On some applications of Galilean electrodynamics of moving bodies

Marc de Montigny, Germain Rousseaux

On some applications of Galilean electrodynamics of moving bodies [1]

M. de Montigny[a,b] **and G. Rousseaux**[c]

[a]*Campus Saint-Jean, University of Alberta*
8406 - 91 Street
Edmonton, Alberta, Canada T6C 4G9
[b]*Theoretical Physics Institute, University of Alberta*
Edmonton, Alberta, Canada T6G 2J1
[c]*Université de Nice Sophia-Antipolis, Institut Non-Linéaire de Nice,*
INLN-UMR 6618 CNRS-UNICE, 1361 route des Lucioles,
06560 Valbonne, France

Abstract

We discuss the seminal article in which Le Bellac and Lévy-Leblond have identified two Galilean limits of electromagnetism [1], and its modern implications. Recent works have shed a new light on the choice of gauge conditions in classical electromagnetism. We discuss various applications and experiments, such as in quantum mechanics, superconductivity, electrodynamics of continuous media, etc. Much of the current technology, where waves are not taken into account, is actually based on Galilean electromagnetism.

Key words: Galilean covariance, special relativity, electromagnetism, four-potential.

---

[1]E-mail: `montigny@phys.ualberta.ca`, `Germain.Rousseaux@inln.cnrs.fr`

1   Introduction

The purpose of this article is to emphasize the relevance of 'Galilean electromagnetism', recognized in 1973 by Michel Le Bellac and Jean-Marc Lévy-Leblond (LBLL). They observed that there exist not only one, but *two* well-defined Galilean limits of electromagnetism: the so-called 'magnetic' and 'electric' limits [1]. Hereafter, 'Galilean' means that the theory satisfies the principle of relativity in its Galilean form (also referred to as the sometimes misleading term 'non-relativistic').

Our purpose is *not* to argue that these limits should be seen as alternatives to Lorentz-covariant electrodynamics. We wish to point out that some physical phenomena, often described with special relativity, can be explained by properly defined Galilean limits. In other words, such phenomena could have been understood without recourse to special relativity, had the Galilean limits of electrodynamics been correctly defined in the first place. Therefore, our general purpose is twofold: first, that one must be careful when investigating alleged 'non-relativistic' limits, and second, that well-defined Galilei-covariant theories might allow one to describe more physical phenomena than usually believed. The later point means that some concepts, which are thought to be 'purely relativistic', can actually be understood within the realm of Galilean physics. A dramatic such example is the concept of spin [2].

We have summarized and discussed various approaches to Galilean electromagnetism in a recent article [3]. Since then, we have learned the existence of several studies emphasing the applications of quasistatic regimes both in research like, for example, in micro-electronics [4], bio-systems engineering, medical engineering, electromagnetic computations [5] and teaching [6].

Now let us briefly review Galilean electromagnetism and set up the main equations that we are going to utilize later on. A *Lorentz transformation* acts on space-time coordinates as follows:

$$
\begin{aligned}
\mathbf{x}' &= \mathbf{x} - \gamma \mathbf{v} t + (\gamma - 1)\frac{\mathbf{v}(\mathbf{v}\cdot\mathbf{x})}{\mathbf{v}^2}, \\
t' &= \gamma\left(t - \frac{\mathbf{v}\cdot\mathbf{x}}{c^2}\right),
\end{aligned}
\tag{1}
$$

where $\mathbf{v}$ is the relative velocity and $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$. When $v << c$, this reduces to a *Galilean transformation* of space-time:

$$
\begin{aligned}
\mathbf{x}' &= \mathbf{x} - \mathbf{v}t, \\
t' &= t.
\end{aligned}
\tag{2}
$$

Since Galilean kinematics involves the time-like condition

$$
c\Delta t >> \Delta x,
\tag{3}
$$

there is no other possible limit than the one given in Eq. (2). As we shall see below, this is not the same for transformations of electric and magnetic fields.

Under a Lorentz transformation, Eq. (1), the electric and magnetic fields in vacuum transform as

$$\mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + (1 - \gamma)\frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{E})}{\mathbf{v}^2},$$
$$\mathbf{B}' = \gamma(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}) + (1 - \gamma)\frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{B})}{\mathbf{v}^2}. \tag{4}$$

If we take the limit $v/c \to 0$, we find

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B},$$
$$\mathbf{B}' = \mathbf{B}. \tag{5}$$

As we will see shortly, this is a legitimate limit called the 'magnetic limit' of electromagnetism. (One might be tempted to consider the limit $\gamma \to 1$ which leads to

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B},$$
$$\mathbf{B}' = \mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}. \tag{6}$$

However, it is not a valid transformation; in particular, it does not even satisfy the group composition law [1].) However, Eq. (4) allows us to obtain, in addition to Eq. (5), another perfectly well-defined Galilean limit. In order to do so, one must compare the modules of the electric field E and the magnetic field cB, in analogy with Eq. (3). For large magnetic fields, Eq. (4) reduces to the so-called *magnetic limit* of electromagnetism:

$$\mathbf{E}'_m = \mathbf{E}_m + \mathbf{v} \times \mathbf{B}_m, \qquad E_m << cB_m,$$
$$\mathbf{B}'_m = \mathbf{B}_m. \tag{7}$$

The alternative, for which the electric field dominates, leads to the *electric limit*:

$$\mathbf{E}'_e = \mathbf{E}_e, \qquad E_e >> cB_e,$$
$$\mathbf{B}'_e = \mathbf{B}_e - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}_e. \tag{8}$$

Indeed, the approximations $E_e/c >> B_e$ and $v << c$ together imply that $E_e/v >> E_e/c >> B_e$, so that we take $E_e >> vB_e$ in Eq. (4).

