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Identification of modal parameters using the wavelet transform

Joseph Lardies*, Stéphane Gouttebroze

*Applied Mechanics Laboratory, University of Franche-Comte, R. Chal'eat, UMR 6604 24 rue de l'Épithaphe, 25
000 Besançon, France*

The wavelet transform is used as a time-frequency representation for the determination of modal parameters such as natural frequencies, damping ratios and mode shapes of a vibrating system. It is shown that using a particular form of the son wavelet function, results are improved compared to those obtained with the traditionally Morlet wavelet function. The accuracy of this new technique is confirmed by applying it to a numerical example and to ambient vibration measurements of a tower excited by wind.

Keywords: Wavelet transform; Modal parameters; Ambient vibration; Random decrement technique

1. Introduction

In recent years, researchers in applied mathematics, theoretical physics and signal processing have developed powerful wavelet techniques, called wavelet transform, for the multiscale representation and analysis of signals [1–3]. In contrast to Fourier-transform-based approaches where a window is used uniformly for spreaded frequencies, the wavelet transform localises the information in the time-frequency plane, using short windows at high frequencies and long windows at low frequencies. By this way, the characteristics of nonstationary signals can be more closely monitored and the transient behaviour and discontinuities in the signals can be better investigated. For example, if there is an instantaneous impulse disturbance at a certain time interval, it may contribute to the Fourier transform but its location on the time axis will be lost. However, by the wavelet transform, both time and frequency information can be obtained. The wavelet transform is much more local: instead of transforming a pure time description into a pure frequency description, the wavelet transform finds a good compromise, a time-frequency description. An important area of application where these properties are relevant is the analysis of mechanical structures under ambient excitation.

* Corresponding author.

E-mail addresses: joseph.lardies@univ-fcomte.fr (J. Lardies), stephane.gouttebroze@univ-fcomte.fr (S. Gouttebroze).

Studies have been conducted on identification of modal parameters including natural frequencies, damping coefficients and mode shapes of structural systems using ambient vibration records without input measurements (see Ref. [4] for an important literature review, Refs. [5,6]). Solutions are usually based on smooth spectral curves obtained from the random decrement method associated with signal processing techniques. Recently, stochastic sub-space methods [7–9] have been developed in modal analysis of mechanical systems excited randomly and without knowledge of the excitation. These methods use an important number of matrices and the computation time is important.

In this paper, a method applying the wavelet transform for the determination of modal parameters of a structure under ambient excitation is presented. The structure is a 310 m tall TV tower constructed in the city of Nanjing in China. Acceleration responses of this structural system are used in the wavelet transform to identify its dynamic characteristics. The wavelet transform is a time-frequency representation and decouples automatically the modal components. Thus, in the analysis of a multidegrees of freedom system, the wavelet transform does not need any band-pass filtering procedure. In practice, damping coefficients in vibrating systems are the most critical to estimate since they are the most sensitive to noise and measurement errors. The paper shows an efficient method to estimate the damping coefficients using the wavelet transform, with the modified Morlet wavelet function. These damping coefficients are also estimated using autocorrelation functions of signals filtered around natural frequencies. Finally, the wavelet transform is applied to determine the mode shapes of the structure.

The contribution of the paper to other techniques presented in Refs. [10,11] consists essentially in the use of the random decrement method to transform random signals in free vibration responses and the use of a modified Morlet wavelet function to improve the resolution: close modes can be identified.

2. Review of the wavelet transform

Mathematically, wavelet transforms are inner products of the signal $x(t)$ and a family of wavelets [1–3]. Let $\psi(t)$ be the analysing wavelet called also the mother wavelet of the analysis or the wavelet ‘prototype’. The corresponding family of wavelets consists of a series of son wavelets, which are generated by dilatation and translation from the mother wavelet $\psi(t)$ shown as follows:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \left(\frac{t-b}{a} \right), \quad a > 0, \quad b \in R, \quad (1)$$

where a is the dilatation or scale parameter defining the support width of the son wavelet and b the translation parameter localising the son wavelet function in the time domain. The factor $1/\sqrt{a}$ is used to ensure energy preservation in the wavelet transform. The difference between these wavelets is mainly due to the different lengths of filters that define the wavelet and scaling functions. Wavelets must be oscillatory, must decay quickly to zero (can only be nonzero for a short period) and must integrate to zero [1].

The idea of the wavelet transform is to decompose a signal $x(t)$ into wavelet coefficients $W_\psi(a,b)$ using the basis of son wavelets $\psi_{a,b}(t)$. Under the hypothesis that $x(t)$ satisfy the condition:

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty, \quad (2)$$

which implies that $x(t)$ decays to zero as $t \rightarrow \pm\infty$, the wavelet transform of $x(t)$ is expressed by the following inner product in the Hilbert space:

$$W_\psi(a, b) = \langle x(t), \psi_{a,b}(t) \rangle = \int_{-\infty}^{+\infty} x(t) \psi_{a,b}^*(t) dt, \quad (3)$$

where the asterisk stand for complex conjugate. This shows that the wavelet transform is a linear scalar product normalised by the factor $1/\sqrt{a}$ and this scalar product is a measure of the fluctuation of the signal $x(t)$ around the point b at the scale a . The scaling operation is nothing more than performing stretching and compressing operations on the son wavelet, which in turn can be used to obtain the different frequency information of the signal to be analysed. The compressed version is used to satisfy the high-frequency needs, and the dilated version is used to meet low-frequency requirements. Then, the translated version is used to obtain the time information of the signal to be analysed. In this way, a family of scaled and translated wavelets is created and serves as the base, the base for representing the signal to be analysed. In other words, the wavelet transform $W_\psi(a, b)$ can be considered as functions of translation b with each fixed scale a . It gives the information of $x(t)$ at different levels of resolution and also measures the similarity between the signal $x(t)$ and each son wavelet $\psi_{a,b}(t)$. Note that the wavelet transform represents also the convolution between the signal $x(t)$ and the wavelet function. This implies that a wavelet can be used for feature discovery if the wavelet used is similar to the feature components (eventually eigenfrequencies and damping coefficients) hidden in the analysed signal. For the function $\psi(t)$ to qualify as an analysing wavelet, it must satisfy the admissibility condition [1–3]:

$$0 < c_\psi = \int_{-\infty}^{+\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty, \quad (4)$$

where $\psi(\omega)$ is the Fourier transform of $\psi(t)$. Then the wavelet transform can be inverted and the signal $x(t)$ recovered:

$$x(t) = \frac{1}{c} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_\psi(a, b) \psi_{a,b}(t) \frac{da db}{a^2}. \quad (5)$$

Note that since $|\psi(\omega)|$ tends to 0 when ω tends to $\pm\infty$, the Fourier transform of the wavelet can be considered as a band-pass filter.

For practical purposes, the possibility of time-frequency localisation arises if the wavelet $g(t)$ is a window function, which means that $\psi(t)$ decays to zero as $t \rightarrow \pm\infty$:

$$\int_{-\infty}^{+\infty} |\psi(t)| dt < \infty \quad (6)$$

and the wavelet transform analyses a signal $x(t)$ only at windows defined by the wavelet function $\psi(t)$. If one assumes a fast decay of $\psi(t)$: the values of $\psi(t)$ are negligible outside a given time domain interval, the transform becomes local in time domain, in this interval.

