by rf-induced magnetic transition [2] between the $|1, -1\rangle$ state and the $|1, 0\rangle$ state, which is submitted to the same transverse confinement due to the optical guide, but is not sensitive (at first order) to the magnetic trapping. We thus obtain a quasicontinuous guided coherent matter wave freely propagating along the optical guide [Fig. 1(b)]. This configuration, where the optical guide dominates the transverse trapping of both the source BEC and the atom laser, enables to collect the outcoupled atoms into the optical guide with 100% efficiency.

As will be explained in more details below, the propagation of the guided atom laser, after leaving the region of interaction with the remaining BEC, is dominated by a potential $V_{\text{guide}}(z)$ resulting from the repulsive second order Zeeman effect $V_{\text{ZQ}}(z) = -m\omega_{\text{ZQ}}^2(z - z_m)^2/2$ and the weakly axially trapping optical potential $V_{\text{op}}(z) = m\omega_{\text{op}}^2(z - z_0)^2/2$, where z_m and z_0 are respectively the magnetic and optical traps centers relative to the BEC center [20]. For our parameters the curvatures of $V_{\text{ZQ}}(z)$ and $V_{\text{op}}(z)$ cancel each other ($\omega_{\text{op}}/2\pi \simeq \omega_{\text{ZQ}}/2\pi = 2$ Hz), so that $V_{\text{guide}}(z)$ is nearly linear, with a slope corresponding to an acceleration $a_{\text{guide}} = \omega_{\text{op}}^2 z_0$, several orders of magnitude smaller than gravity for a proper tuning of the optical guide parameters [Fig. 2]. Then the atom-laser velocity remains almost constant with a value of $v = 9$ $\text{mm}\cdot\text{s}^{-1}$, corresponding to a de Broglie wavelength $\lambda_{\text{db}} = h/mv$ of 0.5 μm .

Besides its de Broglie wavelength, an atom laser is characterized by its flux. In quasicontinuous rf outcoupling and in the weak coupling regime [4, 21], this flux can be controlled by adjusting the rf power. We work at a flux $\mathcal{F} = 5 \times 10^5$ $\text{at}\cdot\text{s}^{-1}$ which is appropriate for efficient absorption imaging of the atom laser. In this situation, the dimensionless parameter $n_{1\text{D}}a_s$ characterizing the interactions [22] is about 0.25. In this expression, $a_s = 5.3$ nm is the (3D) atomic scattering length and $n_{1\text{D}}$ is the linear density ($n_{1\text{D}} = \mathcal{F}/v \simeq 45$ $\text{at}\cdot\mu\text{m}^{-1}$ at $v = 9$ $\text{mm}\cdot\text{s}^{-1}$). Since $n_{1\text{D}}a_s$ is smaller than 1, we are in the “1D mean field” regime [23], where the mean-field intralaser interaction may influence the longitudinal dynamics but not the transverse one.

Let us now give an overview of our modeling of the dynamics of the guided atom laser, based on the formalism used in [22]. The strong transverse confinement allows us to assume that the quantized transverse dynamics adiabatically follows the slowly varying transverse potential as the laser propagates along the z axis. In this “quasi-

1D regime”, the laser wave function takes the form:

$$\Psi(\vec{r}, t) = \phi(z, t)\psi_{\perp}(\vec{r}_{\perp}, z) \quad (1)$$

with the normalization condition $\int |\psi_{\perp}|^2 d\vec{r}_{\perp} = 1$ so that the linear density is $n_{1\text{D}} = \int |\Psi|^2 d\vec{r}_{\perp} = |\phi(z, t)|^2$. In the following we will assume that $\psi_{\perp}(\vec{r}_{\perp}, z)$ is the ground state of the local transverse potential including the mean-field interaction due to the BEC, so that it matches perfectly the BEC transverse shape in the overlap region and evolves smoothly to a gaussian afterwards. The longitudinal dynamics can then be described in terms of hydrodynamical equations, bearing on $n_{1\text{D}}$ and the phase velocity $v = \hbar\nabla S/m$ such that $\phi = \sqrt{n_{1\text{D}}} e^{iS}$. In the stationary regime, for an atom laser of energy E_{AL} , these equations reduce to the atomic flux and energy conser-