Since we will emphasize the use of scalar and vector potentials $(V, \mathbf{A})$, let us consider their transformation properties. Under a Lorentz transformation, Eq. (1), they become

$$\mathbf{A}' = \mathbf{A} - \frac{\gamma \mathbf{v}V}{c^2} + (\gamma - 1)\frac{\mathbf{v}(\mathbf{v} \cdot \mathbf{A})}{\mathbf{v}^2},$$
$$V' = \gamma\left(V - \mathbf{v} \cdot \mathbf{A}\right). \tag{9}$$

When $v << c$ and $A << cV$, this reduces to the *electric limit* of potential transformations:

$$\mathbf{A}'_e = \mathbf{A}_e - \frac{\mathbf{v}V_e}{c^2},$$
$$V'_e = V_e. \tag{10}$$

The electric and magnetic fields are expressed in terms of the potentials as follows

$$\mathbf{E}_e = -\nabla V_e, \qquad \mathbf{B}_e = \nabla \times A_e. \tag{11}$$

Whereas there exists only one possible condition, Eq. (3), for the space-time manifold, here we find a second limit, obtained by $v << c$ and $A >> cV$, such that Eq. (9) reduces to the *magnetic limit* of potential transformations:

$$\begin{aligned}
\mathbf{A}'_m &= \mathbf{A}_m, \\
V'_m &= V_m - \mathbf{v} \cdot \mathbf{A}_m.
\end{aligned} \tag{12}$$

In this limit, the electromagnetic field components are given by

$$\mathbf{E}_m = -\nabla V_m - \partial_t \mathbf{A}_m, \qquad \mathbf{B}_m = \nabla \times A_m. \tag{13}$$

Finally, let us recall the two Galilean limits of the Maxwell equations. Their relativistic form is written as

$$\begin{array}{llll}
\nabla \times \mathbf{E} &=& -\partial_t \mathbf{B}, & \text{Faraday}, \\
\nabla \cdot \mathbf{B} &=& 0, & \text{Thomson}, \\
\nabla \times \mathbf{B} &=& \mu_0 \mathbf{j} + \frac{1}{c^2}\partial_t \mathbf{E}, & \text{Ampere}, \\
\nabla \cdot \mathbf{E} &=& \frac{1}{\epsilon_0}\rho, & \text{Gauss},
\end{array} \tag{14}$$

The existence of two Galilean limits is not so obvious if one naively takes the limit $c \to \infty$. LBLL have found in Ref. [1] that, in the electric limit, the Maxwell equations reduce to:

$$\begin{aligned}
\nabla \times \mathbf{E}_e &= \mathbf{0}, \\
\nabla \cdot \mathbf{B}_e &= 0, \\
\nabla \times \mathbf{B}_e - \frac{1}{c^2}\partial_t \mathbf{E}_e &= \mu_0 \mathbf{j}_e, \\
\nabla \cdot \mathbf{E}_e &= \frac{1}{\epsilon_0}\rho_e.
\end{aligned} \tag{15}$$

Clearly, the main difference with the relativistic Maxwell equations is that here the electric field has zero curl in Faraday's law. In the magnetic limit, the Maxwell equations become

$$\begin{aligned}
\nabla \times \mathbf{E}_m &= -\partial_t \mathbf{B}_m, \\
\nabla \cdot \mathbf{B}_m &= 0, \\
\nabla \times \mathbf{B}_m &= \mu_0 \mathbf{j}_m, \\
\nabla \cdot \mathbf{E}_m &= \frac{1}{\epsilon_0}\rho_m.
\end{aligned} \tag{16}$$

The displacement current term is absent in Ampère's law.

Hereafter, we illustrate some applications of the Galilean electrodynamics of moving bodies. In the next section, we reexamine the gauge conditions and their compatibility with Lorentz and Galilean covariance. Then we comment briefly on the connection between the two limits and the Faraday tensor (and its dual). In Section 4, we discuss Feynman's proof of (the magnetic limit of) the Maxwell equations, and Section 5 contains a few comments about superconductivity seen as a magnetic limit, and gauge potentials. In Sections 6 and 7, we question our current understanding of the electrodynamics of moving bodies by examining the Trouton-Noble experiment in a Galilean context as well the introductory example used by Einstein in his famous work on special relativity. We conclude with some comments on the intrinsic use by Maxwell of both limits, one century before LBLL.

3

2 Gauge conditions and Galilean electromagnetism

Hereafter, we use the Riemann-Lorenz formulation of classical electromagnetism (i.e. in terms of scalar and vector potentials instead of fields [7]) to describe the two Galilean limits. Let us recall how the electric and magnetic limits may be retrieved in this formulation by a careful consideration of orders of magnitude [8, 3]. It is quite natural to define the following dimensionless parameters:

$$\varepsilon \equiv \frac{L}{cT} \qquad \text{and} \qquad \xi \equiv \frac{j}{c\rho}, \tag{17}$$

where L, T, j and ρ represent the orders of magnitude of length, time, current density, and charge density, respectively.

The equations of classical electromagnetism, written in terms of potentials, are cast into the following form [7]:

$$\begin{aligned} \nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} &= -\frac{\rho}{\epsilon_0}, \qquad \text{Riemann equations,} \\ \nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mu_0 \mathbf{j}, \end{aligned} \tag{18}$$

$$\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0, \qquad \text{Lorenz equation,} \tag{19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v} + q\mathbf{A}) = -q\nabla_{\mathbf{A}}(V - \mathbf{v} \cdot \mathbf{A}), \qquad \text{Lorentz force.} \tag{20}$$

The quasistatic approximation, $\varepsilon << 1$, of Eq. (18) leads to

$$\nabla^2 V \simeq -\frac{\rho}{\epsilon_0} \qquad \text{and} \qquad \nabla^2 \mathbf{A} \simeq -\mu_0 \mathbf{j}, \tag{21}$$

from which we can define a further dimensionless ratio, $\frac{cA}{V} \simeq \frac{j}{\rho c}$, so that

$$\frac{cA}{V} \simeq \xi. \tag{22}$$

This echoes LBLL's prescription for the fields [1]: in the magnetic limit, the spacelike quantity cA is dominant, whereas in the electric limit, it is the timelike quantity V that dominates.

The definition $\mathbf{E} = -\partial_t \mathbf{A} - \nabla V$ of the electric field takes different forms in the Galilean limits, depending on the order of magnitude of each term, because the Galilean transformations for the potentials differ for the electric and the magnetic limits [1]. Let us evaluate the order of magnitude of the ratio between its two terms:

$$\frac{\partial_t \mathbf{A}}{\nabla V} \simeq \frac{\frac{A}{T}}{\frac{V}{L}} \simeq \frac{L}{cT}\frac{cA}{V} \simeq \varepsilon\xi.$$

In the magnetic limit, for which $\xi >> 1$, this equation leads to Eq. (13). By computing the curl, we find $\partial_t \mathbf{B}_m = -\nabla \times \mathbf{E}_m$. Likewise, in the electric limit, for

which $\xi << 1$, we can neglect $\partial_t \mathbf{A}$, so that we obtain Eq. (11). The curl of this expression leads to $\nabla \times \mathbf{E}_e \simeq \mathbf{0}$.

