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Abstract

We have developed the full theory of a synchronously pumped type I optical parametric oscillator (SPOPO). We derive expressions for the oscillation threshold and the characteristics of the generated mode-locked signal beam. We calculate the output quantum fluctuations of the device, and find that, in the degenerate case (coincident signal and idler set of frequencies), perfect squeezing is obtained when one approaches threshold from below for a well defined "super-mode', or frequency comb, consisting of a coherent linear superposition of signal modes of different frequencies which are resonant in the cavity.

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Optical Parametric Oscillators are among the best sources of squeezed [1], correlated [2] and entangled [3] light in the so-called continuous variable regime. They have allowed physicists to successfully implement demonstration experiments for high sensitivity optical measurements and quantum information protocols. In order to maximize the quantum effects, one needs to optimize the parametric down-conversion process. This has been achieved so far by using either intense pump lasers or resonant cavities. Having in mind that the parametric process is an almost instantaneous one, femtosecond mode-locked lasers are the best pump sources in this respect, as they generate very high peak optical powers with high coherence properties. Furthermore, they minimize the thermal effects in the linear crystal which often hamper the normal operation of parametric devices. Mode-locked lasers have been already used extensively to generate non classical light, either to pump a parametric crystal [4] or an optical fiber [5]. However in such single-path configurations, perfect quantum properties are only obtained when the pump power goes to infinity. This is the reason why mode-locking is often associated to Q-switching and pulse amplification [6] in order to reach even higher peak powers, at the expense of a loss in the coherence properties between the successive pump pulses. In contrast, intracavity devices produce perfect quantum properties for a finite power, namely the oscillation threshold of the device. It is therefore tempting to consider devices in which one takes advantage of the beneficial effects of both high peak powers and resonant cavity build-up. Such devices exist: they are the so-called synchronously pumped OPOs or SPOPOs. In a SPOPO, the cavity round-trip time is equal to the repetition rate of the mode-locked laser, so that the effect of the successive intense pump pulses add coherently, thus reducing considerably its oscillation threshold. Such SPOPOs have already been implemented as efficient sources of tunable ultra-short pulses [7, 8, 9, 10, 11, 12] and their temporal properties have been theoretically investigated [13, 14, 15]. To the best of our knowledge, they have never been used so far to generate genuine quantum effects. Performing a quantum analysis of this device, we theoretically show in this paper that perfect squeezing can indeed be obtained in SPOPOs as in cw OPOs. The squeezed mode is not a usual single frequency mode, but instead a "super-mode", which is a well defined linear combination of signal modes of different frequencies, forming the frequency comb that will oscillate above the SPOPO oscillation threshold.

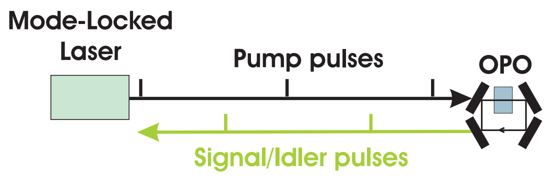

FIG. 1: Synchronously pumped OPO

Let us first precise the model that we use (figure 1). We consider a ring cavity of optical length L containing a type I parametric crystal of thickness l. Degenerate phase matching is assumed, what means that the phase-matching condition is fulfilled for frequencies $2\omega_0$ and ω_0. This amounts to saying that $n\left(2\omega_0\right) = n\left(\omega_0\right) \equiv n_0$, $n\left(\omega\right)$ being the refractive index of the crystal at frequency ω. The mode-locked pump laser, having a repetition rate $\Omega/2\pi = c/L$, is tuned so that the frequency of one of its modes is equal to $2\omega_0$. The electric field generated by the pump mode-locked laser can be expressed as:

$$E_{\mathrm{ext}}\left(t\right) = \left(\frac{P}{2\varepsilon_0 c}\right)^{\frac{1}{2}} \sum_m i\alpha_m e^{-i\left(2\omega_0 + m\Omega\right)t} + \mathrm{c.c.}, \qquad (1)$$