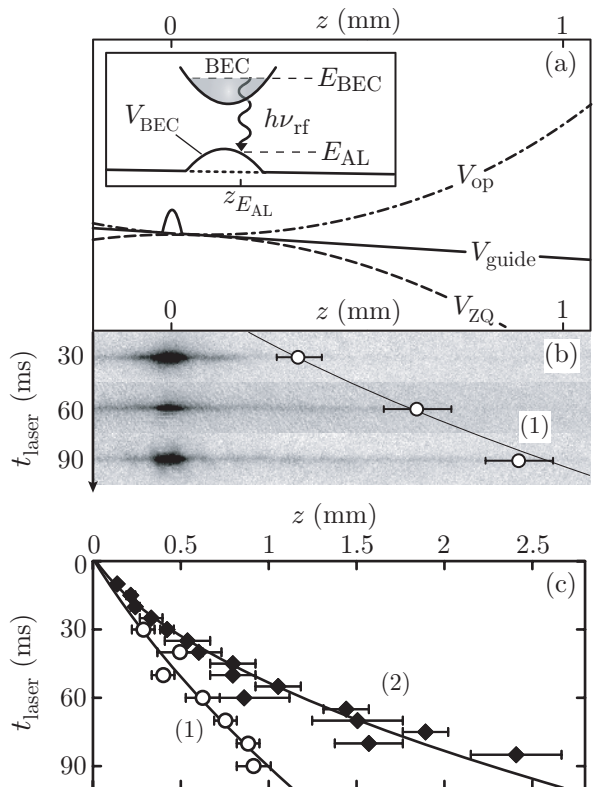


FIG. 2: Longitudinal dynamics of the guided atom laser. (a) Plot of the longitudinal potential $V_{\text{guide}} + V_{\text{BEC}}$, sum of the quadratic Zeeman (dashed), optical (dash-dot) and BEC mean-field (inset) potentials. (b) Propagation of the guided laser for different lasing times t_{laser} . These images allow us to determine the wavefront position (estimated error bars are shown). (c) Wavefront position versus t_{laser} for two different adjustments of the optical potential. Each set of data is fitted by a second degree polynomial, yielding the same initial velocity $v_0 = 9 \pm 2$ $\text{mm}\cdot\text{s}^{-1}$, and different accelerations $a_1 = 0.07 \pm 0.06$ $\text{m}\cdot\text{s}^{-2}$ (1) and $a_2 = 0.36 \pm 0.04$ $\text{m}\cdot\text{s}^{-2}$ (2).

vations:

$$n_{1D}(z) v(z) = \mathcal{F}, \quad (2)$$

$$\frac{1}{2}mv(z)^2 + V_{\text{guide}}(z) + \mu(z) = E_{\text{AL}}. \quad (3)$$

The quantity $\mu(z)$ is an effective local chemical potential that takes into account both intralaser interaction and transverse confinement [22]. Inside the BEC, $\mu(z)$ is dominated by the interaction with the trapped BEC and we can rewrite $\mu(z) = V_{\text{BEC}}(z) = \mu_{\text{BEC}}(1 - z^2/R_z^2)$. Outside the BEC and in the ‘‘1D mean field’’ regime, one has $\mu(z) = \hbar\omega_{\perp}(1 + 2a_s n_{1D}(z))$.

To write Eq. (3), we have neglected the longitudinal quantum pressure since the density n_{1D} varies smoothly along z . With this simplification, Eqs. (2) and (3) are equivalent to the standard WKB approximation. The amplitude of $\phi(z, t)$ is determined by the flux \mathcal{F} [Eq. (2)] and its phase $\mathcal{S}(z)$ can be derived from the classical motion of an atom of energy E_{AL} submitted to the 1D potential $V_{\text{AL}}(z) = V_{\text{guide}}(z) + \mu(z)$. The two parameters E_{AL} and \mathcal{F} , determining the atom-laser wave function, are fixed by the frequency and power of the output coupler, as discussed in the next paragraph.

