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## **AMPLIFICATION AND INCREASED DURATION OF EARTHQUAKE MOTION ON UNEVEN STRESS-FREE GROUND**

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**ABSTRACT-** When a flat stress-free surface (i.e., the ground in seismological applications) separating air from a isotropic, homogeneous or horizontally-layered, solid substratum is solicited by a SH plane body wave incident in the substratum, the response in the substratum is a single specularly-reflected body wave. When the stress-free condition, equivalent to vanishing impedance, is relaxed by the introduction of a spatially-modulated, non-vanishing impedance, the response turns out to take the form of a spectrum of plane body waves and surface waves. It is shown that, in a great variety of situations, resonances are produced at the frequencies of which one or several surface wave amplitudes can become large. Furthermore, at resonance, the amplitude of the motion on the surface is shown to be amplified with respect to the situation in which the surface impedance vanishes. A subsidiary, but all-important, effect of this resonant response is that, when the solicitation is pulse-like, the peak value of the time history of surface motion is larger, and the duration of the signal is considerably longer, for a spatially-modulated impedance surface than for a constant, or vanishing, impedance surface.

### **1 General introduction**

An important question in seismology, civil engineering, urban planning, and natural disaster risk assessment is: to what extent does surface topography of different length and height scales (ranging from those of mountains and hills to city blocks and buildings) modify the seismic response on the ground?

There exists experimental evidence (Singh and Ordaz, 1993; Davis and West, 1973; Griffiths and Bollinger, 1979) that this modification is real and can attain considerable proportions as regards increases in peak ground motion and signal duration. Some theoretical studies (Wirgin, 1989; Wirgin 1990; Wirgin and Kouoh-Bille, 1993; Groby, 2005) seem to indicate that such effects are indeed possible, but various numerical studies ( Bouchon, 1973; Bard, 1982; Sanchez-Sesma, 1987, Geli et al., 1988; Wirgin and Bard, 1996; Guéguen, 2000; Clouteau and Aubry, 2001; Guéguen et al., 2002; Semblat et al., 2003; Tsogka and Wirgin, 2003; Boutin and Roussillon, 2004; Kham, 2004; Groby and Tsogka, 2005) yield conflicting results in that some of these point to amplification, while others to very weak effects, or even to de-amplification. Contradictory results are also obtained regarding the duration of the earthquakes.

The present contribution is devoted to establishing, in a hopefully-decisive manner, whether the aggravation of earthquake effects can or cannot be induced by uneven topography and/or

the presence of buildings on the ground. Furthermore, if such aggravated seismic phenomena are produced, we want to know whether they are rare or of systematic nature, and what their underlying causes may be.

We provide herein a theoretical and numerical analysis which supports the conclusion that such substantial deleterious effects can, and will, indeed occur repeatedly if the irregularity of the ground is spatially-periodic, as often occurs in portions of modern cities and in various geological formations.

## 2 Space-time and space-frequency formulations

In the following, we shall be concerned with the determination of the vectorial displacement field  $\mathbf{u}$  on, and underneath, the ground in response to a seismic solicitation. In general,  $\mathbf{u}$  is a function of the spatial coordinates, incarnated in the vector  $\mathbf{x}$  and time  $t$ , so that  $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ .

Since we shall employ the concept of *surface impedance* (Biot, 1968; Wait, 1971; Guéguen, 2000), and since the latter is defined, *stricto sensu*, only in the space-frequency domain, we shall carry out our analysis therein, and thus search for  $\mathbf{u}(\mathbf{x}, \omega)$ , with  $\omega$  the angular frequency. The time history of response  $\mathbf{u}(\mathbf{x}, t)$  will then be computed via the Fourier transform

$$\mathbf{u}(\mathbf{x}, t) = \int_{-\infty}^{\infty} \mathbf{u}(\mathbf{x}, \omega) \exp(-i\omega t) d\omega, \quad (1)$$

wherein  $\mathbf{u}(\mathbf{x}, \omega)$  is a generally-complex function, and  $\mathbf{u}(\mathbf{x}, t)$  a real function.