where P is the average laser power per unit area, α_m the normalized ($\sum_m \left|\alpha_m\right|^2 = 1$) complex spectral component of longitudinal mode labelled by the integer index m, and $m = 0$ corresponds to the phase-matched mode. For the sake of simplicity in this first approach to the problem, we will take the modal coefficients α_m as real numbers, thus excluding chirped pump pulses. As already mentioned, the SPOPO cavity length is adjusted so that its free spectral range coincides with that of the pumping laser. In the nonlinear crystal, pump photons belonging to all the different longitudinal pump modes are converted into signal and idler photons via the parametric interaction. In addition we will assume here that we are in the ideal case of *doubly resonant degenerate operation*, meaning that the among all the OPO cavity resonant frequencies, they are all the pump mode frequencies $\omega_{p,m} = 2\omega_0 + m\Omega$ but also all the frequencies $\omega_{s,q} = \omega_0 + q\Omega$ around the phase-matched subharmonic frequency ω_0. The intracavity electric field generated by the parametric interaction will then be a superposition of fields oscillating at frequencies $\omega_{s,q}$. We will finally call γ_p and γ_s, the cavity damping rates for the pump and signal modes. Note that the free spectral range Ω is assumed to be the same in the pump and in the signal spectral regions. This is necessary for an efficient intracavity parametric down conversion and requires, from the experimental viewpoint, the use of extra dispersive elements inside the cavity that compensate for the dispersion of the crystal. At the quantum level, the signal field, taken at the middle of the crystal, is represented by the quantum operator $\hat{E}_s$ which can be written as:

$$\hat{E}_s(t) = \sum\nolimits_q i\mathcal{E}_{s,q}\hat{s}_q(t)e^{-i\omega_{s,q}t} + \text{H.c.}, \qquad (2)$$

where $\hat{s}_q$ are the annihilation operators for the q^{th} signal mode in the interaction picture. $\mathcal{E}_{s,q}$ is the single photon field amplitude, equal to $\sqrt{\hbar\omega_{s,q}/2\varepsilon_0 n\left(\omega_{s,q}\right)AL}$, and A its effective transverse area. The following Heisenberg equations for the field operators can be derived using the standard methods. The detail of the derivation will be given in a forthcoming publication. Below threshold, and in the linearized regime for the pump fluctuations, they read:

$$\frac{d\hat{s}_m}{dt} = -\gamma_s\hat{s}_m + \gamma_s\sigma\sum\nolimits_q \mathcal{L}_{m,q}\hat{s}_q^\dagger + \sqrt{2\gamma_s}\hat{s}_{\text{in},m}, \qquad (3)$$

where σ is the normalized pump amplitude

$$\sigma = \sqrt{P/P_0} \qquad (4)$$

in which P_0 is the single mode c.w. oscillation threshold:

$$P_0 = 2\gamma_s^2\gamma_p n_0^3 c^3\varepsilon_0/\left(4\sqrt{2}\chi l\omega_0\right)^2 \qquad (5)$$

with χ the crystal nonlinear susceptibility. $\mathcal{L}_{m,q}$ is the product of a phase-mismatch factor by the pump spectral normalized amplitude α_{m+q}:

$$\mathcal{L}_{m,q} = \frac{\sin\phi_{m,q}}{\phi_{m,q}}\alpha_{m+q}, \qquad (6)$$