In the weak coupling regime, the coupling between the trapped BEC and the continuum of propagating atom-laser wave functions can be described by the Fermi Golden Rule (see [4] and references *therein*). The atom-laser energy is thus given by the resonance condition

$$E_{\text{AL}} = E_{\text{BEC}} - \hbar\nu_{\text{rf}}, \quad (4)$$

and the coupling rate, which determines \mathcal{F} , depends on the overlap integral between the BEC and the atom-laser wave functions. For a uniformly accelerated atom laser, the longitudinal wave function $\phi(z, t)$ is an Airy function with a narrow lobe around the classical turning point $z_{E_{\text{AL}}}$, defined by $v(z_{E_{\text{AL}}}) = 0$ in Eq. (3), and the overlap integral is proportional to the BEC wave function at $z_{E_{\text{AL}}}$ [4]. This can be interpreted by the so-called Franck-Condon principle, which states that the rf coupler selects, via the resonance condition, the atom laser extraction position $z_{E_{\text{AL}}}$ [24]. In contrast to the case where the atom laser is extracted by gravity, here the acceleration due to $V_{\text{guide}}(z)$ is small enough that the potential $V_{\text{AL}}(z)$ is dominated by the bump $V_{\text{BEC}}(z)$ [Fig. 2(a)], so that there are in general two outcoupling points corresponding to two atom lasers emitted on both sides of the trapped condensate [Fig. 2(b)]. Moreover the slope of the potential $ma(z_{E_{\text{AL}}})$ is not constant. Nevertheless, if it varies slowly around the outcoupling point at the scale of the first lobe of the corresponding Airy function, the atom-laser wave function can be locally well approximated by the Airy function and we can use the result of [4] where gravity acceleration is changed to $a(z_{E_{\text{AL}}})$:

$$\mathcal{F} = \frac{\pi\hbar\Omega_{\text{rf}}^2 n_{1D}^{\text{BEC}}(z_{E_{\text{AL}}})}{2 ma(z_{E_{\text{AL}}})}. \quad (5)$$

Here Ω_{rf} is the Rabi frequency characterizing the rf coupling between the different atomic internal states, and $n_{1D}^{\text{BEC}}(z) = \int d\vec{r}_{\perp} |\psi_{\text{BEC}}(\vec{r}_{\perp}, z)|^2$ is the condensate linear density. More rigorously, one can solve the Schrödinger equation in a parabolic antitrapping potential [25]. We checked that the two calculations give the same result when the local slope approximation is valid, and the second approach is necessary only when the coupling is close to the maximum of the potential bump, when the right hand side of Eq. (5) diverges. As expected, the flux is then predicted to reach its maximum value.

The modeling presented above allows us to analyze our experimental data. Firstly, for a Rabi frequency of $\Omega_{\text{rf}}/2\pi = 40$ Hz, a total number of atoms in the BEC of $N_{\text{BEC}} \simeq 10^5$ atoms and assuming a coupling at about $5 \mu\text{m}$ from the center of the BEC, Eq. (5) gives $\mathcal{F} = 5 \times 10^5 \text{ at.s}^{-1}$, in agreement with the observed decay of the atom number in the BEC. Secondly, this modeling shows that with our parameters, the axial dynamics of the atom laser associated to Eqs. (2) and (3) is revealed by the propagation of the wavefront of the atom laser after turning on the outcoupler [Fig. 2(b)]. Indeed, out of the region of overlap with the trapped BEC, and for a coupling close to the potential maximum, the atoms have a kinetic energy of the order of the BEC chemical potential ($\mu_{\text{BEC}}/h \simeq 3.2$ kHz), which is large compared to $\mu(z)$ ($\mu(z)/h \sim \omega_{\perp}/2\pi = 360$ Hz). We can thus neglect $\mu(z)$ in Eq. (3), and out of the BEC the wavefront acceleration is dominated by $V_{\text{guide}}(z)$, while the atomic velocity just leaving the BEC is determined by $V_{\text{BEC}}(z_{E_{\text{AL}}})$. For an outcoupling at the center of the BEC, the expected value is $v_0 \simeq 5.4 \text{ mm.s}^{-1}$, somewhat less than the observed value $v_0 = 9 \pm 2 \text{ mm.s}^{-1}$. The discrepancy will be discussed below.