The frequency localisation can be explained when the wavelet transform is expressed in terms of the Fourier transform. Note $X(\omega)$ the Fourier transform of the signal $x(t)$ and $a\psi^*(a\omega)e^{j\omega b}$ the

Fourier transform of the son wavelet $\psi^*(t - b)/a$. Using the Parseval's theorem [2], we obtain

$$W_\psi(a, b) = \frac{\sqrt{a}}{2\pi} \int_{-\infty}^{+\infty} X(\omega) \psi^*(a\omega) e^{j\omega b} d\omega \quad (7)$$

and the frequency localisation depends on the scale parameter a . Note that this operation is equivalent to a particular filter band analysis in which the relative frequency bandwidth $\Delta\omega/\omega$ are constant and related to the parameters a, b and to the frequency properties of the wavelet. The local resolution of the wavelet transform in time and frequency is determined by the duration and bandwidth of analysing functions given by $\Delta t = a \Delta t_\psi$ and $\Delta f = \Delta f_\psi/a$, where Δt_ψ and Δf_ψ are the duration and bandwidth of the wavelet function. The resolution of the analysis is therefore good for high dilatation in the frequency domain and for low dilatation in time domain.

Before the calculation, the relevant parameters must be discretised for being computed by a computer. Instead of continuous dilatation and translation, the wavelet may be dilated and translated discretely by selecting $a = a_0^m$ and $b = nb_0 a_0^m$ where a_0 and b_0 are fixed values with $a_0 > 1, b_0 > 0, m, n \in \mathbb{Z}$ and \mathbb{Z} is the set of positive integers. We obtain then a discretised son wavelet and a corresponding discrete wavelet transform which provides a decomposition of a signal into sub-bands with a bandwidth that increases linearly with frequency. In the case of dyadic discretisation, the most popular method, we have $a_0 = 2$ and $b_0 = 1$ and each spectral band is approximately one octave wide. In this form, the wavelet transform can be viewed as a special kind of spectral analyser.

3. Application of the wavelet transform to modulated signals

Consider the case of a signal $x(t)$ modulated in amplitude:

$$x(t) = A(t) \cos(\omega t) \quad (8)$$

and $x_a(t)$ its analytic signal defined as

$$x_a(t) = x(t) + jH[x(t)], \quad (9)$$

where $H[x(t)]$ is the Hilbert transform of $x(t)$:

$$H[x(t)] = \frac{1}{\pi} \int_{-\infty}^{+\infty} x(\tau) \frac{1}{t - \tau} d\tau. \quad (10)$$

If $x(t)$ is assumed asymptotic, that is if the phase of the signal varies much faster than the amplitude, it is shown [3] that the complex signal:

$$z(t) = A(t) e^{j\omega t} \quad (11)$$

constitutes a good approximation of the analytic signal and is used for the determination of the wavelet transform. Following Torresani [3] the wavelet transform of the signal $x(t)$ is

$$W_\psi(a, b) = \langle x(t), \psi_{a,b}(t) \rangle = \frac{1}{2} \langle x_a(t), \psi_{a,b}(t) \rangle = \frac{1}{2\sqrt{a}} \int_{-\infty}^{+\infty} A(t) e^{j\omega t} \psi^* \left(\frac{t - b}{a} \right) dt. \quad (12)$$

Now, the basic idea is to develop the amplitude $A(t)$ around $t = b$ (point where the son wavelet $\psi_{a,b}(t)$ is maximum) using Taylor's formula. Neglecting terms of order superior to one,

we have

$$W_\psi(a, b) = \frac{1}{2\sqrt{a}} \int_{-\infty}^{+\infty} (A(b) + o(A'(b))) e^{j\omega t} \psi^* \left(\frac{t-b}{a} \right) dt \quad (13)$$

the primes indicating a derivative. Using the Fourier transform of $\psi^*(t-b)/a$ and neglecting terms of order one (infinitely small), we obtain

$$W_\psi(a, b) = \frac{\sqrt{a}}{2} A(b) \psi^*(a\omega) e^{j\omega b}. \quad (14)$$

We generalise to signals modulated in amplitude and frequency:

$$x(t) = A(t) \cos(\varphi(t)). \quad (15)$$

The wavelet transform of these signals can be approximated by means of asymptotic techniques as

$$W_\psi(a, b) = \frac{\sqrt{a}}{2} A(b) \psi^*(a\varphi'(b)) e^{j\varphi(b)}. \quad (16)$$

The linearity of the wavelet transform can be applied to P signals modulated in amplitude and frequency:

$$x(t) = \sum_{k=1}^P A_k(t) \cos(\varphi_k(t)) \quad (17)$$

and we obtain

$$W_\psi(a, b) = \frac{\sqrt{a}}{2} \sum_{k=1}^P A_k(b) \psi^*(a\varphi'_k(b)) e^{j\varphi_k(b)}. \quad (18)$$

As a consequence, assuming that the wavelet amplitude is localised at a certain value $\omega = \omega_0$ in the frequency domain (central frequency of the wavelet), the wavelet transform modulus $|W_\psi(a, b)|$ is localised near the P curves with equations

$$a = \delta_k(b) = \omega_0 / \varphi'_k(b). \quad (19)$$

The dilatation parameter a has been calculated in order to maximise the amplitude of $\psi^*(a\varphi'_k(b))$.

4. Modal parameters determination

The free response of a viscously damped single degree of freedom system is

$$x(t) = B e^{-\zeta\omega_n t} \cos(\omega_d t + \psi_0) \quad (20)$$

with ω_n the undamped natural frequency, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ the damped natural frequency and ζ the damping ratio. If the system is underdamped, that is if the damping ratio is smaller than 1, the signal $x(t)$ can be considered asymptotic, and therefore the results obtained previously can be used considering:

$$A(t) = B e^{-\zeta\omega_n t}, \quad (21)$$

$$\varphi(t) = \omega_d t + \psi_0 \Rightarrow \varphi'(t) = \omega_d. \quad (22)$$

The wavelet transform of the damped sinusoid is

$$W_\psi(a, b) = \frac{\sqrt{a}}{2} B e^{-\zeta \omega_n b} \psi^*(a \omega_d) e^{j(\omega_d b + \psi_0)} \quad (23)$$

and its modulus is localised from Eq. (19) at a constant value of the dilatation parameter noted a_0 :

$$a = a_0 = \omega_0 / \omega_d. \quad (24)$$

The dilatation parameter a_0 , which maximises the amplitude of the wavelet transform, corresponds and is related to the analysed modal frequency of the system. For a fixed value of the dilatation parameter ($a = a_0$), the wavelet transform modulus is

$$|W_\psi(a_0, b)| = \frac{\sqrt{a_0}}{2} B e^{-\zeta \omega_n b} |\psi^*(a_0 \omega_d)| \quad (25)$$

and applying the logarithm to this function we obtain

$$\ln |W_\psi(a_0, b)| = -\zeta \omega_n b + \ln \left(\frac{\sqrt{a_0}}{2} B |\psi^*(a_0 \omega_d)| \right). \quad (26)$$

Thus the damping ratio ζ of the system can be estimated from the slope of the straight line of the logarithm of the wavelet transform modulus, assuming that the natural frequency ω_n has been previously estimated (for example, by FFT plot $X(\omega)$ of the signal $x(t)$).

Assuming the Fourier transform of the dilated wavelet real, condition which is fulfilled currently, the wavelet transform phase is given by

$$\text{Arg}(W_\psi(a_0, b)) = \omega_d b + \psi_0 \Rightarrow \frac{d}{db} \text{Arg}(W_\psi(a_0, b)) = \omega_d \quad (27)$$

and the plot of $(d/db) \text{Arg}(W_\psi(a_0, b))$ should be constant in time and equal to the damped natural frequency ω_d .