The phase mismatch angle

$$\phi_{m,q} = \frac{l}{2}\left(k_{p,m+q} - k_{s,m} - k_{s,q}\right) \qquad (7)$$

can be computed using a Taylor expansion around $2\omega_0$ for the pump wave vectors $k_{p,m}$ and around ω_0 for the signal wave vectors $k_{s,q}$:

$$\phi_{m,q} \simeq \beta_1\left(m+q\right) + \beta_{2p}\left(m+q\right)^2 - \beta_{2s}\left(m^2+q^2\right), \qquad (8)$$

where $\beta_1 = \frac{1}{2}\Omega\left(k_p' - k_s'\right)l$, $\beta_{2p} = \frac{1}{4}\Omega^2 k_p'' l$, $\beta_{2s} = \frac{1}{4}\Omega^2 k_s'' l$. k' and k'' are the first and second derivative of the wave vector with respect to frequency. Finally $\hat{s}_{\text{in},m}$ are the input signal field operators at frequency $\omega_{s,m}$ transmitted through the coupling mirror. When the input is the vacuum state, which we consider here, their only non-null correlations are:

$$\left\langle \hat{s}_{\text{in},m_1}\left(t_1\right)\hat{s}_{\text{in},m_2}^\dagger\left(t_2\right)\right\rangle = \delta_{m_1,m_2}\delta\left(t_1-t_2\right). \qquad (9)$$

In order to get Eqs. (3), we had to assume that $\mathcal{E}_{s,m} \simeq \mathcal{E}_{s,0}$ for all m and to neglect the dispersion of the nonlinear susceptibility. Both approximations require that pulses are not too short. In usual practical conditions, pulse durations should be longer than 20-30 fs. Let us first determine the average values of the generated fields. They are determined by the "classical" counterpart of Eq. (3), removing the input noise terms, and replacing the operators by complex numbers. The solution of these equations is of the form $s_m(t) = S_{k,m}e^{\lambda_k t}$, where k is an index labelling the different solutions. The parameters $S_{k,m}$ and λ_k obey the following eigenvalue equation:

$$\lambda_k S_{k,m} = -\gamma_s S_{k,m} + \gamma_s\sigma\sum\nolimits_q \mathcal{L}_{m,q}S_{k,q}^*. \qquad (10)$$

As matrix $\mathcal{L}$ is both self-adjoint and real ($\mathcal{L}_{m,q} = \mathcal{L}_{q,m}$ real, see Eqs. (6)–(8)), its eigenvalues Λ_k and eigenvectors $\vec{L}_k$, of components $L_{k,m}$, are all real. As γ_s and σ are also real, there exist two sets of solutions of Eqs. (10), that we will call $S_{k,m}^{(+)}$ and $S_{k,m}^{(-)}$. The first set is given by $S_{k,m}^{(+)} = L_{k,m}$ and the second one is $S_{k,m}^{(-)} = iL_{k,m}$, with corresponding eigenvalues:

$$\lambda_k^{(\pm)} = \gamma_s\left(-1 \pm \sigma\Lambda_k\right), \qquad (11)$$

Let us now label by index $k = 0$ the solution of maximum value of $|\Lambda_k|$: $|\Lambda_0| = \max\{|\Lambda_k|\}$. When $\sigma|\Lambda_0|$ is smaller than 1, all the rates $\lambda_k^\pm$ are negative, which implies that the null solution for the steady state signal field is stable. For the simplicity of notations, we will take Λ_0 positive in the following[16]. The SPOPO reaches its oscillation threshold when σ takes the value $1/\Lambda_0$, i.e. for a pump power $P = P_{\text{thr}}$ equal to:

