



HAL
open science

Phonon Squeezing in a Superconducting Molecular Transistor

A. Zazunov, Denis Feinberg, Thierry Martin

► **To cite this version:**

A. Zazunov, Denis Feinberg, Thierry Martin. Phonon Squeezing in a Superconducting Molecular Transistor. 2006. hal-00080934v1

HAL Id: hal-00080934

<https://hal.science/hal-00080934v1>

Preprint submitted on 21 Jun 2006 (v1), last revised 28 Sep 2006 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Phonon Squeezing in a Superconducting Molecular Transistor

A. Zazunov ^{a)b)c)}, D. Feinberg ^{b)} and T. Martin ^{c)}

^{a)} *Laboratoire de Physique et Modélisation des Milieux Condensés,
Université Joseph Fourier, BP 166, 38042 Grenoble, France*

^{b)} *Laboratoire d'Etudes des Propriétés Electroniques des Solides, CNRS, BP 166, 38042 Grenoble, France and*

^{c)} *Centre de Physique Théorique, CNRS and Université de la Méditerranée, Case 907 Luminy, 13288 Marseille Cedex 9, France*

(Dated: June 21, 2006)

Josephson transport through a double junction formed by a single molecule or carbon nanotube is considered in the presence of a local vibrational mode which is linearly coupled to the charge on the molecule. An exact solution is obtained in the limit of a large superconducting gap, and it is complemented by a variational analysis in the general case. Coherent charge fluctuations involve polaron dressing and are entangled with non-classical phonon states. The flow of a Josephson current induces squeezing of the phonon mode, controlled by the superconducting phase difference and by the junction asymmetry. Squeezing is maximal in the polaron crossover regime, where a nearly minimum-uncertainty state with about 40 per cent squeezing can be obtained. Optical probes of non-classical states are briefly discussed.

PACS numbers:

Nanoelectromechanical systems (NEMS) offer a way to reach the quantum regime of mechanical oscillators. In particular, a challenging goal consists in creating non-classical vibrational states, similar to non-classical states of light [1, 2]. Squeezed phonon states exhibit reduced quantum fluctuations in one of the mode quadratures. The generation of such states has been suggested in bulk materials [3, 4]. In nanosystems, they may improve the sensitivity of force measurements, or allow the storing of quantum information [5]. Recently, squeezing scenarios have been suggested for resonators driven by nonlinear coupling [6] and for a superconducting island with two charge states (qubit) coupled to a cantilever [7]. Here we consider a superconducting molecular transistor, where the molecule carries both electronic and vibrational degrees of freedom. We show that squeezed states occur at equilibrium when a dc Josephson current crosses the molecule. Compared to the frequency range of micro-cantilevers (less than 1 GHz), here the frequency range goes from 1 GHz to 10 THz depending on the system (single wall carbon nanotube (SWCNT) or single molecule).

The excitation of molecular vibrations (phonons) by an electronic current has been observed with normal metallic leads in several molecules including fullerenes and carbon nanotubes [8]. The latter are in fact NEMS, where charge fluctuations couple to bending [9], stretching [10] or radial breathing (RBM) modes [11]. Incoherent polaron-like charge fluctuations due to transport trigger a non-equilibrium distribution of the phonon mode [12]. Here we address the superconducting regime and show that the presence of a supercurrent through the molecule generates non-classical phonon states, by exploiting the coherent regime of polaron dynamics [13]. Recently, the proximity effect and the Josephson effect [14, 15] have been observed in gated carbon nanotubes. Yet, in the tunneling regime and for strong enough electron-phonon ($e-ph$) interaction, the Josephson current can be suppressed due to the Franck-Condon dressing factor [16]. In contrast, we show here that for transparent lead-molecule contacts, the Josephson effect survives a quite strong $e-ph$ interaction and triggers coherent phonon fluctuations. As a striking

consequence, the conjugate momentum of the molecular distortion displays reduced zero-point fluctuations. The superconducting phase difference and the junction asymmetry allows to control non-classical squeezed phonon states, including nearly (gaussian) minimum-uncertainty states.