The damping and frequency estimation procedures, based on the wavelet transform presented above, can be extended to multidegrees of freedom systems. Consider now the free response of a, P degrees of freedom system

$$x(t) = \sum_{k=1}^P B_k e^{-\zeta_k \omega_{nk} t} \cos(\omega_{dk} t + \psi_{0k}), \quad (28)$$

where ζ_k is the damping ratio, ω_{nk} the undamped natural frequency and ω_{dk} the damped natural frequency associated to the k th mode. From Eq. (18), the wavelet transform of the multidegrees of freedom system is

$$W_\psi(a, b) = \frac{\sqrt{a}}{2} \sum_{k=1}^P B_k e^{-\zeta_k \omega_{nk} b} \psi^*(a \omega_{dk}) e^{j(\omega_{dk} b + \psi_{0k})}. \quad (29)$$

The wavelet transform is a signal decomposition procedure working as a filter in the time-frequency domain: it analyses a signal only locally at windows defined by the son wavelet. Thus, multidegrees of freedom system can be decoupled into single degrees of freedom. For a fixed value of the dilatation parameter ($a = a_i$), which maximises $\psi^*(a \omega_d)$, only the mode associated with a_i gives

a relevant contribution in the wavelet transform, while all the other terms are negligible. Thus the wavelet transform of each separated mode $i = 1, 2, \dots, P$ becomes

$$W_\psi(a_i, b) = \frac{\sqrt{a_i}}{2} B_i e^{-\zeta_i \omega_{ni} b} \psi^*(a_i \omega_{di}) e^{j(\omega_{di} b + \psi_{oi})}. \quad (30)$$

Clearly, the wavelet transform offers a decoupling of multidegrees of freedom systems into single modes. Using Eq. (30) associated with Eqs. (26) and (27), it is possible to follow the amplitude and the phase variations in the time domain of each modal component and to estimate the corresponding damping ratio and natural damped frequency associated to the isolated mode. We obtain for each i th mode:

$$\ln|W_\psi(a_i, b)| = -\zeta_i \omega_{ni} b + \ln\left(\frac{\sqrt{a_i}}{2} B |\psi^*(a_i \omega_{di})|\right), \quad (31)$$

$$\text{Arg}(W_\psi(a_i, b)) = \omega_{di} b + \psi_{oi} \Rightarrow \frac{d}{db} \text{Arg}(W_\psi(a_i, b)) = \omega_{di}. \quad (32)$$

This technique requires a previous choice of the value of the dilatation parameter a_i corresponding to the analysed mode and the resolution of the wavelet transform depends on the value of this scale parameter. For a given wavelet function, the resolution of the wavelet transform is determined by the dilatation parameter a_i , thus the choice of the analysing wavelet is important.

5. Choice of the analysing wavelet

There are a number of different complex and real-valued functions used as analysing wavelets and satisfying the admissibility condition. One of the most known and widely used is the Morlet wavelet defined by

$$\psi(t) = e^{j\omega_0 t} e^{-t^2/2}, \quad (33)$$

where ω_0 is the central wavelet frequency. The dilated version of the Fourier transform of $\psi(t)$ is real and is given by

$$(a\omega) = \sqrt{2\pi} e^{-1/2(a\omega - \omega_0)^2}. \quad (34)$$

In practice, to verify the admissibility condition, the value of ω_0 is chosen superior to 5. Note that $\psi(a\omega)$ is maximum at the central frequency $\omega = \omega_0/a$ and the Morlet wavelet can be viewed as a linear bandpass filter whose bandwidth is proportional to $1/a$ or to the central frequency. Thus, the value of the dilatation parameter a at which the wavelet filter is focused on the wavelet frequency ω can be determined from

$$a = \omega_0/\omega. \quad (35)$$

For a given value of the dilatation parameter a , the spectrum of the Morlet wavelet has a fixed bandwidth. If the analysed frequency is important, the dilatation parameter becomes small and the spectrum of the Morlet wavelet function is wide. There is then a bad spectral resolution. An alternative is to modify the Morlet wavelet function introducing a parameter N which controls the shape of the basic wavelet: this parameter balances the time resolution and the frequency resolution of the Morlet wavelet. The modified Morlet wavelet function used in this paper is

$$\psi(t) = e^{j\omega_0 t} e^{-t^2/N} \quad (36)$$

with $N > 0$ and whose dilated version of its Fourier transform is

$$(a\omega) = \sqrt{N\pi} e^{-N/4(a\omega - \omega_0)^2}. \quad (37)$$

An important value of N gives a narrower spectrum allowing a better resolution of closely spaced modes, but at the expense of time resolution. Indeed, increasing N will increase the frequency resolution but it decreases the time resolution. This modified Morlet wavelet function offers a better compromise in terms of localisation, in both time and frequency for a signal, than the traditionally Morlet wavelet function.

6. Applications

6.1. Simulated results

In order to show the usefulness of the wavelet transform method in modal analysis, a simulation is performed with a 2 degrees of freedom system viscously damped. The parameters of the system are chosen as follows: $f_1 = 25$ Hz, $f_2 = 30$ Hz, $\zeta_1 = 0.02$, $\zeta_2 = 0.06$. The free time response and the frequency response functions of this system are given in Fig. 1. The natural frequencies can be obtained directly by inspection of peaks of resonance. To estimate the damping coefficients we use

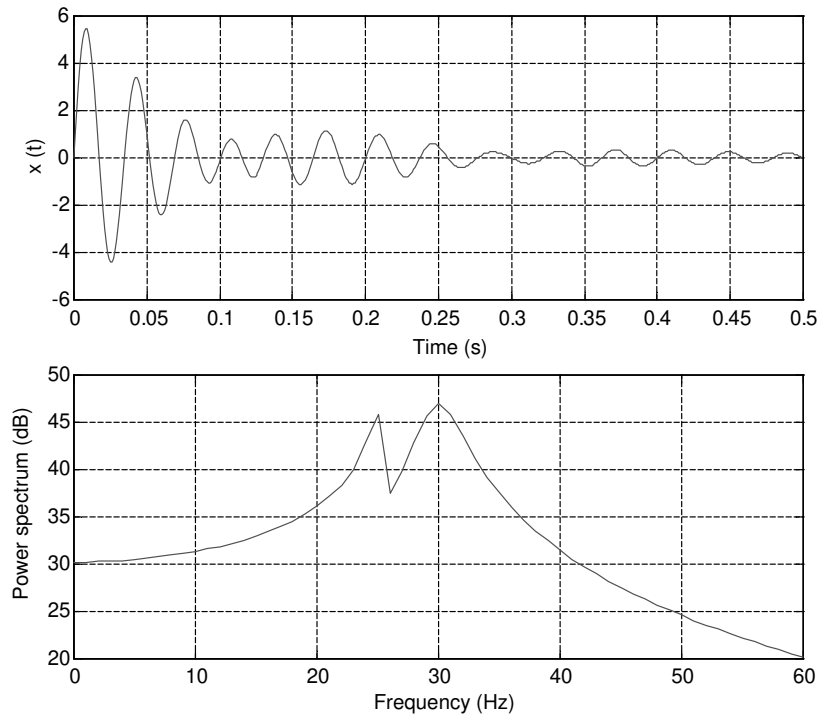


Fig. 1. Time and frequency response of a two degrees of freedom system.