## 3 Reflection of a SH plane wave from a planar spatially-modulated impedance boundary

### 3.1 Features of the problem

The uneven ground is replaced by the flat, horizontal, planar boundary  $I \{x_2 = 0; \forall x_1 \in \mathbb{R}; \forall x_3 \in \mathbb{R}\}$  which separates the lower half-infinite region  $x_2 < 0$  from the upper half-infinite region  $x_2 > 0$ . The medium filling  $x_2 > 0$  is air, assumed for the purpose of the analysis, to be the vacuum. The unevenness of the ground is accounted for by a suitably-chosen surface impedance function.

The incident plane body seismic wave propagates in  $x_2 < 0$  towards  $I$ . Attention will be restricted to the displacement field exclusively in  $x_2 \leq 0$  (in fact, this is the purpose of employing the concept of surface impedance).

We choose the cartesian coordinate system so that the wavevector associated with the incident shear wave lies in the  $x_1 - x_2$  plane. This signifies that the displacement associated with this wave is perpendicular to the  $x_1 - x_2$  plane and therefore lies in a horizontal plane. Thus, the incident wave is a *shear-horizontal (SH) wave*. Moreover, the motion associated with this wave is, due to the choice of the cartesian reference system, independent of the coordinate  $x_3$ . This implies that the resultant total motion induced by this incident wave is independent of  $x_3$ , i.e., the boundary value problem is 2D, so that it is sufficient to look for the displacement field in the  $x_1 - x_2$  plane. Actually, due to previous comments, we look for the total displacement field (hereafter designated by  $\mathbf{u}(\mathbf{x}, \omega) := (0, 0, u(\mathbf{x}, \omega))$ ) only in the lower half of the  $x_1 - x_2$  plane, i.e., in  $\Omega$ . The trace of the boundary  $I$  in the  $x_1 - x_2$  plane is designated by  $\Gamma$ .

Hereafter, we designate the (real) density and (real) Lamé parameters in  $\Omega$  by  $\rho > 0$  and  $\lambda \geq 0, \mu \geq 0$  respectively.

### 3.2 Governing equations

The mathematical translation of the boundary value problem in the *space-frequency domain* is:

$$\mu[u_{,11}(\mathbf{x}, \omega) + u_{,22}(\mathbf{x}, \omega)] + \rho\omega^2 u(\mathbf{x}, \omega) = 0 ; \forall \mathbf{x} = (x_1, x_2) \in \Omega , \quad (2)$$

$$i\omega Z(x_1, \omega)u(\mathbf{x}, \omega) + \mu u_{,2}(\mathbf{x}, \omega) = 0 ; \forall \mathbf{x} \in \Gamma , \quad (3)$$

$$u^d(\mathbf{x}, \omega) := u^0(\mathbf{x}, \omega) - u^i(\mathbf{x}, \omega) \sim \text{outgoing waves} ; \|\mathbf{x}\| \rightarrow \infty , \mathbf{x} \in \Omega , \quad (4)$$

wherein  $u_j(\mathbf{x}, \omega)$  is the total displacement field,  $u_j^d(\mathbf{x}, \omega)$  the (unknown) diffracted field and  $u^i(\mathbf{x}, \omega)$  the (known) incident field in  $\Omega$  given by:

$$u^i(\mathbf{x}, \omega) = A^i(\omega) \exp[i(k_1^i x_1 - k_2^i x_2)] ; \forall \mathbf{x} \in \Omega , \quad k_1^i = k \sin \theta^i , \quad k_2^i = k \cos \theta^i , \quad k = \frac{\omega}{c} , \quad (5)$$

$\theta^i$  being the angle of incidence with respect to the  $x_2$  axis, and  $c = \sqrt{\mu/\rho}$  the shear wave velocity in the substratum.

Eq. (2) is the space-frequency domain equation of motion, (3) the impedance boundary condition, (4) the radiation condition, and (5) the description of the incident (plane) body wave. The impedance  $Z$  is a generally-complex function of  $x_1$  and  $\omega$ , i.e.,

$$Z(x_1, \omega) := R(x_1, \omega) - iX(x_1, \omega) , \quad (6)$$

wherein  $R(x_1, \omega)$  and  $X(x_1, \omega)$  are real functions. When the impedance vanishes for all  $x_1$ , the boundary condition becomes that of a flat, stress-free surface. Otherwise, the impedance boundary condition is supposed to simulate the presence of a topographically-uneven stress-free surface.

Until further notice, we drop the  $\omega$ -dependence on all field quantities and consider it to be implicit.

