Controversy about the applicability of Tsallis statistics to the HMF model

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Comment to "Nonextensive Thermodynamics and Glassy Behaviour in Hamiltonian Systems" by A. Rapisarda and A. Pluchino, Europhysics News 36, 202 (2005).

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The dynamics of the Hamiltonian Mean-Field (HMF) model [1] shows many intriguing non-equilibrium behaviors. In particular, it has been reported several times that the system gets stuck into *quasi-stationary states* (QSS), whose lifetime increases with system size. As correctly pointed out by C. Tsallis and coworkers (see e.g. Refs. [2]), the presence of such non-equilibrium states is tightly linked to the fact that the infinite time limit and the thermodynamic limit do not commute in systems with long-range interactions. However, contrary to what is claimed in Ref. [3], the non-extensive statistics approach does not convincingly "explain" any of these non-equilibrium behaviors.

Two main quantities have been tested up to now: velocity distribution functions and correlation functions. In Ref. [4], the authors fit single particle velocity distribution functions in QSS using q-exponentials. They obtain a value of the index $q = 7$. In Ref. [3], an analogous fit of correlation functions with q-exponentials gives values of q between 1.1 and 1.5. It is questionable that different values of q are used for the same model and the same physical conditions.

The fact of being in a non-equilibrium state could in principle allow the use of an entropy other than Boltzmann-Gibbs. However, there is up to now not a single paper which gives a rigorous justification of the use of non-extensive entropy for the HMF model. Hence, there is no compelling reason of using q-exponentials as a fitting function.

A general alternative approach has been introduced to *explain* the presence of QSS in systems with long-range interactions. This approach begins by performing first the thermodynamic limit and then looking at the time evolution. This procedure amounts to associate to the HMF model appropriate Vlasov and kinetic equations. This method is fully predictive and has been extensively exploited in Ref. [5] to obtain the Vlasov equation predictions for the HMF model.

Restricting to *homogeneous* QSS, velocity distribution functions of QSS have been analysed, reaching the conclusion that they cannot be fitted by q-exponentials. This conclusion has not been questioned so far in the literature. Moreover, the kinetic approach also allows to derive properties of the correlation functions, deducing them directly from the HMF model [6]. Such homogeneous states are of paramount importance, since they appear to be "attractive" for a large class of initial conditions. For instance, it can be shown that the plateau values of the magnetization M_0 shown in Fig. 1 of Ref. [3], all converge to $M_0 = 0$ when N increases, which is a distinctive sign of homogeneity.

The Vlasov equation approach is just in a beginning stage. However, the already existing results are encouraging and we believe that the difficulty of treating inhomogeneous QSS is of technical nature. This problem will be solved in the near future.

Hence, the conclusion of Ref. [3]: "However the actual state of the art favours the application of non-extensive thermostatistics to explain most of the anomalies observed in the QSS regime" is highly questionable.

As a final remark, we think that, as physicists, we should pay great attention to the difference between "fitting" and "explaining".

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