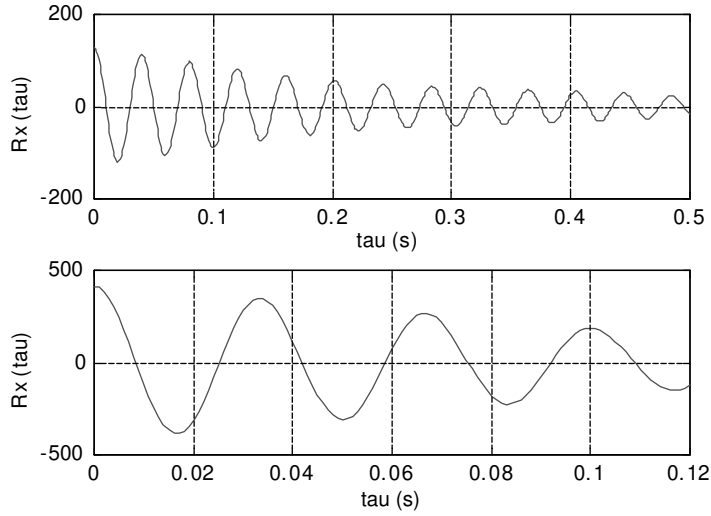


Fig. 2. Autocorrelation functions filtered around the 1st and 2nd mode.

two different methods: the autocorrelation method [9] and the wavelet transform method presented in this paper. In the autocorrelation method, each mode is isolated and filtered around its natural frequency. The autocorrelation function is then computed for each isolated mode and the damping ratio is estimated by

$$\zeta = \frac{1}{2\pi k} \ln \frac{R_x(\tau)}{R_x(\tau + k\tau)}, \quad (38)$$

where $R_x(\tau)$ is the value of the autocorrelation function at time τ and k is the number of cycles. Autocorrelation functions for the two different modes are given in Fig. 2.

In order to estimate damping ratios ζ_i ($i=1,2$) from the wavelet transform, we have to determine the dilatation parameter a_i for each eigen-mode. This can be done by plotting the variations of the scale factor in time. If we use the traditionally Morlet wavelet function where $N=2$, like Fasana et al. [10] and Staszewski [11], the results presented in Fig. 3 (upper) are not exploitable: we cannot extract rigorously the coefficients a_i and present modes are very difficult to observe. The resolution of the wavelet transform is bad to separate the two different modes. Although, increasing N in Eqs. (36) or (37), we obtain for $N=17$ horizontal bands (Fig. 3 below) representing appropriate dilatation parameters and the two present modes can be easily observed. The resolution of the wavelet transform is now accurate enough. Once the coefficients a_1 and a_2 have been obtained, we plot for each a_i the logarithm of the wavelet modulus as a function of time, and use Eq. (31). Thus the damping ratio ζ_1 of the first mode can be estimated from the slope of the straight line of the wavelet modulus, as shown in Fig. 4. The damped eigenfrequency in Hz ($f_{d1}=\omega_{d1}/2\pi$) is obtained from Eq. (32) and plotted: we obtain practically a horizontal line corresponding to f_{d1} . The damping ratio ζ_2 of the second mode can be obtained similarly from Fig. 5. Natural frequencies and damping ratios using the autocorrelation method and the wavelet transform method are given in Table 1.

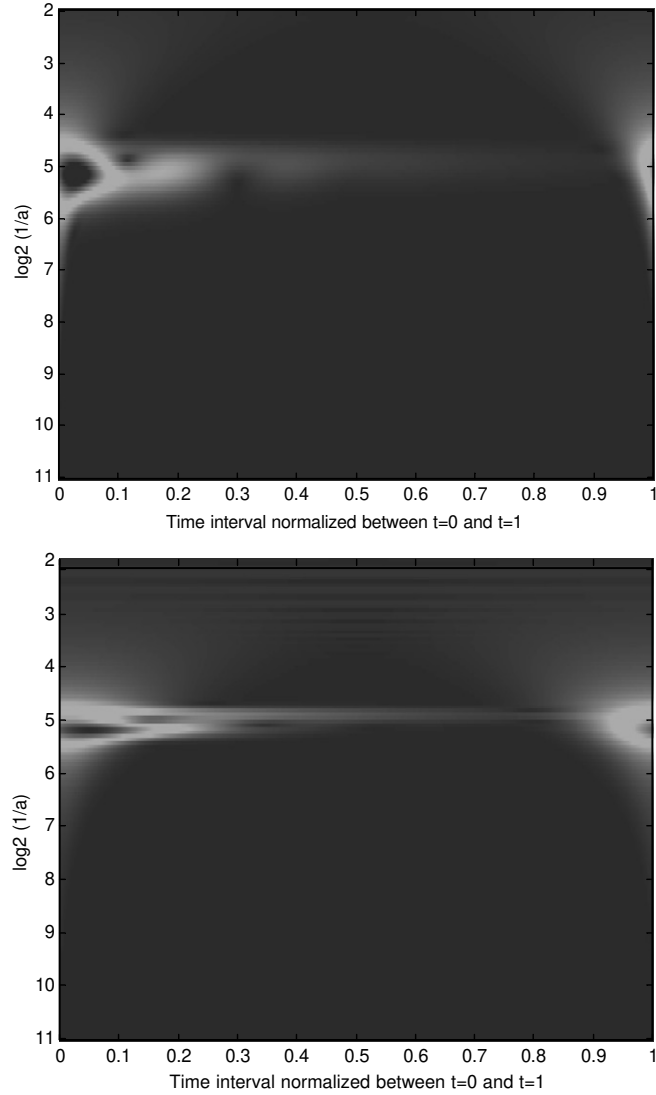


Fig. 3. Time scale for $N = 2$ and for $N = 17$.

6.2. Application to real data

Fig. 6 shows the main structure of the TV tower in the city of Nanjing in China. This tower is 310 m high and the acceleration response of the structural system measured under ambient conditions is used to identify its dynamic characteristics. System identification using ambient vibration measurements is a challenge requiring the use of special identification techniques, which can deal with very small magnitudes of ambient vibration contaminated by noise in the situation where input forces cannot be measured. The accelerometers are installed on the tower at two sets of different locations, as shown in Fig. 6, to measure the ambient vibrations of the system. The sensors at

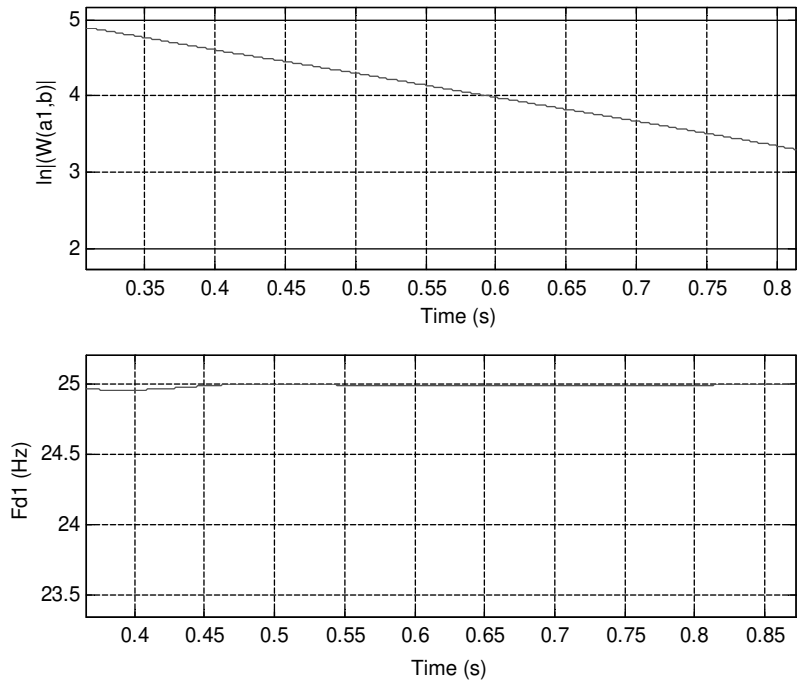


Fig. 4. Wavelet transform envelope and damped frequency for the first mode.