### 3.3 Field representations incorporating the radiation condition

Separation of variables (in the cartesian coordinate system), and the outgoing wave condition (4) lead to the field representation:

$$u^d(\mathbf{x}) = \int_{-\infty}^{\infty} A(k_1) \exp[i(k_1 x_1 + k_2 x_2)] dk_1 ; \quad k_2 = \sqrt{k^2 - k_1^2} , \quad \Re k_2 \geq 0 , \quad \Im k_2 \geq 0 ; \quad \omega \geq 0 . \quad (7)$$

### 3.4 Application of the boundary condition(s)

Inserting the field representation into the impedance boundary condition gives rise, after Fourier projection, to the integral equation:

$$\frac{\mu K_2}{\omega} B(K_1) + \int_{-\infty}^{\infty} B(k_1) Z(K_1 - k_1) dk_1 = -2Z(K_1 - k_1^i) A^i ; \quad \forall K_1 \in \mathbb{R} , \quad (8)$$

wherein

$$B(k_1) := A(k_1) - A^i \delta(k_1 - k_1^i) ; \quad \forall k_1 \in \mathbb{R} , \quad Z(k_1) = \int_{-\infty}^{\infty} Z(x_1) \exp(-ik_1 x_1) dx_1 , \quad (9)$$

and  $\delta(\cdot)$  is the Dirac distribution.

In the case of *constant impedance*  $Z(x_1) = \zeta = \text{const.}$  ;  $\forall x_1 \in \mathbb{R}$ , we obtain  $Z(k_1) = \zeta \delta(k_1)$ , which, after introduction into (8), gives rise to the solution

$$B(k_1) = \left( \frac{-2\zeta}{\frac{\mu k_2^i}{\omega} + \zeta} \right) A^i \delta(k_1 - k_1^i) ; \quad \forall k_1 \in \mathbb{R} . \quad (10)$$

For vanishing impedance (i.e.,  $\zeta = 0$ ),  $B(k_1) = 0$ .

Now consider the *diffracted field* for the case of constant impedance. The introduction of (10) into (9) and (7) leads to

$$u_3^d(\mathbf{x}) = \mathcal{R} u^i(\mathbf{x}^*) , \quad \mathcal{R} = \frac{\frac{\mu k_2^i}{\omega} - \zeta}{\frac{\mu k_2^i}{\omega} + \zeta} , \quad \mathbf{x}^* = (x_1, -x_2) , \quad (11)$$

which indicates that the scattered wavefield reduces to a single, specularly-reflected plane body wave.

The ratio of the amplitude of this wave with respect to that of the incident wave is  $\mathcal{R}$ , and due to the fact that  $k_2^i$  is real,  $\|\mathcal{R}\| < 1$  if  $\zeta$  is (as is assumed herein): i) real, which is the case of a purely-dissipative impedance and ii) positive, which is the case of a passive system. If, on the other hand,  $\zeta$  is complex, i.e.,  $\zeta = \zeta' - i\zeta''$ , then

$$\|\mathcal{R}\| = \sqrt{\frac{\gamma^2 + (\zeta')^2 + (\zeta'')^2 - 2\gamma\zeta'}{\gamma^2 + (\zeta')^2 + (\zeta'')^2 + 2\gamma\zeta'}} , \quad (12)$$

wherein  $\gamma = \frac{\mu k_2^i}{\omega}$ , so that once again we see that  $\|\mathcal{R}\| < 1$  provided (as is assumed herein) that  $\zeta' > 0$ . *This shows that it is impossible to amplify the field in  $\Omega$  and on  $\Gamma$  in a passive configuration with constant surface impedance.*

We shall now show that the situation changes radically when the surface impedance is non-constant, i.e., depends on  $x_1$ .

## 4 Scattering of a SH plane wave from a periodically-modulated impedance ground plane

### 4.1 Preliminaries

As previously, the incident plane wave propagates in  $\Omega$  toward the ground plane which is assumed to be flat and to be the locus of a *periodically-varying (in the  $x_1$  direction) surface impedance*, i.e.,

$$Z(x_1 + d) = Z(x_1) ; \quad \forall x_1 \in \mathbb{R} , \quad (13)$$

wherein  $d$  is the period of the modulation.