The choice of gauge conditions allows one to retrieve the two sets of Galilean Maxwell equations in terms of fields, as stated by LBLL [1]. Moreover, as we now proceed to show, the gauge conditions are closely related to the nature of kinematic transformations. In the magnetic limit, the condition $\xi >> 1$ leads to the Coulomb gauge condition: $\nabla \cdot \mathbf{A}_m = 0$. From the definition of $\mathbf{B}_m$ together with the identity

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \tag{23}$$

we see that

$$\nabla \times \mathbf{B}_m = \nabla \times (\nabla \times \mathbf{A}_m) = \nabla(\overbrace{\nabla \cdot \mathbf{A}_m}^{0}) - \nabla^2 \mathbf{A}_m = \mu_0 \mathbf{j}_m.$$

The last term follows from Eq. (24). We point out that the displacement current term is missing. The divergence of the electric field, in the magnetic limit, gives

$$\nabla \cdot \mathbf{E}_m = \nabla \cdot (-\partial_t \mathbf{A}_m - \nabla V_m) = -\partial_t(\overbrace{\nabla \cdot \mathbf{A}_m}^{0}) - \nabla^2 V_m = \frac{\rho_m}{\epsilon_0},$$

where we have utilized Eq. (23). This is the second inhomogeneous equation, in the last line of Eq. (16). In the electric limit, the condition $\xi << 1$ leads similarly to the Lorenz condition. Proceeding as in the magnetic limit, we begin with the curl of $\mathbf{B}_e$:

$$\nabla \times \mathbf{B}_e = \nabla \times (\nabla \times \mathbf{A}_e) = \nabla(\overbrace{\nabla \cdot \mathbf{A}_e}^{-(\partial_t V_e)/c^2}) - \nabla^2 \mathbf{A}_e = \frac{1}{c^2}\partial_t \mathbf{E}_e + \mu_0 \mathbf{j}_e.$$

¿From the divergence of $\mathbf{E}_e$, we find

$$\nabla \cdot \mathbf{E}_e = \nabla \cdot (-\nabla V_e) = -\nabla^2 V_e = \frac{\rho_e}{\epsilon_0},$$

where we have used (24).

To summarize the preceding discussion, we point out forcefully that the choice of a gauge condition is dictated by the relativistic versus Galilean nature of the problem. The role of potentials and gauge conditions in quasistatic regimes was pointed out only recently by Dirks [4] and Larsson [6], although the problem was not handled correctly, as we have done with the Riemann-Lorenz formulation. Indeed, both of them use the (erroneous) equation :

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{j} + \frac{1}{c^2}\nabla\frac{\partial V}{\partial t}, \tag{24}$$

obtained by plunging the (magnetic Galilean-covariant: $\xi >> 1$) Coulomb gauge condition into the full set of (Lorentz-covariant:$\xi \simeq 1$) Maxwell equations in terms of

5

the fields without remarking that the temporal terms are negligible with respect to the spatial terms as $\epsilon << 1$...

The Lorenz gauge condition is compatible with the relativistic context as well as the electric Galilean limit. The Coulomb gauge condition, however, is compatible with the Galilean magnetic limit only because it is not covariant with respect to neither the Lorentz transformations nor the Galilean electric transformations. We refer the interested reader to a discussion of the physical meaning that one can ascribe to the various 'gauge conditions' [9].

Galilean electromagnetism sheds a new light on the pre-relativity era. Indeed, a careful reading of James Clerk Maxwell's famous *Treatise on Electricity and Magnetism* reveals that he was actually working with the electric limit in the discussion of dielectric materials in his first volume (see Chapters II to V) [10]. Likewise, in his treatment of ohmic conductors and induced magnetic fields, in his second volume, the magnetic limit was employed implicitly, except in the chapters on the theory of light propagation, where he introduced 'by hand' the displacement current term into the magnetic limit equations in order to demonstrate that light is a transverse electromagnetic wave [10]. But, as we have seen in the particular case of the electric limit (and it is also valid in relativity), the displacement current follows from choosing the Lorenz gauge, and Maxwell (wrongfully) kept the Coulomb gauge within the relativistic context for the fields (for more details, see the forthcoming [11]). This problem prompted Hertz and Heaviside to relinquish potentials and to rather cast the Maxwell equations in terms of fields. Following Hertz's approach, Einstein subsequently expressed the Maxwell equations in terms of fields (i.e. in the Heaviside-Hertz formulation), whereas Henri Poincaré wrote the Maxwell equations in terms of the potentials (i.e. the Riemann-Lorenz formulation) by adopting the Lorenz condition in a relativistic context [11].

3 The Faraday tensor and its dual

In special relativity, it is well known that the Faraday tensor:

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \mu, \nu = 0, 1, 2, 3,$$

and its dual:

$$^*F_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma},$$

do have the same physical meaning. This is not the case within Galilean electromagnetism. As pointed out in Earman's book, and recently discussed by Rynasiewicz, the Galilean tranformations of the Faraday tensor and its dual tensor lead to the electric or the magnetic limit, respectively [12]. The effect of the duality operation amounts to exchanging **E** and **B**:

$$\mathbf{E} \to c\mathbf{B} \quad \text{and} \quad \mathbf{B} \to -\mathbf{E}/c.$$

One recovers the magnetic and electric limits, Eqs. (7) and (8), by applying the duality transformations directly to the electric transformations of the fields in order to get the magnetic transformations, and vice versa.