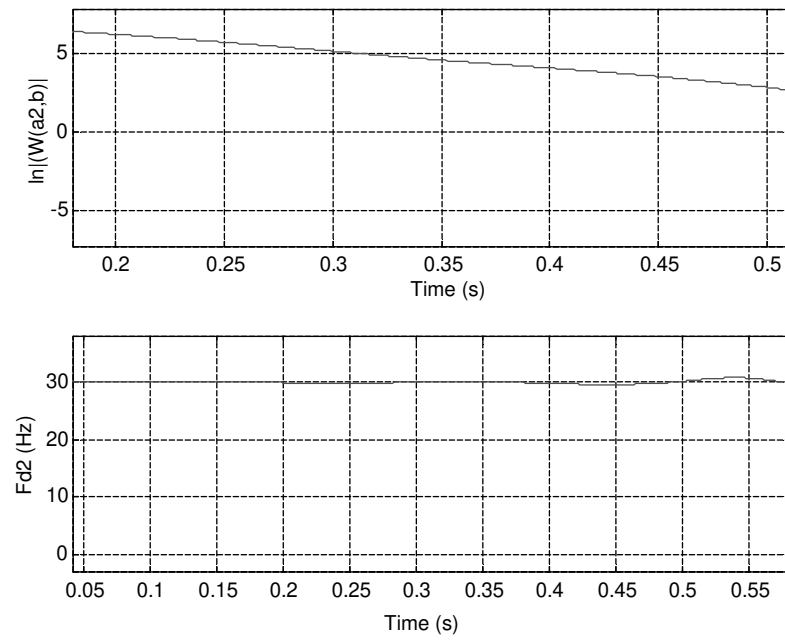


Fig. 5. Wavelet transform envelope and damped frequency for the second mode.

Table 1
Estimation of frequencies and damping ratios of the simulated two degrees of freedom system

| Mode | Frequency (Hz) | Damping ratio by the autocorrelation method | Damping ratio by the wavelet transform |
|------|----------------|---|--|
| 1 | 25 | 0.0201 | 0.0200 |
| 2 | 30 | 0.0597 | 0.0604 |

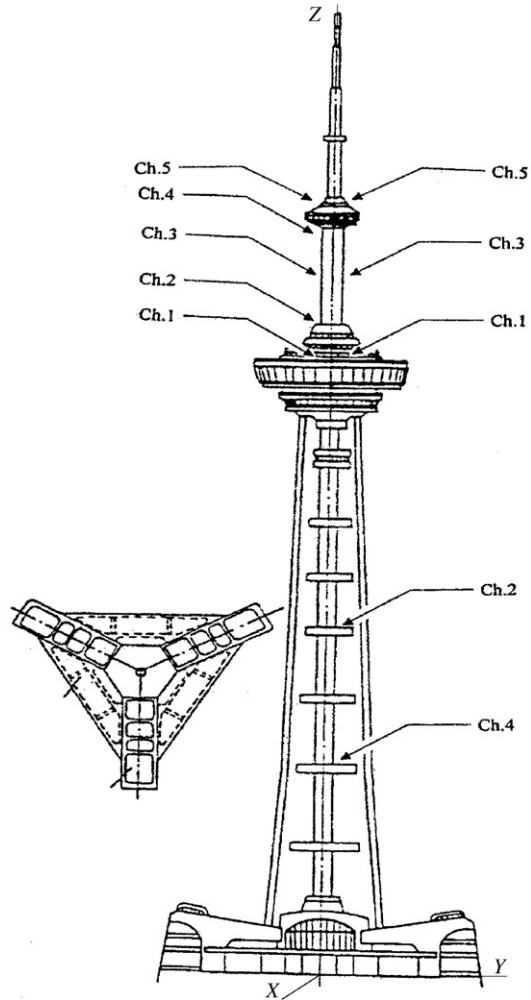


Fig. 6. Main structure of the tower and sensor locations.

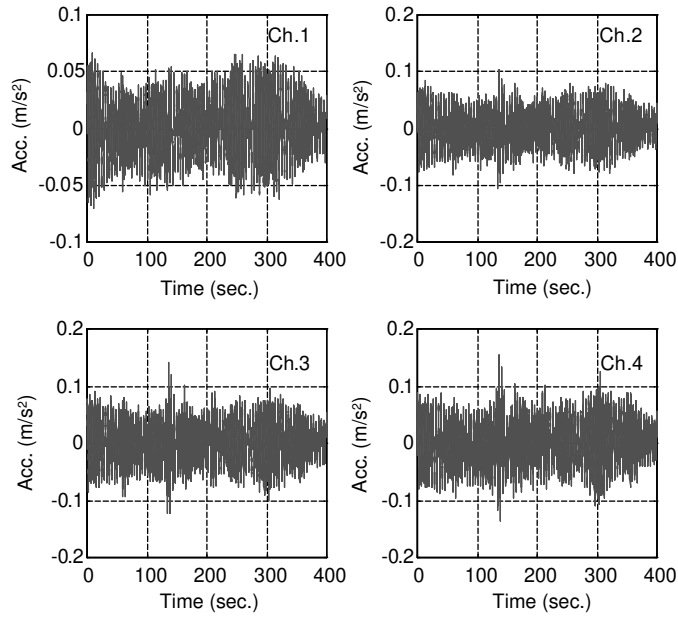


Fig. 7. Time histories of acceleration records.

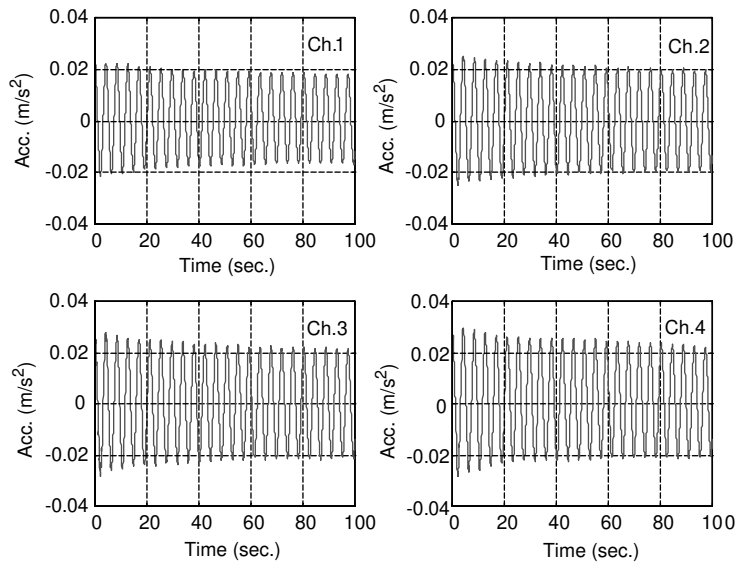


Fig. 8. Time histories of randomdec acceleration signals.