### 4.2 Field representation

Due to the periodicity of  $Z$ , we have

$$Z(x_1) = \sum_{l=-\infty}^{\infty} Z_l \exp\left(i\frac{2l\pi}{d}x_1\right) ; \quad \forall x_1 \in \mathbb{R} , \quad Z_l = \int_{-\frac{d}{2}}^{\frac{d}{2}} Z(x_1) \exp\left(-i\frac{2l\pi}{d}x_1\right) \frac{dx_1}{d} ; \quad \forall l \in \mathbb{Z} . \quad (14)$$

The impedance boundary condition, the periodicity of  $Z(x_1)$ , and the plane wave nature of the solicitation entail the Floquet relation

$$u(x_1 + d, x_2) = u(x_1, x_2)e^{ik_1^i d} ; \quad \forall x_1 \in \mathbb{R} , \quad (15)$$

so that the diffracted field admits the representation

$$u^d(x_1, x_2) = \sum_{n=-\infty}^{\infty} A_n \exp[i(k_{1n}x_1 + k_{2n}x_2)] ; \quad \forall (x_1, x_2) \in \Omega , \quad (16)$$

wherein

$$k_{1n} = k_1^i + \frac{2n\pi}{d} , \quad k_{2n} = \sqrt{k^2 - k_{1n}^2} ; \quad \Re k_{2n} \geq 0 ; \quad \Im k_{2n} \geq 0 ; \quad \forall n \in \mathbb{Z} ; \quad \omega \geq 0 . \quad (17)$$

Eq. (16) shows that the diffracted field is expressed as a sum of plane waves. The plane waves for which  $k_{2n}$  is real are *propagative* (or homogeneous) and those for which  $k_{2n}$  is imaginary are *evanescent* or (inhomogeneous). As shown further on, the number of evanescent waves is zero and the number of propagative waves is one when the surface impedance is constant, so that we can expect anomalous effects (amplification and increased duration of ground motion) to be tied up with the existence of evanescent waves in the diffracted wavefield and to be possible only for spatially-modulated surface impedances.

### 4.3 Application of the impedance boundary condition

Introducing (16) into the impedance boundary condition gives, after projection,

$$\sum_{n=-\infty}^{\infty} [Z_{j-n} + \gamma_n \delta_{nj}] B_n = -2Z_j A^i ; \quad \forall j \in \mathbb{Z} , \quad (18)$$

with  $\delta_{nj}$  the Kronecker delta symbol and:

$$B_n = A_n - A^i \delta_{n0} , \quad \gamma_n := \frac{\mu k_{2n}}{\omega} ; \quad \forall n \in \mathbb{Z} . \quad (19)$$

The linear system of equations (18) is of the form of a matrix equation  $\mathbf{E}\mathbf{f} = \mathbf{g}$ , with  $\mathbf{f}$  the set of scattering coefficients  $B_n$ .

### 4.4 Solution of the matrix equation in the general case

Eq. (18) can be written as

$$B_n = \frac{-2Z_n A^i - \sum_{j \in \mathbb{Z} \setminus \{n\}} Z_{n-j} B_j}{Z_0 + \gamma_n} ; \quad \forall n \in \mathbb{Z} . \quad (20)$$

This suggests a Neumann-series iterative procedure for solving the system:

$$B_n^{(0)} = \frac{-2Z_n A^i}{Z_0 + \gamma_n} , \quad B_n^{(l)} = \frac{-2Z_n A^i - \sum_{j \in \mathbb{Z} \setminus \{n\}} Z_{n-j} B_j^{(l-1)}}{Z_0 + \gamma_n} ; \quad \forall n \in \mathbb{Z} ; \quad l = 1, 2, \dots . \quad (21)$$

It is easy to verify that the previously-obtained solution is recovered for constant (spatially-speaking) impedance. This means, in particular, that no evanescent waves are present in the wavefield for constant surface impedance.

#### 4.5 On the possibility of amplified fields in the general case of a non-constant, periodic, passive surface impedance

The natural frequencies of the *modes* of the configuration are obtained by turning off the solicitation in the matrix equation (Wirgin, 1996), i.e.,  $\mathbf{g} = \mathbf{0}$ , wherein  $\mathbf{0}$  is the null vector. The solution of  $\mathbf{E}\mathbf{f} = \mathbf{0}$  is trivial (i.e.,  $\mathbf{f} = \mathbf{0}$ ) unless  $\det(\mathbf{E}) = 0$ , so that a natural frequency is a value  $\omega = \tilde{\omega}$  for which (the so-called general dispersion relation)  $\det(\mathbf{E}) = 0$  at a given  $s^i = \sin\theta^i = \tilde{s}^i$ . When the configuration is such that the couple  $(\omega, s^i)$  of the incident wave (note that  $\omega$  of the incident wave is necessarily real) is equal to  $(\tilde{\omega}, \tilde{s}^i)$ , the system is said to be in a state of *resonance*.