Earman also noted [12] that the field transformations of the magnetic limit are obtained when $\mathbf{E}$ and $\mathbf{B}$ are expressed in terms of 'covariant', or $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, tensor $F_{\mu\nu}$, whereas the electric limit is obtained when the fields transformations are calculated by using the 'contravariant', or $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, tensor $F^{\mu\nu}$. Let us illustrate it briefly, with

$$A^\mu = \left(\frac{V}{c}, \mathbf{A} \right), \qquad A_\mu = \left(\frac{V}{c}, -\mathbf{A} \right),$$

as well as

$$\partial^\mu = \left(\frac{1}{c}\partial_t, -\nabla \right), \qquad \partial_\mu = \left(\frac{1}{c}\partial_t, \nabla \right).$$

The magnetic limit follows from the relation

$$F'_{\mu\nu} = \Lambda_\mu{}^\rho \Lambda_\nu{}^\sigma \, F_{\rho\sigma},$$

where the Galilean transformation matrix $\Lambda_\mu{}^\nu$ is defined by the four-gradient transformation, $\partial'_\mu = \Lambda_\mu{}^\nu \partial_\nu$, so that

$$\Lambda_\mu{}^\nu = \begin{pmatrix} 1 & \frac{v_x}{c} & \frac{v_y}{c} & \frac{v_z}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The index μ denotes the line of each entry. We find, for example,

$$\begin{aligned} \frac{E'_x}{c} = F'_{01} &= \Lambda_0{}^\mu \Lambda_1{}^\nu \, F_{\mu\nu}, \\ &= \Lambda_0{}^0 F_{01} + \Lambda_0{}^2 F_{21} + \Lambda_0{}^3 F_{31}, \\ &= \tfrac{1}{c}(E_x + v_y B_z - v_z B_y), \end{aligned}$$

and

$$-B'_z = F'_{12} = \Lambda_1{}^\mu \Lambda_2{}^\nu F_{\mu\nu} = -B_z,$$

which is Eq. (7).

The electric limit transformations follows from

$$F'^{\mu\nu} = \Lambda^\mu{}_\rho \Lambda^\nu{}_\sigma \, F^{\rho\sigma}.$$

The transformation matrix $\Lambda^\mu{}_\nu$ is now defined by the coordinate transformation, $x^\mu = \Lambda^\mu{}_\nu \, x^\nu$, with $x^\mu = (ct, x, y, z)$, so that

$$\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{v_x}{c} & 1 & 0 & 0 \\ -\frac{v_y}{c} & 0 & 1 & 0 \\ -\frac{v_z}{c} & 0 & 0 & 1 \end{pmatrix}.$$

Again, the first index μ denotes the matrix line. For instance, we compute

$$-\frac{E'_x}{c} = F'^{01} = \Lambda^0{}_0 \Lambda^1{}_\nu F^{0\nu} = -\frac{E_x}{c},$$

and

$$
\begin{aligned}
B'_z = -F'^{12} &= -\Lambda^1{}_\mu \Lambda^2{}_\nu \, F^{\mu\nu}, \\
&= -\Lambda^1{}_0 F^{02} - \Lambda^2{}_0 F^{10} - F^{12}, \\
&= B_z - \tfrac{1}{c^2}(v_x E_y - v_y E_x),
\end{aligned}
$$

which is Eq. (8).

4 Quantum mechanics with external potentials

In 1990, Dyson published a demonstration of the Maxwell equations due to Richard Feynman [13]. The demonstration dates back to the forties and had remained hitherto unpublished. It was believed to be incomplete because Feynman considered only the homogeneous Maxwell equations, given by the first two lines of Eq. (14):

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \qquad \nabla \cdot \mathbf{B} = 0.$$

During the nineties, some authors revisited this demonstration and noted that the Schrödinger equation admitted external potentials only if they were compatible with the *magnetic* limit of LBLL and, therefore, with the Coulomb gauge condition (see [14, 15] and references therein).

Indeed, from Eq. (16), it is clear that the homogeneous Maxwell equations given above are valid only within the magnetic limit because the electric field has zero curl in the electric limit, Eq. (15). This is a consequence of the Galilean magnetic limit of the four-potential which does enter into the Schrödinger equation. Let us recall the statement more precisely (more details can be found in reference [14]). The Schrödinger equation with external fields $V(\mathbf{x}, t)$ and $\mathbf{A}(\mathbf{x}, t)$ is written as

$$i\hbar \partial_t \Psi(\mathbf{x}, t) = \frac{1}{2m}(-i\hbar\nabla - \mathbf{A}(\mathbf{x}, t))^2 \Psi(\mathbf{x}, t) + V(\mathbf{x}, t)\Psi(\mathbf{x}, t).$$

It is covariant under Galilean transformations, Eq. (2), with

$$
\begin{aligned}
&\Psi(\mathbf{x}, t) \to \Psi'(\mathbf{x}', t') = \text{const } \exp[(i/\hbar)(-m\mathbf{v} \cdot \mathbf{x} + \tfrac{1}{2}m\mathbf{v}^2 t + \phi(\mathbf{x}, t))] \, \Psi(\mathbf{x}, t), \\
&V(\mathbf{x}, t) \to V'(\mathbf{x}', t') = V(\mathbf{x}, t) - \partial_t \phi(\mathbf{x}, t) - \mathbf{v} \cdot (\mathbf{A}(\mathbf{x}, t) + \nabla\phi(\mathbf{x}, t)), \\
&\mathbf{A}(\mathbf{x}, t) \to \mathbf{A}'(\mathbf{x}', t') = \mathbf{A}(\mathbf{x}, t) + \nabla\phi(\mathbf{x}, t),
\end{aligned}
$$

where $\phi(\mathbf{x}, t)$ is some scalar function. In the case $\phi(\mathbf{x}, t) = 0$, which corresponds to pure Galilean boosts, the equations above reduce to the magnetic limit of Galilean transformations of the potentials, Eq. (12). Hence, one can say that 'Galilean covariance selects the gauge'.

In a subsequent study, Holland and Brown have shown that the Maxwell equations admit an electric limit only if the source is a Dirac current [16]. In addition, they have shown that the Dirac equation admits both Galilean limits, just like the Maxwell equations, corroborating thereby earlier results by Lévy-Leblond [2]. To summarize, what Feynmann did not (actually, could not) realize is that he had derived only the part of the Maxwell equations compatible with the Galilean covariant magnetic limit, that is to say, the homogeneous equations.