the first set of locations are concentrated on the upper part of the structure since this part is more flexible resulting in more vibration than the lower part, while those at the second set of locations are distributed along the height of the whole structure. The accelerometers are placed as close as possible to the centre of the cross section of the tower in order to minimise the response component

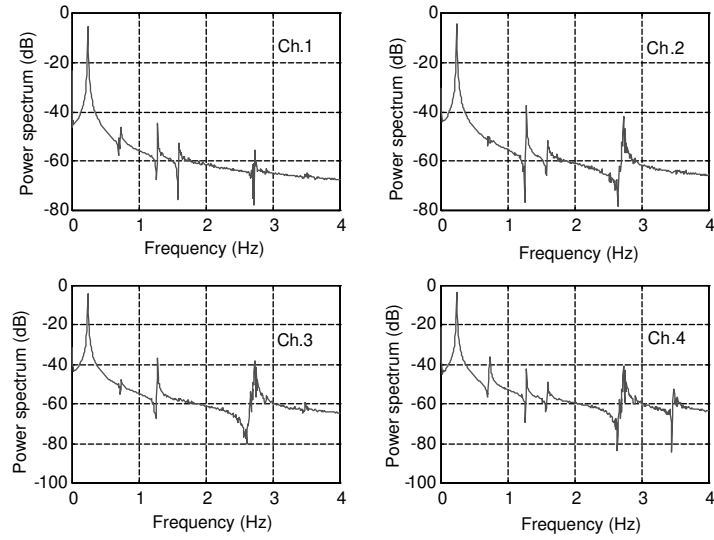


Fig. 9. Frequency response of randomdec acceleration signals.

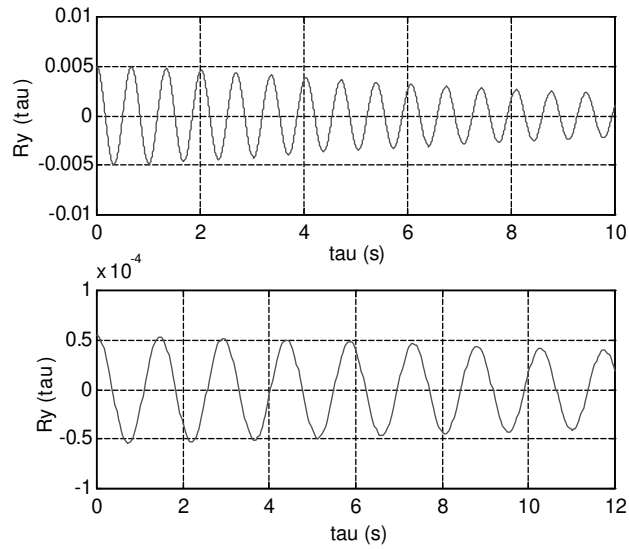


Fig. 10. Autocorrelation functions of signals filtered around the 1st and 2nd mode of the tower.

due to torsional vibration. Acceleration records are obtained simultaneously in one direction each time, with a sampling time interval of 0.03125 s and a total recording time of approximately 600 s. Fig. 7 shows some time histories (400 s) recorded.

The wavelet estimation technique operates on the free response of the analysed system. A well-established method to convert random responses to free decay responses is the random decrement technique [5,6]. Its basic concept is that the acceleration response $y(t)$ measured on the structure can be decomposed into free vibration component and forced vibration component. The free vibration

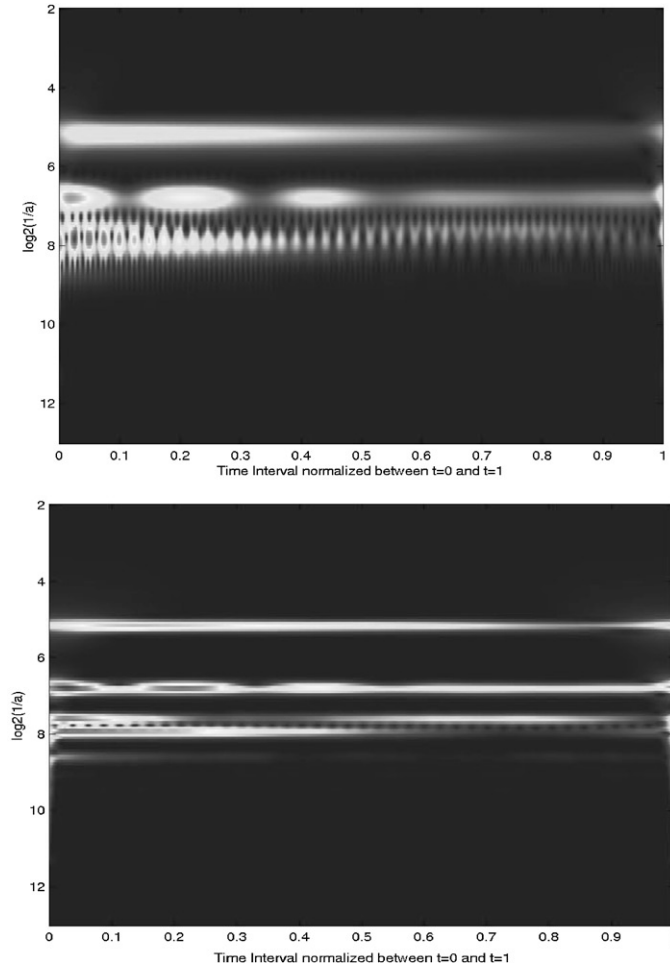


Fig. 11. Time-scale for $N = 2$ (upper) and for $N = 15$.

component contains an impulse or step response, while the forced vibration component represents response to the random wind load. The free vibration component can be obtained by a special averaging procedure of measurements which remove the random part, leaving its deterministic part. The randomdec signal (or free response) $x(\tau)$ of the measured signal is defined by

$$x(\tau) = \frac{1}{N} \sum_{m=1}^N y(t_m + \tau), \quad (39)$$

where N is the number of time samples averaged, τ is the free response time length and t_m is determined from original data as all the time instants satisfying initial conditions; t_m 's are taken as those of zero crossing time with positive slope on the acceleration time history record, we have a

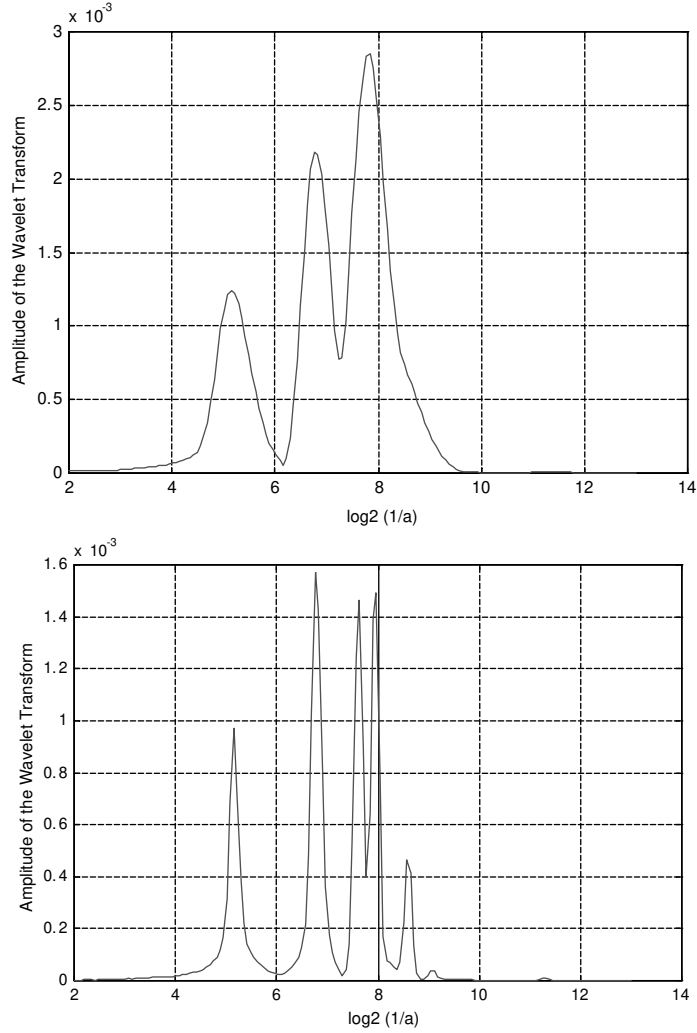


Fig. 12. Amplitude of the wavelet transform for $N = 2$ (upper) and for $N = 15$.