The model Hamiltonian reads $H = H_M + H_L + H_R + H_T$, where H_M , $H_{L,R}$ and H_T describe, respectively, the uncoupled molecule, two superconducting leads and the molecule-lead coupling. Explicitly,

$$H_M = [\epsilon_0 - \lambda (b + b^\dagger)] \sum_{\sigma=\uparrow,\downarrow} (n_\sigma - \frac{1}{2}) + U n_\uparrow n_\downarrow + \Omega b^\dagger b, \quad (1)$$

where $n_\sigma = d_\sigma^\dagger d_\sigma$ with d_σ , d_σ^\dagger being the fermion operators for spin $\sigma = \uparrow, \downarrow$ on the molecular level $\epsilon_0 = \epsilon_0(V_g)$, the position of which can be tuned by the gate, U is the repulsive Coulomb interaction, and b, b^\dagger are the boson operators for the local mode of frequency Ω . The leads are described by a pair of standard BCS Hamiltonians ($j = L, R$ is the lead index),

$$H_j = \sum_k \Psi_{jk}^\dagger (\xi_k \sigma_z + \Delta \sigma_x) \Psi_{jk}, \quad \Psi_{jk} = \begin{pmatrix} \psi_{jk,\uparrow} \\ \psi_{j(-k),\downarrow}^\dagger \end{pmatrix}, \quad (2)$$

with the energy dispersion ξ_k ; the Pauli matrices $\sigma_{x,z}$ act in Nambu space. The molecule-lead coupling is given by

$$H_T = \sum_{jk} \Psi_{jk}^\dagger \mathcal{T}_j d + \text{H.c.}, \quad d = \begin{pmatrix} d_\uparrow \\ d_\downarrow \end{pmatrix}, \quad (3)$$

where $\mathcal{T}_{j=L/R} = t_j \sigma_z e^{\pm i \sigma_z \phi/4}$, t_j is the j th lead-molecule tunneling amplitude, and ϕ is the superconducting phase difference across the molecule. Averaging over the leads by using the Wick theorem yields a partition function for the molecular site, $Z[\phi] = \text{Tr} \{ e^{-\beta H_M} W(\beta, 0) \}$, with

$$W(\beta, 0) = T_\tau \exp \left\{ - \int_0^\beta d\tau d\tau' d^\dagger(\tau) \Sigma(\tau - \tau') d(\tau') \right\}, \quad (4)$$

where T_τ is the imaginary-time ordering operator, β is the inverse temperature, $\Sigma(\tau - \tau') = \sum_j T_j^\dagger(\tau) g(\tau - \tau') T_j(\tau')$, and $g(\tau) = -\sum_k (\partial_\tau + \xi_k \sigma_z + \Delta \sigma_x)^{-1} \delta(\tau)$ is the Green function of the uncoupled leads. Assuming a constant density of states near the Fermi level in the (normal) leads, ν , one obtains:

$$\Sigma(\tau - \tau') = \Gamma \left[\partial_\tau + \Delta \left(\cos \frac{\phi}{2} \sigma_x + \gamma \sin \frac{\phi}{2} \sigma_y \right) \right] Q(\tau - \tau'), \quad (5)$$

with $Q(\tau) = \beta^{-1} \sum_{\omega_n} e^{-i\omega_n \tau} / \sqrt{\omega_n^2 + \Delta^2}$, where $\omega_n = \pi(2n + 1)/\beta$ is a fermionic Matsubara frequency, $\Gamma = \Gamma_L + \Gamma_R$, $\gamma = (\Gamma_L - \Gamma_R)/\Gamma$ and $\Gamma_j = \pi \nu t_j^2$.

First we focus on the case where the relevant molecule dynamics is restricted to the low-frequency domain on the scale of the superconducting gap (in particular, $\Gamma \ll \Delta$). In this limit, the Σ term in Eq. (4) becomes local in time with $Q(\tau - \tau') \rightarrow \Delta^{-1} \delta(\tau - \tau')$ at low frequencies, $\omega_n \ll \Delta$. As a result, one can write an effective Hamiltonian for the molecule as

$$H_{eff} = H_M + \Gamma d^\dagger \left(\cos \frac{\phi}{2} \sigma_x + \gamma \sin \frac{\phi}{2} \sigma_y \right) d. \quad (6)$$