5   Superconductivity

Superconductivity also enters into the realm of the magnetic limit; indeed, it selects the Coulomb gauge condition as a necessary consequence of Galilean covariance. To illustrate this, consider the London equation, which states that the current density is proportional to the vector potential:

$$\mathbf{p} = m^*\mathbf{v} + q^*\mathbf{A} = \mathbf{0}.$$

The star denotes a quantity describing Cooper pairs [17]. This implies that there is a perfect transfer of electromagnetic momentum to kinetic momentum. Hence, contrary to what is usually stated, gauge invariance is not broken by superconductivity since the Coulomb gauge condition is implied. Moreover, the Meissner effect can be explained by starting with Ampère's equation written as $\nabla \times \mathbf{B} = \mu_0\mathbf{j}$, that is, without the displacement current term as in the magnetic case, third line of Eq. (16). Hence, this expression (or more directly $\nabla^2\mathbf{A} \simeq -\mu_0\mathbf{j}$ in the Riemann-Lorenz formulation) together with $\nabla \cdot \mathbf{A} = 0$ and London equation, lead to solutions (in one dimension x) of the type $A \simeq \exp -\lambda x$ (where λ is a constant) so that the vector potential (hence the magnetic field) only penetrates the superconductor to a depth $1/\lambda$ [17].

We point out that the current density in the magnetic limit (hence in superconductivity) is divergenceless: by taking the divergence of

$$\nabla^2\mathbf{A} \simeq -\mu_0\mathbf{j}$$

and using $\nabla \cdot \mathbf{A} = 0$, dictated by Galilean covariance, we end up with $\nabla \cdot \mathbf{j} = 0$. It does not mean, as often assumed, that the current is constant in time. Indeed, only the time derivative of the charge density is negligible with respect to the divergence of the current [3].

As a consequence, superconductivity cannot be associated with a symmetry breaking of gauge invariance but is magnetic Galilean covariant. This unusual statement has been recently advocated by Martin Greiter using a different approach [18]. As a matter of fact, it is the global U(1) phase rotation symmetry that is spontaneously violated. A striking consequence is that the Higgs mechanism for providing mass to particles becomes doubtful, since it was believed to be analogous to the assumed symmetry breaking of gauge invariance in superconductivity.

6 Electrodynamics of continuous media at low velocities

In 1904, Lorentz claimed that a moving magnet could become electrically polarized [19]. In 1908, Einstein and Laub noted that the Minkowski transformations for the fields and the excitations [20] predict that a moving magnetic dipole induces an electric dipole moment [21]. It is interesting to reexamine these predictions in the light of the Galilean electrodynamics of continuous media. Indeed if one starts from the Minkowski transformations that relate the polarization and the magnetization [20], one would expect two Galilean limits: one with $\mathbf{M}' = \mathbf{M}$ and $\mathbf{P}' = \mathbf{P} - \mathbf{v} \times \mathbf{M}/c^2$ and the other with $\mathbf{M}' = \mathbf{M} + \mathbf{v} \times \mathbf{P}$ and $\mathbf{P}' = \mathbf{P}$ (see Chapter 9 of Ref. [22]).

In reference [3], we derived the following fields transformations:

Magnetic Limit	Electric Limit
$\mathbf{B} = \mathbf{B}'$	$\mathbf{E} = \mathbf{E}'$
	$\rho = \rho'$
$\mathbf{j} = \mathbf{j}'$	$\mathbf{j} = \mathbf{j}' + \rho'\mathbf{v}$
$\mathbf{H} = \mathbf{H}'$	$\mathbf{H} = \mathbf{H}' + \mathbf{v} \times \mathbf{D}'$
$\mathbf{E} = \mathbf{E}' - \mathbf{v} \times \mathbf{B}'$	$\mathbf{D} = \mathbf{D}'$
$\mathbf{M} = \mathbf{M}'$	$\mathbf{P} = \mathbf{P}'$
$\mathbf{P} = \mathbf{P}' + \mathbf{v} \times \mathbf{M}'/c^2$	$\mathbf{M} = \mathbf{M}' - \mathbf{v} \times \mathbf{P}'$

We can infer the following boundary conditions for moving media, with $\mathbf{n}$ being the unit vector between two media denoted by 1 and 2, $\mathbf{K}$ the surface current, σ the surface charge, Σ the surface separating both media, and v_n the projection of the relative velocity on the normal of Σ:

Magnetic Limit	Electric Limit
$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}$	$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$
$\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = \mathbf{0}$	$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma$
$\mathbf{n} \cdot (\mathbf{j}_2 - \mathbf{j}_1) + \nabla_\Sigma \cdot \mathbf{K} = \mathbf{0}$	$\mathbf{n} \cdot (\mathbf{j}_2 - \mathbf{j}_1) + \nabla_\Sigma \cdot \mathbf{K} = v_n(\rho_2 - \rho_1) - \partial_t \sigma$
$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = v_n(\mathbf{B}_2 - \mathbf{B}_1)$	$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K} + v_n\mathbf{n} \times [\mathbf{n} \times (\mathbf{D}_2 - \mathbf{D}_1)]$

Therefore, as we presumed, the effects in continuous media predicted by Lorentz as by Einstein and Laub are not purely relativistic since they can be described in a Galilean framework.

7 Electrodynamics of moving bodies at low velocities

Galilean electromagnetism raises doubts about our current understanding of the electrodynamics of moving media. For instance, several experiments (like the ones by Roentgen [23], Eichenwald [24], Wilson [25], Wilson and Wilson [26], Trouton and Noble [27], etc.) are generally believed to corroborate special relativity. However, as we will show hereafter for the Trouton-Noble experiment, there is not always a need for special relativity because the typical relative velocity in these experiments is much smaller than the speed of light. As we have emphasized previously, the Galilean framework must involve the two limits of electromagnetism. A question that arises is which of the experiments mentioned above can be explained by either the electric limit, the magnetic limit or a coherent combination of both.

It is interesting to notice that Ernest Carvallo, a notorious anti-relativist, used both quasistatic limits as early as 1921 in order to deny the success of Einstein's theory of relativity [28]. In some sense, he was right to point out that the electrodynamics of moving bodies at low velocities could be described in a Galilean-covariant manner by distinguishing the conductors and dielectrics. However, he was wrong to think that the optical properties of moving bodies can be described along the same lines.

7.1 The Trouton-Noble experiment

The Trouton-Noble experiment can be thought of as the electromagnetic analogue of the optical Michelson and Morley experiment [27]. It was designed to verify whether one can observe a mechanical velocity of the ether if one considers the luminiferous medium as having parts which can be followed mechanically. Like the Michelson-Morley optical experiment, the Trouton-Noble experiment led to a negative results in the sense that no one was able to detect either an absolute motion with respect to the ether, or a partial entrainment like in Fizeau experiment.