$$P_{\text{thr}} = P_0/\Lambda_0^2, \qquad (12)$$

We can now define the normalized amplitude pumping rate r by $r = \sigma\Lambda_0$, so that threshold occurs at $r = 1$. We will call eigen-spectrum the set of $S_{k,m}$ values for a given k, which corresponds physically to the different spectral components of the signal field, and critical eigen-spectrum $S_{0,m}^{(+)}$, the one associated with $\lambda_0^{(+)}$, which changes sign at threshold. Above threshold, this critical mode will be the "lasing" one, i.e. the one having a non-zero mean amplitude when $r > 1$. Let us note that the eigen-spectrum in quadrature with respect to the critical one, $S_0^{(-)} = iS_0^{(+)}$, has an associated eigenvalue $\lambda_0^{(-)} = -2\gamma_s$ at threshold. Furthermore, equation (11) implies that all the damping rates $\lambda_k^{(\pm)}$ are comprised below threshold between $-2\gamma_s$ and 0, and that, whatever the pump intensity, all the eigenvalues $\lambda_k^{(\pm)}(r)$ lie between $\lambda_0^{(+)}(r)$ and $\lambda_0^{(-)}(r)$. These properties will be useful for the study of squeezing. To determine the threshold, we must find Λ_0, and therefore diagonalize $\mathcal{L}$, which cannot be done analytically in the general case. The detailed study of the SPOPO threshold will be made in a forthcoming publication. Here we will consider a special case that leads to simple calculations and corresponds to an optimized situation. We assume first that $\beta_1 = 0$ (equal group velocities) and $\beta_{2s} = 2\beta_{2p}$ (matched group velocity dispersions). We take simple forms for the phase matching coefficient

$$\mathcal{L}_{mq} = e^{-\frac{1}{2}\eta|\phi_{m,q}|}\alpha_{m+q} \tag{13}$$

and for the normalized pump spectrum:

$$\alpha_m = \pi^{-1/4}\Delta_p^{-1/2}e^{-\frac{m^2}{2\Delta_p^2}}. \tag{14}$$

If in addition we impose the relation

$$\eta\,|\beta_{2p}| = \Delta_p^{-2}, \tag{15}$$

giving comparable widths for the phase matching curve and the laser spectrum, we can analytically diagonalize the matrix $\mathcal{L}_{m,q}$. We find that its (unnormalized) eigenvectors have the following components:

$$L_{k,m} = e^{-\frac{m^2}{2\Delta_p^2}}H_k\left(m/\Delta_p\right), \tag{16}$$

H_k being the Hermite polynomial of order k, and that its eigenvalues are:

$$\Lambda_k = \delta_{k,0}\quad \pi^{1/4}\sqrt{\frac{\Delta_p}{2}}, \tag{17}$$

This gives finally a SPOPO threshold (Eq. (12)) equal to:

$$P_{\mathrm{thr}} = \frac{2P_0}{\sqrt{\pi}\Delta_p}, \tag{18}$$

the associated eigenspectrum $S_{0,m}^{(+)}$ being a Gaussian of width Δ_p. Relation (18) means that the usual c.w. OPO oscillation threshold is divided by a quantity roughly equal to the number of pump modes, at least in the fully optimized SPOPO that we have considered here. This has a simple explanation: since the parametric coupling is very fast, the OPO actually oscillates when the instantaneous peak power of the pump exceeds the c.w. threshold. Having in mind that P_0 is on the order of a few tens of mW and that the number of coherently oscillating modes in a mode-locked laser can easily exceed 10^4, one can then envision SPOPO thresholds of the order of a microWatt. Experimental implementations of such SPOPOs have not reached such ultra-low thresholds because of the difficulty to fulfill simultaneously, in a real experiment, all these precise conditions. We can now determine the squeezing properties of the signal field in a SPOPO below threshold. This is done by using the SPOPO linearized quantum equations. Let us introduce the operator $\hat{S}_{in,k}(t)$ by:

$$\hat{S}_{in,k}(t) = \sum\nolimits_m L_{k,m}\hat{s}_{in,m}(t) \tag{19}$$

Because $\sum_m |L_{k,m}|^2 = 1$, one has $\left[S_{in,k}(t), S_{in,k}^\dagger(t')\right] = \delta(t-t')$: $\hat{S}_{in,k}$ is the annihilation operator of a combination of modes of different frequencies, which are the eigen-modes of the linearized evolution equation (3). The corresponding creation operator applied to vacuum state creates a photon in a single mode, which can be labelled as "super-mode", which globally describes a frequency comb. Defining in the same way as in (19) the intracavity operator $\hat{S}_k(t)$, one can then write:

$$\frac{d}{dt}\hat{S}_k = -\gamma_s\hat{S}_k + \gamma_s\sigma\Lambda_k\hat{S}_k^\dagger + \sqrt{2\gamma_s}\hat{S}_{\mathrm{in},k}, \tag{20}$$