very simple condition:

$$y(t_m) = 0 \quad \text{and} \quad \left(\frac{dy}{dt} \right)_{t=t_m} > 0. \quad (40)$$

The total length of the randomdec signal is 4096 points. In Fig. 8, the time histories of four randomdec signals are plotted and are similar to free vibration responses. Their spectra are plotted in Fig. 9 and natural frequencies can be estimated directly from these plots. Autocorrelation functions for the two first modes are plotted in Fig. 10 and used to determine their damping ratios from Eq. (38). Similar results are obtained if we consider other modes and are not reported in the paper. Using the wavelet transform to estimate ζ_i from Eq. (31), we have to determine the dilatation

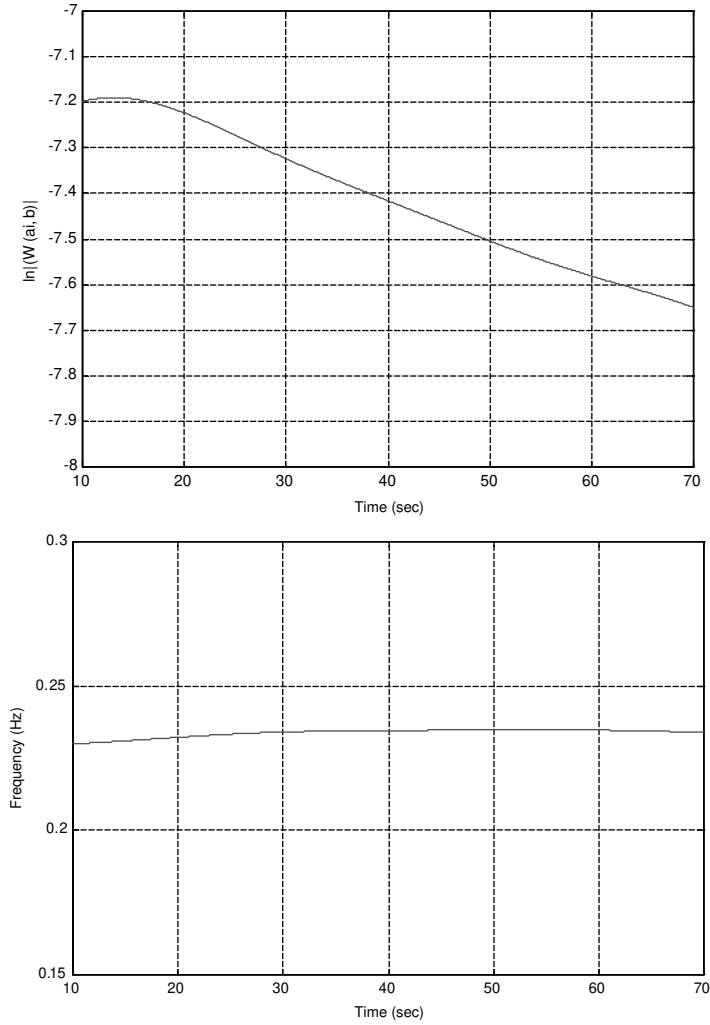


Fig. 13. Wavelet transform envelope and damped frequency for the first mode with $N = 15$, using channel 1.

parameter a_i for each eigen-mode. This can be obtained by plotting the variations of the scale factor in time. We use the traditionally Morlet wavelet function with $N = 2$. The results presented in Figs. 11 (upper) and 12 (upper) are not exploitable: we cannot extract rigorously the coefficients a_i and present modes are impossible to observe. The resolution of the wavelet transform is not sharp enough to separate different modes of the system. Although, by increasing N in the modified Morlet wavelet function, we obtain for $N = 15$ horizontal bands (Fig. 11) representing appropriate dilatation parameters and present modes can be easily observed. The resolution of the wavelet transform is now accurate enough. This can be confirmed if we plot the amplitude of the wavelet transform for different values of the dilatation parameter as shown in Fig. 12. Six values of the dilatation parameter are predominant in this plot and they correspond to the number of modes present in the frequency band.

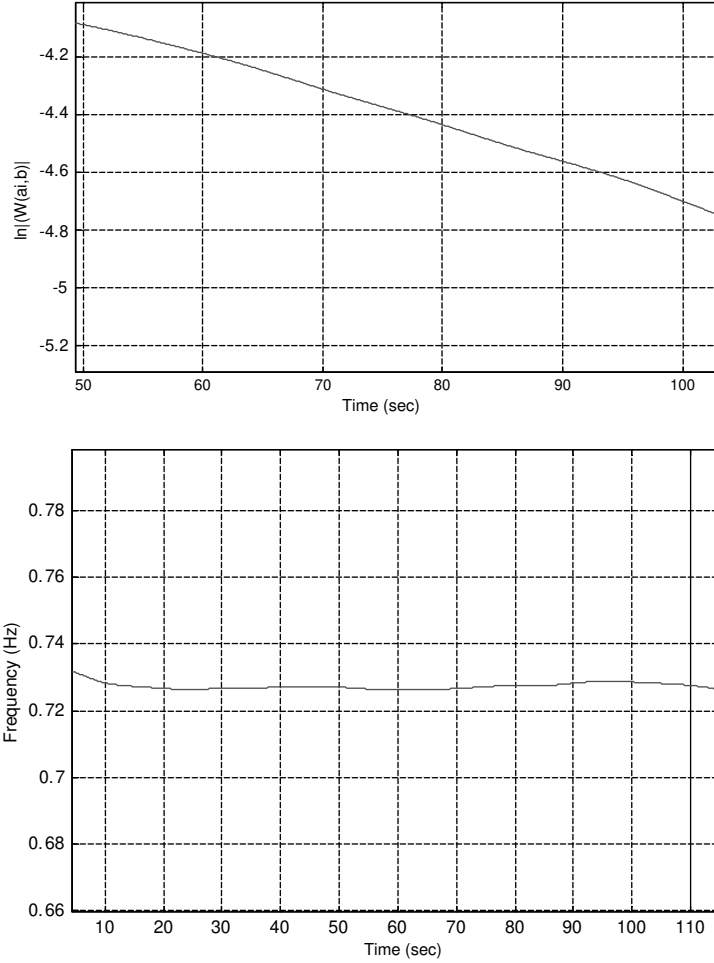


Fig. 14. Wavelet transform envelope and damped frequency for the second mode with $N = 15$, using channel 1.