In 1905, Albert Einstein suggested to consider the ether as superfluous, since its mechanical motion was not detected experimentally. Some theorists, such as H. Poincaré and H.A. Lorentz, were reluctant to relinquish ether as the bearer of the electromagnetic field, despite the fact that they had adopted the relativity principle. In 1920, at a conference in Leyden, Einstein himself recoursed to ether as the medium allowing the propagation of gravitational waves, although it cannot be endowed with the characteristics of a material medium [29]. Today, even though the ether is a banished word in modern science, one can use it as did the older Einstein in order to describe the vacuum with physical (though not mechanical) properties.

Before the advent of special relativity, Hertz, Wien, Abraham, Lorentz, Cohn and all the specialists of the electrodynamics of moving bodies have used the transformations given in Eq. (6), which is an incoherent mixture of the electric and magnetic Galilean limits [30]. As mentioned previously, these expressions do not even obey the

group property of composition of transformations.

The purpose of the Trouton-Noble experiment was to observe the effect of a charged capacitor in motion with an angle θ between the plates and the motion through the ether [27]. The electric field in the reference frame of the capacitor generates a magnetic field in the ether frame given by

$$\mathbf{B}' = -\frac{1}{c^2}\mathbf{v}\times\mathbf{E},$$

where $\mathbf{v}$ is the absolute velocity. Thus we have

$$B' = \frac{1}{c^2}vE\sin\theta.$$

Consequently, there is a localization of magnetic energy density inside a volume $\mathrm{d}V$:

$$\mathrm{d}W = \frac{1}{2}\frac{B'}{\mu_0}\mathrm{d}V = \frac{1}{2}\frac{v^2}{c^2}\epsilon_0 E^2 \sin^2\theta \ \mathrm{d}V.$$

The volume of the capacitor being written as Sl, the total energy between the plates is

$$W = \frac{1}{2}\frac{v^2}{c^2}\epsilon_0 E^2 \sin^2\theta \ Sl.$$

If one denotes by $V = E/l$ the difference of potential between the plates, then the capacitor is submitted to the electrical torque

$$\Gamma = -\frac{\mathrm{d}W}{\mathrm{d}\theta} = -\frac{\epsilon_0}{2}\frac{V^2 S}{l}\frac{v^2}{c^2}\sin(2\theta),$$

which is maximal for $\theta = 45°$, and zero for $\theta = 90°$. Hence, the plates are expected to be perpendicular to the velocity. However, this effect has not been observed experimentally.

In order to understand what is wrong with the above demonstration, let us first consider the electric limit transformation, given in Eq. (8). A consequence of this transformation is that the Biot-Savart law follows from the Coulomb law associated with the electric transformation of the magnetic field. Contrary to the transformations (6), used by Trouton and Noble [27], they do respect the group additivity. Besides, these transformations are only compatible with the approximate set of the Maxwell equations where the time derivative in the Faraday equation vanishes, as in Eq. (15).

We can derive the following 'electric limit' approximation of the Poynting theorem:

$$\partial_t\left(\frac{1}{2}\epsilon_0 E^2\right) + \nabla\cdot\left(\frac{\mathbf{E}\times\mathbf{B}}{\mu_0}\right) \simeq -\mathbf{j}\cdot\mathbf{E}. \tag{25}$$

This shows that the energy density is of electric origin only. Hence, no electric energy associated with the motional magnetic field can be taken into account within the

electric limit, because it is of order $(v/c)^2$ with respect to the static, or quasistatic, electric one. Thus, the Trouton-Noble experiment does not show any effect as soon as we are in the realm of the electric limit. Recall that the electric limit is such that the relative velocity is small compared to the velocity of light c, and the order of magnitude of the electric field is large compared to the product of c by the magnetic field. Of course, for larger velocities, special relativity is needed and we must take into account the additional mechanical torque [31] due to the length variation in order to explain the negative result (i.e. no torque).

In the very last paragraph of their 1903 article, Trouton and Noble comment on the source of the negative result being caused by the fact that they considered the energy of the motional magnetic field [27]. They suggested that the energy of the magnetic field must have had some origin, and that the electrostatic energy of the capacitor had to dimininish by $1/2\epsilon_0 E^2 v^2/c^2$ when it is moving with a velocity v at right angles to its electrostatic lines of force (the electrostatic energy then being $1/2\epsilon_0 E^2$). We may assert that the converse situation of a solenoid or magnet in motion will not create a motional magnetic torque because the magnetic energy associated with the motional electric field is negligible compared to the magnetic energy of the static, or quasistatic, magnetic field.

7.2 'Einstein's asymmetry'

In his famous article on the electrodynamics of moving media, Albert Einstein pointed out the importance of whether or not one should ascribe energy to the fields when dealing with motion [32]. In the introduction of his paper, he recalled that Maxwell's electrodynamics, when applied to moving bodies, leads to intrinsic theoretical asymmetries. He illustrated it with the example of the reciprocal electrodynamic action of a magnet and a conductor. Then, the observable phenomenon depends on the relative motion of the conductor and the magnet, unlike the traditional view advocated by Lorentz in which either one or the other of these bodies is in motion: (1) if the magnet is moving with the conductor at rest, an electric field, with a certain definite energy, is induced in the neighbourhood of the magnet, producing a current where parts of the conductor are located; (2) if the conductor is in motion and the magnet at rest, then no electric field arises in the neighbourhood of the magnet. Lorentz therefore argued that the conductor must contain an electromotive force with no intrinsic energy, but which causes electric currents similar to those produced by the electric forces in the former case, assuming the same relative motion in the two cases. This dual representation of the same phenomena was unbearable for Einstein.

By invoking the Lorentz transformation (obtained in the kinematical analysis of his article) to the Maxwell equations (actually, Einstein used the Heaviside-Hertz formulation, whereas Poincaré used the Riemann-Lorenz formulation in his relativity article [11]), Einstein replaced Lorentz's explanation:

1. If a unit electric point charge is in motion in an electromagnetic field, there acts upon it, in addition to the electric force, an electromotive force which, if we neglect the terms multiplied by the second and higher powers of v/c, is equal to the vector product of the velocity of the charge and the magnetic force, divided by the velocity of light.[32]

by the now famous special relativity explanation, valid for all velocities:

2. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge.[32]

Einstein therefore concluded that the analogy is valid with magnetomotive forces, based on the idea that the electromotive force is merely some auxiliary concept owing its existence to the fact that the electric and magnetic forces are related to the relative motion of the coordinate system. Then he pointed out that the asymmetry mentioned in the introduction of his article now disappears.