The Hilbert space of H_{eff} is divided into two subspaces, \mathcal{H}_1 and \mathcal{H}_2 , spanned by electron states $\{|\uparrow\rangle, |\downarrow\rangle\}$ and $\{|0_d\rangle, |\uparrow\downarrow\rangle\}$, respectively. In the large Δ limit, these subspaces are decoupled. In what follows, we assume the repulsion U to be relatively small, $U/2 < \Gamma$, a condition verified for sufficiently good contacts [15]. As a consequence, the ground state of the system lies in the \mathcal{H}_2 subspace [17]. Introducing the Pauli-matrix operators $\tau_z = |0_d\rangle\langle 0_d| - |\uparrow\downarrow\rangle\langle \uparrow\downarrow|$ and $\tau_+ = (\tau_-)^\dagger = |0_d\rangle\langle \uparrow\downarrow|$, and performing the rotation $H_{eff} = e^{i\tau_z \chi/2} \tilde{H} e^{-i\tau_z \chi/2}$ with $\chi = \arctan(\gamma \tan \frac{\phi}{2})$ lead to an effective spin-boson Hamiltonian:

$$\tilde{H} = \Omega b^\dagger b - [\epsilon - \lambda(b + b^\dagger)] \tau_z + \tilde{\Gamma}(\phi) \tau_x, \quad (7)$$

where $\tilde{\Gamma}(\phi) = \rho(\phi)\Gamma$, $\rho(\phi) = \sqrt{\cos^2(\phi/2) + \gamma^2 \sin^2(\phi/2)}$, and $\epsilon = \epsilon_0 + U/2$. Any eigenstate of \tilde{H} , and particularly the ground state $|\tilde{\Psi}_0\rangle$, can be written as

$$|\tilde{\Psi}_0\rangle = e^{-i\tau_z \chi/2} |\Psi_0\rangle = \sum_{n=0}^{\infty} \left(A_n^{(0)} |0_d\rangle + A_n^{(2)} |\uparrow\downarrow\rangle \right) |n\rangle, \quad (8)$$

where $|n\rangle = (b^\dagger)^n / \sqrt{n!} |0_{ph}\rangle$ is the n -phonon Fock state. The state (8) exhibits entangled charge and vibrational states. We notice in passing that in the case of $\epsilon = 0$, the amplitudes in Eq. (8) are related by $A_n^{(0)} = (-1)^{n+1} A_n^{(2)}$, owing to the parity symmetry $[\tilde{H}, \tau_x (-1)^{b^\dagger b}] = 0$.

The Josephson current flowing through the molecule can be obtained by taking a functional derivative of the generating functional $Z_\xi \equiv Z[\phi + \xi(\tau)]$ with respect to the source variable $\xi(\tau)$, $J = -2 \delta \ln Z_\xi / \delta \xi(0)|_{\xi=0}$. After averaging over the leads and neglecting retardation of the kernel Σ in Eq. (4), the equation for the Josephson current

reduces to $J = Z^{-1} \text{Tr} \left\{ e^{-\beta H_M} W(\beta, 0) \hat{J} \right\}$, where $\hat{J} = \Gamma d^\dagger (-\sin(\phi/2) \sigma_x + \gamma \cos(\phi/2) \sigma_y) d$ is an effective current operator [18] in terms of d . In the rotated basis of Eq. (7), the current operator takes the form

$$\tilde{J} = (\Gamma/2\rho(\phi)) \left[(1 - \gamma^2) \sin \phi \tau_x + 2\gamma \tau_y \right]. \quad (9)$$

Note that in the presence of phonons ($\lambda \neq 0$), the current operator does not commute with the Hamiltonian (7), which results in quantum fluctuations of the current even in the case of $\epsilon = \gamma = 0$.

In the ground state, the relevant experimental quantity is the current expectation value, $J(\phi) = \langle \Psi_0 | \hat{J} | \Psi_0 \rangle$, which probes the overlap of phonon states in Eq. (8). To make further progress, we perform a unitary transformation $U_\alpha = e^{-i\alpha P \tau_z}$ on \tilde{H} of Eq. (7), where $P = i(b^\dagger - b)$ and $\alpha = \lambda/\Omega$:

$$\tilde{H}' = \Omega b^\dagger b - \epsilon \tau_z + \tilde{\Gamma} [\cos(\alpha P) \tau_x + \sin(\alpha P) \tau_y], \quad (10)$$

To lowest order in Γ/Ω , for $\epsilon = 0$, one obtains the ground state $|\Psi_0\rangle = e^{i\chi/2} |\alpha\rangle |0_d\rangle - e^{-i\chi/2} |-\alpha\rangle |\uparrow\downarrow\rangle$, where $|\pm\alpha\rangle = U_{\pm\alpha} |0_{ph}\rangle$ are coherent states of the phonon mode. In the limit of large α , the Josephson current $J(\phi) = \Gamma e^{-\alpha^2} |\sin(\phi/2)|$ becomes strongly suppressed [16]. As we show below, in a molecular transistor, a sizeable Josephson current may flow for relatively strong coupling to the leads and generate squeezing without the necessity of an external drive. This is in contrast with Ref. [7] where an external oscillator ($\Omega < 1\text{GHZ}$) is coupled to a Cooper pair box, for which $\Omega \ll \Gamma$ and $\alpha < 1$ holds.