In the case of constant surface impedance, the exact dispersion relation is  $\zeta + \frac{\mu k_{20}}{\omega} = 0$ , and we showed previously that this does not correspond to amplified ground motion for passive impedance, which is another way of saying that no resonances can exist for a constant, passive impedance surface.

However, inspection of (20) shows that it is possible for  $B_n$  to become large when (the approximate dispersion relation)

$$Z_0 + \frac{\mu k_{2n}}{\omega} = 0, \quad (22)$$

is satisfied, and, owing to the fact that this cannot occur for  $n = 0$  for the previously-mentioned reason, it can only occur, if at all, for  $n \neq 0$ . This shows that if one is to have resonant behavior, then, at the very least, he must include scattered plane waves of order  $|n|$  larger than 0 in the plane wave representation of the scattered field, i.e., he must take into account not only the specularly-reflected ( $n = 0$ ) wave, but also non-specularly reflected waves in this representation.

The general form of the surface impedance was  $Z(x_1, \omega) = R(x_1, \omega) - iX(x_1, \omega)$ . Consider the case of vanishing dissipation, i.e.,  $R(x_1) = 0$  (we already took  $\Im\mu = 0$ , and  $\Im k = 0$  at the outset). Then the approximate dispersion relation (for  $\omega \geq 0$ ) is

$$-iX_0(\omega) + \frac{\mu k_{2n}(\omega, s^i)}{\omega} = 0. \quad (23)$$

Now, since  $\omega \geq 0$ ,  $\Re\mu > 0$ ,  $\Re k_{2n} > 0$ , and  $\Im k_{2n} > 0$ , the second term in (23) is either positive real (for real  $k_{2n}$ ) or positive imaginary (for imaginary  $k_{2n}$ ), so that the sum of the two terms can vanish only if

$$X_0 > 0 \quad \text{and} \quad \Re k_{2n} = 0. \quad (24)$$

The second of these requirements means that resonances occur only for the *evanescent* waves in the plane wave representation of the scattered field. In other words: we can expect the amplitude of the  $n$ -th order evanescent wave to become infinite (when  $R = 0$ ) or large (for  $R > 0$ ) at resonance, which is another way of saying that a surface wave (evanescent waves are of this sort) is strongly excited at resonance.

We can anticipate the numerical results: the frequency domain response in  $\Omega$  and on the ground will be amplified at the resonance frequencies, and, in the time domain, the signal will be all the more amplified, and its duration will be all the longer, the larger is the quality factor of the resonance.

## 5 Numerical results

We chose a sinusoidal surface impedance function  $Z(x_1) = \zeta [1 + h \cos(\frac{2\pi}{d}x_1)]$  whose spectrum is  $Z_l = \zeta [\delta_{l0} + \frac{\alpha}{2}(\delta_{l,1} + \delta_{l,-1})]$ , with  $\frac{c}{\mu}\zeta(\omega) = r - ix(\omega)$ ,  $x(\omega) = m\omega - \frac{\kappa}{\omega}$ ,  $\gamma_n = \frac{\mu}{\omega}k_{2n} = \frac{\mu}{c}c_n$ ,

$c_n = \sqrt{1 - s_n^2}$ ,  $s_n = s^i + n\frac{s}{\omega}$ , and  $s = \frac{2\pi c}{d}$ . The real parameters  $r$ ,  $m$  and  $\kappa$  are the normalized mass, stiffness and dashpot constants respectively that enter into the impedance amplitude  $\zeta$ . The amplitude spectrum of the incident plane wave is that of a Ricker pulse, i.e.,  $A^i(\omega) = -\frac{\omega^2}{4\alpha^3\sqrt{\pi}} \exp(-\frac{\omega^2}{4\alpha^2} + i\omega\beta)$ .