We wish to point out forcefully that the transformations of the electromagnetic field given by the Galilean magnetic limit are sufficient to explain Einstein's thought experiment with the magnet and the conductor, without recourse to Lorentz covariance [33]. Indeed, as in our discussion of the Trouton-Noble experiment, the magnetic Poynting theorem can explain why one cannot ascribe an energy to the motional electric field in Einstein's thought experiment,

$$\partial_t \left(\frac{B^2}{2\mu_0} \right) + \nabla \cdot \left(\frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right) \simeq -\mathbf{j} \cdot \mathbf{E},$$

which is the magnetic analogue of Eq. (25).

It means that the second postulate (invariance of the velocity of light) used by Einstein is not required in order to explain the thought experiment. The relativity principle and the magnetic Galilean transformations are sufficient, together with the fact that the relative velocity involved in such an experiment is much smaller than the velocity of light. Hence, in the low-velocity regime, we proposed the following explanation of Einstein's asymmetry:

3. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by a Galilean magnetic transformation of the field to a system of co-ordinates at rest relatively to the electrical charge.

Einstein was correct in replacing Lorentz's explanation because Lorentz thought that the vector product of the velocity with the magnetic field was not an electric field (which is why Lorentz called it the electromotive field). But, because LBLL's

article was not available yet, Einstein could not notice that the same vector product was an effective electric field due to a transformation of the Galilean magnetic limit. Wolfgang Pauli too offered a solution to the asymmetry problem in his textbook on electrodynamics, but he only assumed that his calculations were a first order approximation of the relativistic demonstration [34]. He did not acknowledge the existence of the Galilean magnetic limit.

To summarize, Einstein's explanation to remove the asymmetry is completely valid. However, we have noticed above that special relativity is not necessary to remove it, but only sufficient. It is ironic that the thought experiment that led Einstein to special relativity could have been explained by Galilean relativity if only the magnetic limit had been known by him at that time.

As pointed out by Keswani and Kilminster [35], Maxwell did resolve Einstein's asymmetry within the formalism of the magnetic limit when he stated that for all phenomena related to closed circuits and the current within them, the fact that the coordinate system be at rest or not is immaterial. On p. 346 of the same paper, they go on by explaining that the formula for the electromotive intensity (in its modern sense and not in Lorentz sense above) is of the same type, whether the motion of the conductors refers to fixed axes or to moving axes, because the only differences is that for moving axes the electric potential V becomes $V' = V - \mathbf{v} \cdot \mathbf{A}$. Then they recall that Maxwell claimed that whenever a current is produced within a circuit C, the 'electromotive force' is equal to $\int_C \mathbf{E}' \cdot d\mathbf{s}$, and the value of V therefore disappears from this integral, so that the term $-\mathbf{v} \cdot \mathbf{A}$ has no influence on its value [35].

Concluding remarks

One century after the relativity revolution has taken place, and more than thirty years after the work of Lévy-Leblond and Le Bellac, Galilean electromagnetism is becoming a field of current research, because it allows physicists and engineers to explain much more simply low-energy experiments involving the electrodynamics of moving media without the sophisticated formalism of special relativity.

In this paper, we have reexamined gauge conditions in connection with Lorentz and Galilean covariance. After a brief comment on the two Galilean limits of electromagnetism and the Faraday tensor, we have recalled the importance of the magnetic limit in 'Feynman's proof' of the Maxwell equations as well as in superconductivity. Finally, we have questioned our current understanding of the electrodynamics of moving bodies by examining the Trouton-Noble experiment and the example used by Einstein in the introduction of his famous article on special relativity.

For slow velocities it is clear that effects of special relativity, such as length contraction, cannot explain (as it was believed so far) the corresponding experiments since these effects are negligible. In the realm of mechanics, one might ask what would have happened if Newton were born after Einstein? We are in an analogous situation with respect to electromagnetism.

Acknowledgement

References

[1] Le Bellac M and Lévy-Leblond J M 1973 Galilean electromagnetism *Nuov. Cim. B* **14** 217-233.

[2] Lévy-Leblond J M 1967 Nonrelativistic particles and wave equations *Comm. Math. Phys.* **6** 286-311.

[3] de Montigny M and Rousseaux G 2006 On the electrodynamics of moving bodies at low velocities *Eur. J. Phys.* **27** 755-768.

[4] Dirks H K 1996 Quasi-stationary fields for microelectronic applications *Electrical Engineering (Archiv fur Elektrotechnik)* **79** 145-155

[5] van Rienen U, Flehr J, Schreiber U and Motrescu V 2003 Modeling and Simulation of Electro-Quasistatic Fields *Birkhauser Verlag Basel/Switzerland, International Series of Numerical Mathematics* **146** 17-31;

Schreiber U, Flehr J, Motrescu V and van Rienen U 2004 The Electro-Quasistatic Model in Different Applications *W.H.A. Schilders, E. Jan W. ter Maten, S.H.M.J. Houben (Herausg.) Scientific Computing in Electrical Engineering Proc. of the 4th Conference on Scientific Computing in Electrical Engineering SCEE Springer Verlag Berlin Mathematics in Industry* **4** 356-363;

van Rienen U, Flehr J, Schreiber U, Schulze S, Gimsa U, Baumann W, Weiss D G, Gimsa J, Benecke R, and Pau H-W 2005 Electro-Quasistatic Simulations in Bio-Systems Engineering and Medical Engineering Advances in Radio Science *3* 39-49.

[6] Larsson J 2006 Electromagnetics from a quasistatic perspective, submitted to American Journal of Physics. (Article available at: *http : //arxiv.org/abs/physics/*0606109).

[7] O'Rahilly A 1965 *Electromagnetic Theory, a Critical Examination of Fundamentals* (New York: Dover);

Su C C 2001 Explicit definitions of electric and magnetic fields in potentials and derivation of Maxwell's equations *Eur. J. Phys.* **22** L5-L8;

Yakubov V P 2004 Special theory of relativity in electrodynamics *Russian Phys. J.* **47** 726-738;

Jefimenko O D 2004 Presenting electromagnetic theory in accordance with the principle of causality *Eur. J. Phys.* **25** 287-296.