In order to probe the generic squeezing properties of the molecular phonon, we now compute the mean square fluctuations of both the displacement coordinate $X = b + b^\dagger$ and the conjugate momentum P in the ground state of the system assuming zero temperature. The ground state (8) is found numerically by truncating the phonon Fock space to a maximum number of phonons, $n_{max} \gg \lambda/\Omega$. In the range of parameters considered, $n_{max} = 30$ is sufficient to be sufficient to ensure the convergence of the calculations. When discussing the numerical results, we set $\Omega = 1$.

Coherent charge fluctuations enhance fluctuations of X , $\delta X = (\langle \Psi_0 | X^2 | \Psi_0 \rangle - \langle \Psi_0 | X | \Psi_0 \rangle^2)^{1/2}$, over the quantum zero-point magnitude. Concomitantly, the charge fluctuations *reduce* the fluctuations of the momentum, $\delta P = \langle \Psi_0 | P^2 | \Psi_0 \rangle^{1/2}$. It should be mentioned that $\langle \Psi_0 | P | \Psi_0 \rangle = 0$ always, while $\langle \Psi_0 | X | \Psi_0 \rangle = 0$ for $\epsilon = 0$ only.

The effective polaronic interaction constant $\lambda^2/\tilde{\Gamma}(\phi)$ of the spin-boson Hamiltonian (7) depends on the superconducting phase difference and becomes infinite at $\phi = \pi$ for a symmetric junction, $\gamma = 0$. Fig. 1 displays the ϕ -dependent variation of the momentum fluctuation δP as well as the uncertainty $\delta X \delta P$. (The ground-state value of $\delta X \delta P$ for an harmonic oscillator is 1.) Squeezing ($\delta P < 1$), occurs for a wide range of parameters, and its intensity depends on ϕ , in accordance with the effective polaronic interaction.

For small Γ (strong e - ph interaction), the resulting coherent states $|\pm\alpha\rangle$ are not squeezed, i.e., squeezing marks a deviation from the displaced oscillator (coherent) states. The

generation of gaussian squeezing can be understood from the Hamiltonian (10) by noticing that P^2 is not modified by the transformation U_α . Expanding the third term in Eq. (10) for small λ gives $\frac{1}{2}\alpha^2\Gamma(b^2 + (b^\dagger)^2)\tau_x$, which precisely generates gaussian (two-phonon) squeezed states [2]. The variation of δP between $\phi = 0$ and π , where $\tilde{\Gamma}(\phi)$ is maximum/minimum, shows that squeezing is a crossover phenomenon and is maximal for intermediate coupling parameters of the polaron problem [3]. In general, squeezing does not involve (gaussian) minimum-uncertainty states. Yet, $\delta X \delta P = 1$ can be made very close to 1, as shown in Fig. 1b, for intermediate λ and Γ .

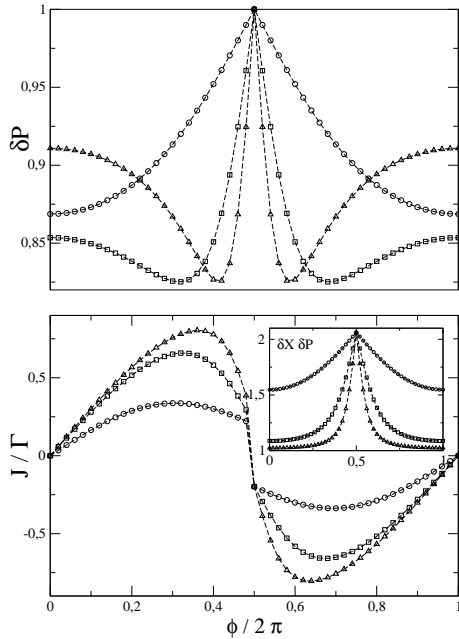


FIG. 1: (a) Squeezing of the momentum, (b) Josephson current, as function of the phase. The inset in (a) displays the Heisenberg uncertainty (1 stands for the harmonic oscillator ground state): $\Gamma = 0.5$ (circles), 2 (squares), and 4 (triangles) for $\lambda = 0.9$, $\epsilon = 0$, $\gamma = 0$.