Once the coefficients a_i ($i = 1, \dots, 6$) have been obtained, we plot for each a_i the logarithm of the wavelet modulus as a function of time, and use Eq. (31). Thus the damping ratio ζ_i can be estimated from the slope of the straight line of the wavelet modulus, as shown in Figs. 13–15, where only the three first modes have been considered (similar results are obtained if we use other modes). The damped angular frequency ω_{di} (or damped frequency in Hz) is given by Eq. (32) and plotted in these figures: we obtain practically horizontal lines, since the ω_{di} 's should be constant in time. Natural frequencies and damping ratios using an average over accelerometers are given in Table 2.

Results using the autocorrelation function are conditioned to the choice of a bandpass filter to isolate the mode under consideration. Here, we have taken a 6th-order Butterworth bandpass filter and results obtained are not optimal. These results vary if we use other bandpass filters. The wavelet transform technique does not use any external bandpass filter and the results obtained can be considered as optimal for the modified Morlet wavelet function, once the value of N has been properly chosen.

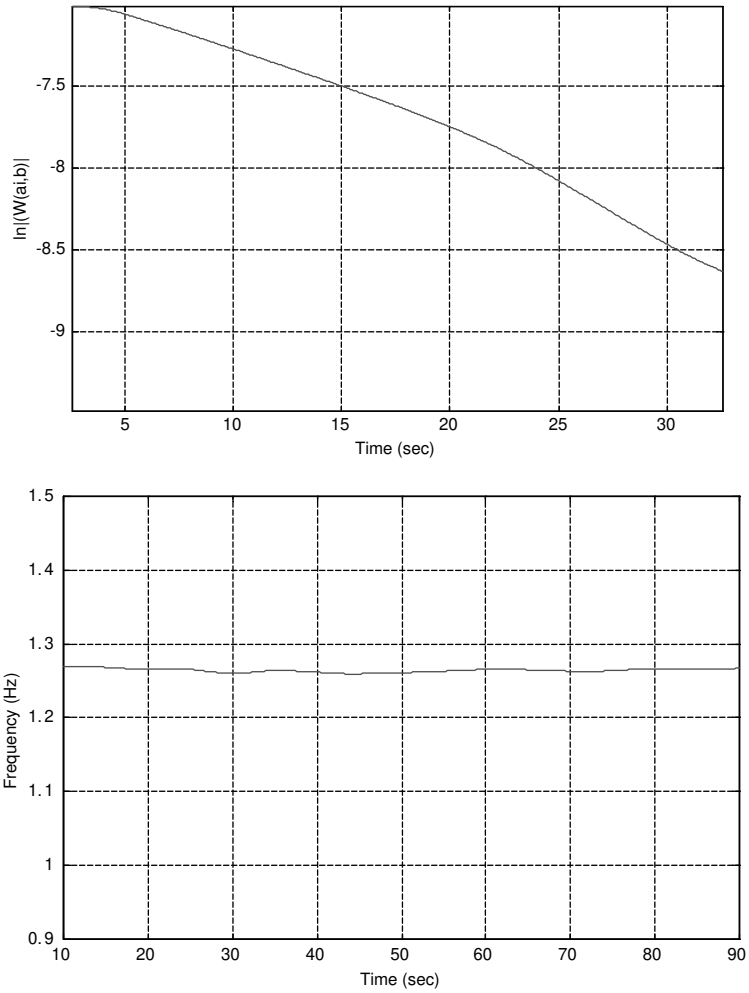


Fig. 15. Wavelet transform envelope and damped frequency for the third mode with $N = 15$, using channel 1.

Table 2
Estimation of frequencies and damping ratios of the tower

| Mode | Frequency (Hz) | Damping ratio by the autocorrelation method | Damping ratio by the wavelet transform |
|------|----------------|---|--|
| 1 | 0.234 | 0.0141 | 0.0137 |
| 2 | 0.734 | 0.0062 | 0.0028 |
| 3 | 1.273 | 0.0014 | 0.0064 |
| 4 | 1.594 | 0.0036 | 0.0030 |
| 5 | 2.726 | 0.0030 | 0.0044 |
| 6 | 3.468 | 0.0012 | 0.0020 |

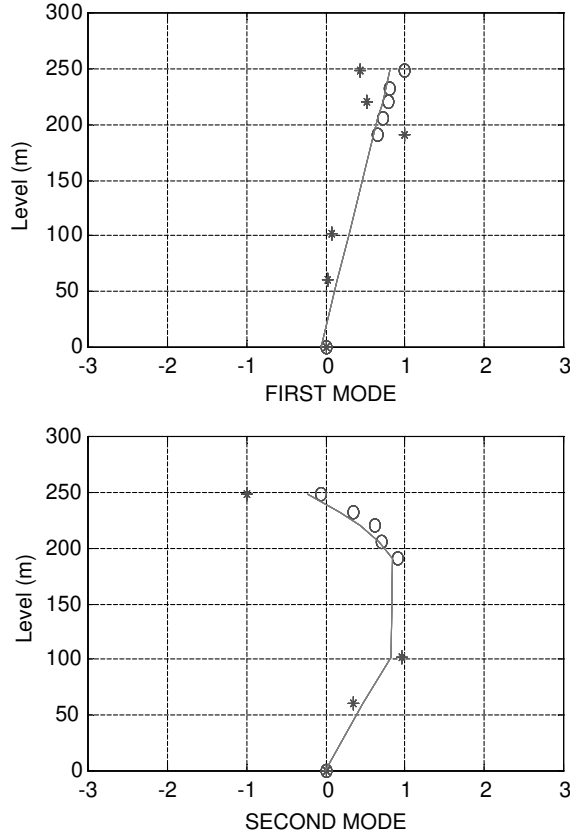


Fig. 16. Mode shapes of the tower: (○) Set 1 location of accelerometers, (*) Set 2 location of accelerometers, continue: theoretical mode shapes of a clamped beam.

When the free time responses recorded from several points of the structure are available, phase and amplitude relationships between the different degrees of freedom of the system can be obtained through the wavelet transform analysis. The i th mode shape of the structure can be estimated by evaluating the wavelet transform of the time signals from all measured points, at the corresponding i th frequency, that is for $a = a_i$. Let $W_{\psi}^j(a_i, b)$ be the wavelet transform of the signal obtained from the accelerometer positioned at point j and let $W_{\psi}^r(a_i, b)$ be the wavelet transform of the signal obtained from the accelerometer of reference, positioned at point r . The quantity

$$\chi_{ij} = W_{\psi}^j(a_i, b) / W_{\psi}^r(a_i, b) \quad (41)$$

represents the j th component of the i th complex mode shape of the structure, referred to point r . The two first experimental mode shapes of the tower are shown in Fig. 16 and are in concordance with the mode shapes of a traditionally clamped beam. Errors are due to the exact positioning of accelerometers, to measurement noise, to the estimate of eigenfrequencies and damping ratios of different modes in the real structure, to the values of Young's module, density and moment of inertia in the theoretical clamped beam.

7. Conclusion

An approach to estimate modal parameters in time domain, from output data only, using the wavelet transform has been presented. The efficiency of the method has been demonstrated using a numerical example and experimental results. The wavelet transform method is very suitable for the analysis of mechanical systems excited by random forces and has been applied to real data. Further work is continuing to determine the optimal value of N , used in the modified Morlet wavelet function. Note that other methods have been used to obtain modal parameters from output only measurements based on the spectral decomposition of a transition matrix [7–9]. A comparison of these methods with the wavelet transform method presented here is under investigation.

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