[8] Rousseaux G 2005 Lorenz or Coulomb in Galilean electromagnetism? *Europhys. Lett.* **71** 15-20.

[9] Rousseaux G 2003 On the physical meaning of the gauge conditions of classical electrodynamics: the hydrodynamics analogue viewpoint *Ann. Fondation Louis de Broglie* **28** 261-9. (Article available at: *http* : *//arxiv.org/abs/physics/*0511047).

[10] Maxwell J C 1954 *A Treatise on Electricity and Magnetism, Vol. I and II* (New York: Dover, 1st ed: Oxford, 1873)

[11] Pierseaux Y and Rousseaux G 2006 Les structures fines de l'électromagnétisme classique et de la relativité restreinte forthcoming. (Article available at: *http* : *//arxiv.org/abs/physics/*0601023).

[12] Earman J 1989 *World Enough and Space-Time, Absolute versus Relational Theories of Space and Time* (Cambridge, Mass.: MIT);

Rynasiewicz R 2003 Field unification in the Maxwell-Lorentz theory with absolute space *Phil. Sci.* **70** 1063-1072. (Article available at: *http* : *//philsci* − *archive.pitt.edu/archive/*00001096*/index.html*)

[13] Dyson F J 1990 Feynman's proof of the Maxwell equations *Am. J. Phys.* **58** 209-211.

[14] Brown H R and Holland P R 1999 The Galilean covariance of quantum mechanics in the case of external fields *Am. J. Phys.* **67** 204-214.

[15] Vaidya A and Farina C 1991 Can Galilean mechanics and full Maxwell equations coexist peacefully? *Phys. Lett. A* **153** 265-267.

[16] Brown H R and Holland P R 2003 The non-relativistic limit of the Maxwell and Dirac equations: The role of Galilean and gauge invariance *Studies in the History and Philosophy of Modern Physics* **34** 161-87.

[17] de Gennes P-G 1966 Superconductivity of Metals and Alloys *Ed. W A Benjamin, Inc. New York, Amsterdam.*

[18] Greiter M 2005 Is electromagnetic gauge invariance spontaneously violated in superconductors? *Ann. Phys. (NY)* **319** 217-249.

[19] Lorentz H A 1904 Remarque au sujet de l'induction unipolaire *Arch. Néer.* **9** (1904) 380.

[20] Minkowski H 1908 Die Grundleichungen fur die elektromagnetischen Vorgange in bewegten Koerpern *Nachr. Ges. Wiss. Gottingen* **53** ; 1910 *Math. Annalen* **68** 472;

Arzeliès H 1959 *Milieux conducteurs ou polarisables en mouvement, Etudes Relativistes* (Paris: Gauthier-Villars).

[21] Einstein A and Laub J 1908 Uber die elektromagnetischen Grundgleichungen fur bewegte Koerper *Annalen Phys.* **26** 532-539;

Einstein A and Laub J 1908 Uber die im elektromagnetischen Felde auf ruhende Koerper ausge?bten ponderomotorischen Krafte *Annalen Phys.* **26** 541-550.

[22] Panofsky W K H and Phillips M 1955 *Classical Electricity and Magnetism* (New York: Adison-Wesley).

[23] Roentgen 1888 Uber die durch Bewegung eines im homogenen elektrischen Felde befindlichen Dielectricums hervorgerufene elektrodynamische Kraft *Annalen Phys.* **35** 264.

[24] Eichenwald A 1903 Uber die magnetischen Wirkungen bewegter Koerper im elektrostatischen Felde *Annalen Phys.* **11** 1-30.

[25] Wilson H A 1904 On the electric effect of rotating a dielectric in a magnetic field *Phil. Trans. Roy. Soc. London* **204** 121-137.

[26] Wilson M and Wilson H A 1913 On the electric effect of rotating a magnetic insulator in a magnetic field *Phil. Trans. Roy. Soc. London* **89** 99-106;

Hertzberg J B, Bickman S R, Hummon M T, Krause D, Peck S K and Hunter L R 2001 Measurement of the relativistic potential difference across a rotating dielectric cylinder *Am. J. Phys.* **69** 648-654.

[27] Trouton F T 1902 The results of an electrical experiment involving the relative motion of the earth and ether *Transactions of the Royal Dublin Society* **7** 379-384;

Trouton F T and Noble H R 1903 The mechanical forces acting on a charged electric condenser moving through space *Phil. Trans. R. Soc.* **A 202** 165-181;

Hayden H C 1994 High sensitivity Trouton-Noble experiment *Rev. Sc. Instrum.* **65** 788-792;

Jefimenko O D 1999 The Trouton-Noble paradox *J. Phys. A: Math. Gen.* **32** 3755-3762.

[28] Carvallo E 1921 L'électromagnétisme et le principe de relativité *Comptes Rendus de l'Académie des Sciences de Paris* **173** 1155-1157;

Carvallo E 1921 Le problème de la relativité dans les diélectriques *Comptes Rendus de l'Académie des Sciences de Paris* **173** 1461-1463.

[29] Einstein A 1922 'Ether and the Theory of Relativity: an Address delivered on May 5th, 1920, in the University of Leyden', in Sidelights on Relativity (New York: Dover).

[30] Darrigol O 1993 The electrodynamics of moving bodies from Faraday to Hertz *Centaurus* **36** 245-260;

Darrigol O 1995 Emil Cohn's electrodynamics of moving bodies *Am. J. Phys.* **63** 908-915.

[31] Pauli W 1921 *Theory of Relativity* (New York: Dover, 1981) 127-130.

[32] Einstein A 1905 Zur Elektrodynamik bewegter Korper *Annalen Phys.* **17** 891;

Miller A I 1981 *Albert Einstein's Special Theory of Relativity* (New York: Addison-Wesley).

[33] Norton J D 2004 Einstein's investigation of Galilean covariant electrodynamics prior to 1905 *Archive for History of Exact Sciences* **59** 45-105;

Hon G and Goldstein B R 2005 How Einstein made asymmetry disappear: symmetry and relativity in 1905 *Archive for History of Exact Sciences* **59** 437-544.

[34] Pauli W 1949 *Electrodynamics, Pauli Lectures on Physics* Volume 1 (New York: Dover, 2000) 106-108.

[35] Keswani G H and Kilmister C W 1983 Intimations of relativity: relativity before Einstein *Brit. J. Phil. Sci.* **34** 343-354.