Squeezed states can only be produced with a sizeable Josephson current. Actually, optimal squeezing (in the sense of minimum-uncertainty) is generated in the parameter range where J is moderately affected by e - ph interaction. To illustrate this, the Josephson current is plotted as a function of the bare interaction $\lambda^2/\Gamma\Omega$ (Fig. 2a): as expected, J decreases but moderately if λ is not too large. The inflexion region corresponds to the polaron crossover where optimum squeezing is achieved. Interestingly, the asymmetry $\gamma \neq 0$ leads to strong (up to 40 percent) and nearly harmonic squeezing, having a weak dependence with the phase difference (Fig. 2b).

In order to substantiate that the squeezing property is generic and not only restricted to the large Δ limit, the case of finite Δ is checked using a variational ansatz approach [3]. A modified polaronic transformation $U_p(\eta) = e^{\eta\alpha(b^\dagger - b)\tau_x}$ is performed on the original Hamiltonian, Eqs. (1)–(3), followed by a harmonic squeezing transformation $U_{sq}(\beta) =$

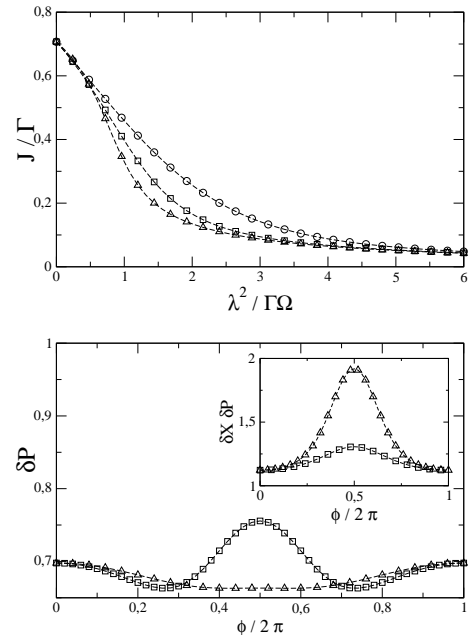


FIG. 2: (a) Josephson current as a function of the electron-phonon interaction, at $\phi = \pi/2$: $\Gamma = 0.5$ (circles), 1 (squares) and 2 (triangles) for $\epsilon = 0$, $\gamma = 0$; (b) Same as Fig. 1a, asymmetric case: $\gamma = 0.5$ (squares) and 0.75 (triangles) for $\lambda = 1.5$, $\Gamma = 4$, $\epsilon = 0$.

$e^{\beta(b^\dagger - b)^2}$, where η and β are variational parameters. Averaging over the zero-phonon state leads to renormalized tunneling amplitudes $T'_j = T_j \exp(-\frac{1}{2}\alpha^2\eta^2 e^{-4\beta})$ and the effective interaction $U' = U - 2\eta(2 - \eta)(\lambda^2/\Omega)$. The degree of squeezing is given by $\delta P = e^{-2\beta}$. For relatively large Γ , using the Hartree approximation, one minimizes the ground state energy of the system with respect to η and β . This yields a good qualitative agreement with the exact large Δ calculation. For finite Δ , squeezing is still present provided that $\lambda > \Omega$. For instance, with the parameters of Fig. 2b ($\lambda = 1.5$, $\Gamma = 4$, $\gamma = 0.75$) and $\phi = 0$, one finds $\delta P = 0.79$ and 0.76 for $\Delta = 100$ and 4, respectively. In the opposite limit of $\Delta \ll \Gamma$, assuming $\Delta = 0.1$, $\lambda = 1.2$, $\Gamma = 2$, $\gamma = 0$, one finds (at $\phi = \pi/2$) $\delta P = 0.77$ with $\delta X \delta P = 1.1$, indicating that even with rather high frequency modes such as RBM in nanotubes ($\Omega \approx 10\text{meV}$), squeezing could be obtained (assuming niobium contacts, $\Delta \approx 1\text{meV}$).