The computations of response were made using the zeroth-order approximation in (21) of the scattering coefficients  $B_m$ . The field was computed by taking into account: i) no scattered wave (the result for a stress-free flat ground), ii) only the  $n = 0$  propagative scattered wave, and iii) all (i.e., the  $n = 0$ ,  $n = -1$  and  $n = 1$ ) the scattered waves (note that higher-order waves do not contribute to the scattered field for the sinusoidally-modulated surface impedance function employed herein). The results of the computations are given in figs. 1-3. Case i) is designated therein by  $\|ui + ur\|$  and  $ui + ur$ , case ii) by  $\|2 + b(0)\|$  and  $u0$ , and case iii) by  $\|2 + b(0) + b(-1) + b(1)\|$  and  $u$ .

Fig. 1 corresponds to a case in which both the  $n = -1$  and  $n = 1$  waves enter into resonance, as manifested by the sharp peaks in the spectra  $B_{-1}(\omega)$  and  $B_{+1}(\omega)$ . It seems that the spectrum function  $B_0(\omega)$  betrays the existence of a resonance for the  $n = 0$  wave also, but this is not so; in fact, we are rather in the presence of a *pseudo-resonance* which is produced at a complex value of  $\omega$ . Such pseudo resonances (also termed leaky wave resonances) are produced in any propagative wave (for  $r = 0$ ) and the imaginary part of  $\omega$  reflects both material dissipation (embodied in  $r$ ) and radiation damping (which exists even for  $r > 0$ ). One notes that the time history obtained by neglecting the contribution of the  $n = -1$  and  $n = 1$  waves is similar to that of the flat stress-free surface, in terms of duration, but different (i.e., de-amplified) from the flat stress-free surface response in terms of peak amplitude. This is similar to various results of Clouteau and Boutin and Roussillon, but clearly does not reflect the real situation (corresponding to the taking into account of all scattered waves) in which we observe a very long signal that is even amplified (in terms of peak value) with respect to the signal on a flat stress-free ground. The beating phenomenon in this long-duration signal is due to the presence of two high-quality factor resonances in the scattered field spectrum.

In Fig. 2 we have reduced the parameter  $s$  so as to obtain a situation in which none of the waves enter into resonance. All the signals are approximately of the same form and duration and are either de-amplified or of the same peak amplitude. In this case, the various results of Clouteau and Boutin and Roussillon would be in agreement with the real situation if they had chosen a configuration similar to ours.

Fig. 3 differs from the previous case by a significant increase in the incident angle of the body wave solicitation. Now a sharp resonance is produced in the  $n = 1$  wave only and the cumulative translation of this is a near monochromatic, amplified, long duration response in the time domain.

## 6 Conclusions

Many other choices of the parameters were found to lead to the types of resonant frequency domain response described above, which leads us to believe that amplified, increased-duration time domain seismic response is a quite common occurrence in a great variety of natural and man-made geological sites.

The cause of this abnormal response was shown to be the resonant excitation of modes of the configuration as manifested by poles in the amplitudes of the scattered waves.



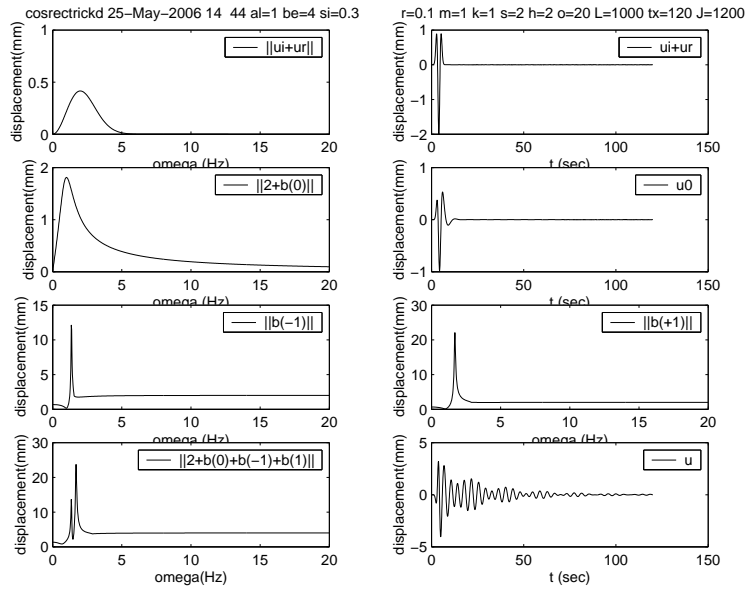


Figure 1: Graphs of spectra and time histories of various approximations of the ground displacement at  $\mathbf{x} = (0, 0)$ .  $\alpha = 1$ ,  $\beta = 4$ ,  $s^i = 0.3$ ,  $r = 0.1$ ,  $m = 1$ ,  $\kappa = 1$ ,  $s = 2$ ,  $h = 2$ .