We now turn to the transverse mode of the guided atom laser. To characterize it experimentally, we measure the transverse energy using the standard time-of-flight technique: after 60 ms of propagation, the optical guide is suddenly switched off and we measure the expansion along the y axis as the beam falls under gravity. The evolution of the rms size is directly related to the transverse kinetic energy according to $\sigma(t) = \sqrt{\sigma_0^2 + \langle v_y^2 \rangle t^2}$ where σ_0 is the resolution of the imaging system ($7.5 \mu\text{m}$) which dominates the initial transverse size ($0.6 \mu\text{m}$). A fit gives $\langle v_y^2 \rangle = 4.5 \pm 0.2 \text{ mm}^2/\text{s}^2$. Assuming a perfect cylindrical symmetry, this corresponds to a total transverse energy $E_{\perp} = (5.5 \pm 0.8) \hbar\omega_{\perp}$, *i.e.* an average excitation quantum number of 2 along each transverse direction. This shows that only a few transverse modes are excited, and we may wonder whether purely single-mode propagation in the zeroth order transverse mode is achievable.

Actually, we expect the atom laser to be outcoupled in its lowest transverse mode. Indeed, the transverse potential experienced by an atom in the atom laser has

the same shape as the one experienced by an atom of the BEC, *i.e.*, in the Thomas-Fermi approximation, quadratic trapping edges and a flat bottom of width $2R_{\perp}(z)$. As z increases, this width decreases monotonically to 0 until the point R_z where the atom laser leaves the BEC and experiences a pure harmonic potential. A numerical simulation shows that this evolution is smooth enough to enable the transverse atom-laser wave function $\psi_{\perp}(r_{\perp}, z)$ to adiabatically adapt itself to the local ground state, resulting in the prediction of almost single-mode emission. The observed multimode behavior may be attributed to different experimental imperfections. Firstly, if the magnetic trap is not perfectly centered on the optical guide, there is a transverse shift of the BEC relatively to the guide axis, leading to imperfect mode matching. Secondly, in our experiment, there could be residual fluctuations in the position of the optical guide. More precise stabilization and control of this position should render these effects negligible. Finally, a numerical simulation of the coupled Gross-Pitaevskii equations suggests that at our value of the atomic flux, the BEC decay is not adiabatic enough [4] so that the outcoupling could induce excitations inside the BEC and thus increase the energy transferred to the atom laser. Note that this might also explain why the observed values of atom-laser velocity correspond to an energy somewhat larger than μ_{BEC} . A more sensitive observation system should allow us to work at a reduced atom-laser flux, and to eliminate this problem.

In conclusion, we have demonstrated a scheme for efficiently coupling an atom laser into a waveguide. We have shown how to obtain a guided atom laser with an almost constant de Broglie wavelength, at a value as large as $0.5 \mu\text{m}$, and by coupling near the boundary of the BEC it should be possible to obtain even larger de Broglie wavelengths. Such values are of interest for experiments in atom interferometry as, for instance, the coherent splitting of matterwaves at the crossing of two optical guides [26, 27]. Furthermore, as the atomic wavelength reaches values similar to visible light wavelength, transport properties through wells, barriers or disordered structures engineered with light should enter the quantum regime [8, 9, 10, 11, 12, 13, 14]. We also note that the control of the atom-laser flux offers the possibility to control the amount of interaction inside the guided atom-laser beam. The noninteracting regime is interesting to study, for instance, Anderson localization [13, 14] or quantum reflection [28, 29]. On the other hand, the interacting regime should allow investigation of effects such as the breakdown of superfluidity through obstacles [10, 12], or nonlinear resonant transport [8, 13]. We thus believe that our scheme, which could be integrated in miniaturized components [30], constitutes a very promising tool for further development of coherent guided atom optics.

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