Nonclassical phonon states (other than squeezed states) can be generated by projecting the ground state on the current eigenstates, which are in principle accessible experimentally. The ground state in the current state basis (defined from Eq. (9) as $|\tilde{\Psi}_0\rangle = \pm J|\pm\rangle$) reads: $|\Psi_0\rangle = |\Phi_+\rangle|+\rangle + |\Phi_-\rangle|-\rangle$, with $|\Phi_\pm\rangle = e^{\mp i\chi'/2} \sum_{n=0}^{\infty} \left(e^{i\chi/2} A_n^{(0)} \pm e^{-i\chi/2} A_n^{(2)} \right) |n\rangle$ and $\chi' = \arctan\left(\gamma \cot\frac{\phi}{2}\right)$. In the small Γ regime, one obtains, for $\epsilon = 0$, the ‘‘phase cat’’ states,

$$|\Phi_\pm\rangle = e^{\mp i\chi'/2} \left(e^{i\chi/2} |\alpha\rangle \pm e^{-i\chi/2} |-\alpha\rangle \right) / \sqrt{2}, \quad (11)$$

which are superpositions of opposite coherent states of the

phonon mode. More generally, states like $|\Phi_{\pm}\rangle$ are similar to those generated in Quantum Optics [1]. They carry nontrivial phases, obtained here by tuning the junction asymmetry γ . In particular, for $\gamma = 0$, they are built from phonon Fock states with even/odd occupation numbers only, being linked, respectively, to the $|\pm\rangle$ current states. At moderate λ it is possible to achieve such nontrivial states. They have a substantial overlap between phonon states and a sizable Josephson current.

We have checked that squeezing is robust against environmental effects induced by coupling of the local vibrational mode to an external phonon bath, using the reduced density matrix formalism. Provided that the quality factor $Q = \Omega/\eta$, with η the dissipative coupling constant is large enough ($Q \sim 10^2 - 10^4$ has been suggested [9, 10, 11]), the squeezing property of the ground state is protected due to the gap ($\sim \min(\Omega, \Delta)$) in the excitation spectrum of the system.

A direct probe of squeezed states, and non-classical phonon states is needed. For low- Ω bending modes in SWCNT, capacitive detection through a single-electron transistor can be envisioned. Optical detection techniques may be used, such as Resonant Raman Scattering (RRS), which has been achieved in carbon nanotubes [20] for RBM modes. In RRS, the incident photon excites an electronic transition within the molecule, and a photon is re-emitted with excitation of the phonon modes. We denote d' the state corresponding to a low lying molecular orbital, which is assumed to be decoupled from the molecular vibrations. We complement the molecule Hamiltonian as

$$H'_M = H_M + \epsilon' d'^{\dagger} d' + (\zeta e^{-i\omega t} d'^{\dagger} d' + H.c.) . \quad (12)$$

One uses the golden rule transition rate from state (i), as a sum

over all final states (f) of the virtual transition. The initial state is the projection of the ground state (8) on the zero-electron subspace, with the phonon part $|\Phi^{(0)}\rangle = \sum_{n=0}^{\infty} A_n^{(0)} |n\rangle$, while the final state with *one* electron in the orbital d (together with a hole in d'), involves an arbitrary number of phonons without a net displacement: $|i\rangle = |\Phi^{(0)}\rangle |0\rangle_d |\uparrow\downarrow\rangle_{d'}$ and $|f\rangle = |n\rangle |\sigma\rangle_d |-\sigma\rangle_{d'}$. Resonant Raman emission lines (Stokes) will appear at energies $\epsilon_0 - \epsilon' + n\Omega - E_0(\phi)$ ($E_0(\phi)$ is the ground state energy), with probabilities given by $\langle n | \Phi^{(0)} \rangle^2 = |A_n^{(0)}|^2$; this yields a spectroscopy of the squeezed state $|\Phi^{(0)}\rangle$. While absorption from a filled molecular orbital (d') probes $|\Phi^{(0)}\rangle$, similarly absorption towards an empty orbital will probe $|\Phi^{(2)}\rangle = \sum_{n=0}^{\infty} A_n^{(2)} |n\rangle$ by taking out an electron from the state $|\uparrow\downarrow\rangle_d$. Concerning "cat states", experimental evidence could be gained by far-infrared optical absorption, involving a transition from the subgap (Andreev) bound state to quasiparticle states in the leads.