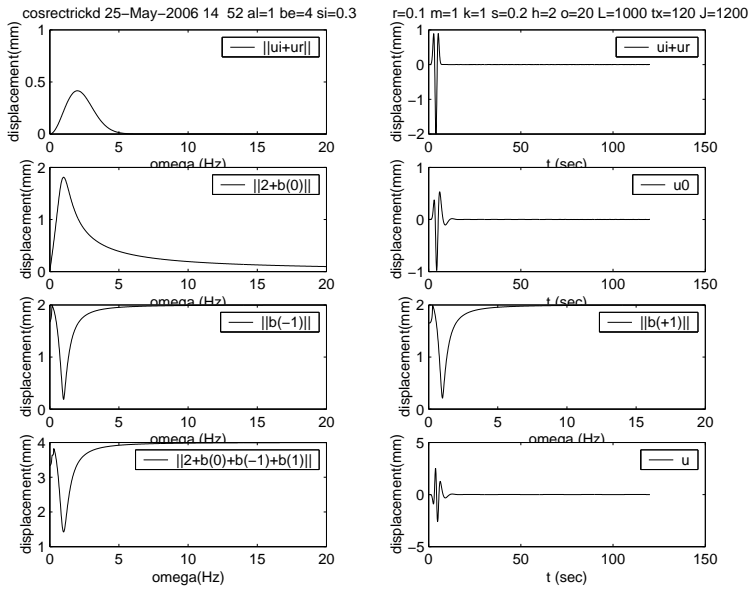


Figure 2: Graphs of spectra and time histories of various approximations of the ground displacement at  $\mathbf{x} = (0, 0)$ .  $\alpha = 1$ ,  $\beta = 4$ ,  $s^i = 0.3$ ,  $r = 0.1$ ,  $m = 1$ ,  $\kappa = 1$ ,  $s = 0.2$ ,  $h = 2$ .

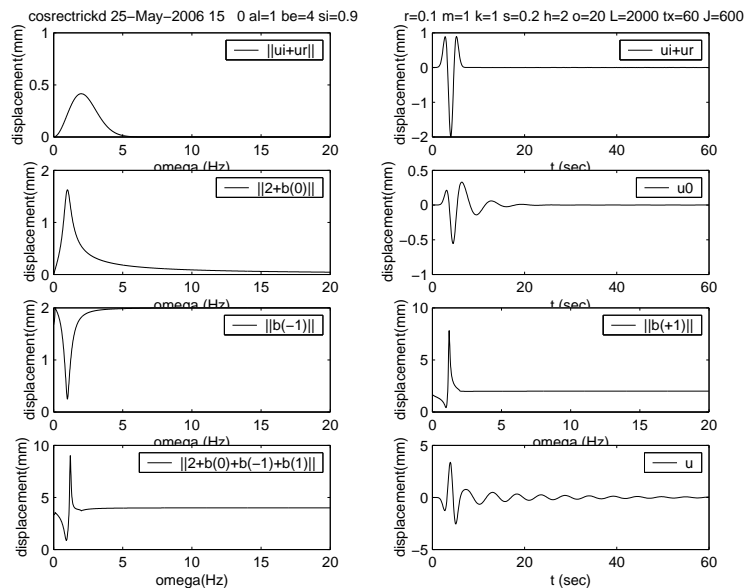


Figure 3: Graphs of spectra and time histories of various approximations of the ground displacement at  $\mathbf{x} = (0, 0)$ .  $\alpha = 1$ ,  $\beta = 4$ ,  $s^i = 0.9$ ,  $r = 0.1$ ,  $m = 1$ ,  $\kappa = 1$ ,  $s = 0.2$ ,  $h = 2$ .