In summary, phonon squeezing results from the nonlinear interaction brought by coherent charge fluctuations in a superconducting molecular junction. Squeezing occurs for a wide range of parameters and is easily controllable, provided that the local vibrational mode is weakly coupled to the environment. For these purposes, suspended nanotubes present the advantage that their various vibrational modes are well characterized and the corresponding quality factors are large. Besides interesting fundamental properties, the generation of non-classical phonon states promises applications to nanoelectromechanics and to quantum information.

D. F. and T. M. acknowledge support from the A.C. Nanosciences Program NR0114 of Ministry of Research.

-
- [1] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986); M. Brune, S. Haroche, J.M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45** 5193 (1992).
- [2] H.P. Yuen, Phys. Rev. A **13**, 2226 (1976).
- [3] H. Zheng, Solid State Commun. **65**, 731 (1988); Phys. Rev. B **37**, 7419 (1988); D. Feinberg, S. Ciuchi, and F. de Pasquale, Int. J. Mod. Phys. B **4** 1317 (1990).
- [4] Xuedong Hu and F. Nori, Phys. Rev. B **53**, 2419 (1996); Phys. Rev. Lett **79**, 4605 (1997).
- [5] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information* (Springer-Verlag, Berlin, 2000).
- [6] M.P. Blencowe and M.N. Wybourne, Physica B **280** 555 (2000); P. Rabl, A. Shnirman, and P. Zoller, Phys. Rev. B **70**, 205304 (2004); Y.D. Wang, Y.B. Gao, and C.P. Sun, Eur. Phys. J. B **40**, 321 (2004); R. Ruskov, K. Schwab, and A. N. Korotkov, Phys. Rev. B **71**, 235407 (2005).
- [7] A.D. Armour, M.P. Blencowe and K.C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).
- [8] H. Park et al., Nature (London) **407**, 57 (2000); A.N. Pasupathy et al., NanoLetters **5**, 203 (2005).
- [9] S. Sapmaz, Ya. M. Blanter, L. Gurevich, and H. S. J. van der Zant, Phys. Rev. B **67**, 235414 (2003); L.M. Jonsson, L.Y. Gorelik, R.I. Shekhter, and M. Jonson, Nano Lett. **5**, 1165 (2005).
- [10] S. Sapmaz, P. Jarillo-Herrero, Ya.M. Blanter, C. Dekker, and H.S.J. van der Zant, Phys. Rev. Lett **96** 026801 (2006).
- [11] B. J. LeRoy, S. G. Lemay, J. Kong, and C. Dekker, Nature **432**, 371 (2004).
- [12] D. Boese and H. Schoeller, Europhys. Lett **54**, 668 (2001); S. Braig and K. Flensberg, Phys. Rev. B **68**, 205324 (2003); A. Mitra, I. Aleiner, and A.J. Millis, Phys. Rev. B **69**, 245302 (2004); J. Koch, M. Semmelhack, F. von Oppen, and A. Nitzan, Phys. Rev. B **73**, 155306 (2006).
- [13] T. Holstein, Ann. Phys. **8**, 325 (1959); **8**, 353 (1959).
- [14] A. Yu. Kasumov et al., Science **284**, 1508 (1999).
- [15] P. Jarillo-Herrero, J. A. van Dam, and L. P. Kouwenhoven, Nature (2006).
- [16] T. Novotny, A. Rossini and K. Flensberg, Phys. Rev. B **72**, 224502 (2005).
- [17] E. Vecino, A. Martin-Rodero, and A. Levy Yeyati, Phys. Rev. B **68**, 035105 (2003).
- [18] A. Zazunov, V.S. Shumeiko, E.N. Bratus', J. Lantz, and G. Wendin, Phys. Rev. Lett. **90**, 087003 (2003); A. Zazunov, V.S. Shumeiko, G. Wendin, and E.N. Bratus', Phys. Rev. B **71**, 214505 (2005).
- [19] D.V. Averin and K.K. Likharev, in B.L. Al'tschuler, P.A. Lee and R.A. Webb (eds.) *Mesoscopic Phenomena in Solids* p. 213 (Elsevier, New York 1991); M. Tinkham, *Introduction to Superconductivity*, (McGraw-Hill 1996), p. 274.
- [20] M.S. Dresselhaus and P.C. Eklund, Adv. Phys. **49**, 705 (2000).