## References

- [1] P.-Y. Bard. Diffracted waves and displacement field over two-dimensional elevated topographies. *Geophys.J.R.Astron.Soc.*, 71:731–760, 1982.
- [2] M.A. Biot. Generalized boundary condition for multiple scatter in acoustic reflection. *J.Acoust.Soc.Am.*, 44:1616–1622, 1968.
- [3] M. Bouchon. Effect of topography on surface motion. *Bull.Seism.Soc.Am.*, 63:615–632, 1973.
- [4] C. Boutin C. and P. Roussillon. Assessment of the urbanization effect on seismic response. *Bull.Seism.Soc.Am.*, 94:251–268, 2004.
- [5] D. D. Clouteau and D. Aubry. Modifications of the ground motion in dense urban areas. *J.Comput.Acoust.*, 9:1659–1675, 2001.
- [6] L.L. Davis and L.R. West. Observed effects of topography on ground motion. *Bull.Seism.Soc.Am.*, 63:283–298, 1979.
- [7] L. Geli, P.-Y. Bard, and B. Jullien. The effect of topography on earthquake ground motion: a review and new results. *Bull.Seism.Soc.Am.*, 78:42–63, 1988.
- [8] D.W. Griffiths and G.A. Bollinger. The effect of the Appalachian mountain topography on seismic waves. *Bull.Seism.Soc.Am.*, 69:1081–1105, 1979.
- [9] J.-P. Groby. *Modélisation de la propagation des ondes élastiques générées par un séisme proche ou éloigné à l'intérieur d'une ville*. PhD thesis, Université de la Méditerranée, Marseille, 2005.

- [10] J.-P. Groby, C. Tsogka, and A. Wirgin. Simulation of seismic response in a city-like environment. *Soil Dynam.Earthquake Engrg.*, 25:487–504, 2005.
- [11] P. Gueguen. *Interaction sismique entre le sol et le bâti: de l'interaction sol-structure à l'interaction site-ville*. PhD thesis, Université Joseph Fourier, Grenoble, 2000.
- [12] P. Gueguen, P.-Y. Bard, and F.J. Chavez-Garcia. Site-city seismic interaction in Mexico city like environments : an analytic study. *Bull.Seism.Soc.Am.*, 92:794–804, 2002.
- [13] M. Kham. *Propagation d'ondes sismiques dans les bassins sédimentaires: des effets de site à l'interaction site-ville*. PhD thesis, Laboratoire Central des Ponts et Chaussées, Paris, 2004.
- [14] F.J. Sanchez-Sesma. Site effects on strong ground motion. *Soil Dynam.Earthqu.Engrg.*, 6:124–132, 1987.
- [15] J.F. Semblat, P. Guéguen, M. Kham, P.-Y. Bard, and A.-M. Duval. Site-city interaction at local and global scales. In *12th European Conference on Earthquake Engineering*, Oxford, 2003. Elsevier. paper no. 807 on CD-ROM.
- [16] S.K. Singh and M. Ordaz. On the origin of long coda observed in the lake-bed strong-motion records of Mexico City. *Bull.Seism.Soc.Am.*, 83:1298–1306, 1993.
- [17] C. Tsogka and A. Wirgin. Simulation of seismic response in an idealized city. *Soil. Dynam.Earthquake Engrg.*, 23:391–402, 2003.
- [18] J.R. Wait. Perturbation analysis for reflection from two-dimensional periodic sea waves. *Radio Sci.*, 6:387–391, 1971.
- [19] A. Wirgin. Amplification résonante du tremblement d'une chaîne de montagnes cylindriques soumise à une onde SH. *C.R.Acad.Sci. Paris II*, 311:651–655, 1989.
- [20] A. Wirgin. Amplification résonante du mouvement du sol sur une montagne cylindrique isolée soumise à une onde sismique SH. *C.R.Acad.Sci. II*, 311:651–655, 1990.
- [21] A. Wirgin. Resonant response of a soft semi-circular cylindrical basin to a SH seismic wave. *Bull.Seism.Soc.Am.*, 86:914–920, 1996.
- [22] A. Wirgin and P.-Y. Bard. Effects of buildings on the duration and amplitude of ground motion in mexico city. *Bull.Seism.Soc.Am.*, 86:914–920, 1996.
- [23] A. Wirgin and L. Kouoh-Bille. Amplification du mouvement du sol au voisinage d'un groupe de montagnes de profil rectangulaire ou triangulaire soumis à une onde sismique SH. In *Génie Parasismique et Aspects Vibratoires dans le Génie Civil*, pages ES28–ES37, Saint-Rémy- lès-Chevreuse, 1993. AFPS.