Let us now define quadrature hermitian operators $\hat{S}_k^{(\pm)}$ by:

$$\hat{S}_k^{(+)} = \hat{S}_k + \hat{S}_k^\dagger \tag{21}$$

$$\hat{S}_k^{(-)} = -i\left(\hat{S}_k - \hat{S}_k^\dagger\right) \tag{22}$$

which obey the following equations:

$$\frac{d}{dt}\hat{S}_k^{(\pm)} = \lambda_k^{(\pm)}\hat{S}_k^{(\pm)} + \sqrt{2\gamma_s}\hat{S}_{\mathrm{in},k}^{(\pm)}, \tag{23}$$

with $\lambda_k^{(\pm)}$ given by Eq. (11). These relations enable us to determine the intracavity quadrature operators in the Fourier domain $\tilde{S}_k^{(\pm)}(\omega)$

$$i\omega\tilde{S}_k^{(\pm)}(\omega) = \lambda_k^{(\pm)}\tilde{S}^{\pm}(\omega) + \sqrt{2\gamma_s}\tilde{S}_{\mathrm{in},k}^{(\pm)}(\omega). \tag{24}$$

Finally, using the usual input-output relation on the coupling mirror:

$$\tilde{s}_{\mathrm{out},m}(\omega) = -\tilde{s}_{\mathrm{in},m}(\omega) + \sqrt{2\gamma_s}\tilde{s}_m(\omega), \tag{25}$$

which extends by linearity to any super-mode operator because the mirror is assumed to have a transmission independent of the mode frequency, one obtains finally the following expression for the output signal super-mode quadrature component in Fourier space:

$$\tilde{S}^{(\pm)}_{\text{out},k}(\omega) = \frac{\gamma_s\left(1 \pm r\Lambda_k/\Lambda_0\right) - i\omega}{\gamma_s\left(-1 \pm r\Lambda_k/\Lambda_0\right) + i\omega}\tilde{S}^{(\pm)}_{\text{in},k}(\omega), \quad (26)$$

These expressions are particularly simple for the critical mode quadrature components ($k = 0$):

$$\tilde{S}^{(\pm)}_{\text{out},0}(\omega) = \frac{\gamma_s\left(1 \pm r\right) - i\omega}{-\gamma_s\left(1 \mp r\right) + i\omega}\tilde{S}^{+}_{\text{in},0}(\omega), \quad (27)$$

The variance of these operators can be indeed measured using the usual balanced homodyne detection scheme: the local oscillator is in the present case a coherent mode-locked multimode field $E_{\text{L}}(t)$ having the same repetition rate as the pump laser:

$$E_{\text{L}}(t) = i\epsilon_L \sum_m e_m e^{-i\omega_{s,m}t} + \text{c.c.}, \quad (28)$$

where $\sum_m |e_m|^2 = 1$, and ϵ_L is the local oscillator field total amplitude factor. Assuming that the photodetectors measure the intensity of the Fourier components of the photocurrent averaged over many successive pulses, the balanced homodyne detection scheme measures the variance of the fluctuations of the projection of the output field on the local oscillator mode when the mean field generated by the OPO is zero, which is the case below threshold. As a result, when the coefficients e_m of the local oscillator field spectral decomposition are equal to the coefficients $L_{k,m}$ of the k-th super-mode, one measures the two following variances, depending on the local oscillator phase:

$$V_k^-(\omega) = \frac{\gamma_s^2\left(1 - r\Lambda_k/\Lambda_0\right)^2 + \omega^2}{\gamma_s^2\left(1 + r\Lambda_k/\Lambda_0\right)^2 + \omega^2} \quad (29)$$

$$V_k^+(\omega) = V_k^-(\omega)^{-1} \quad (30)$$

Eqs. (29,30) shows that quantum noise reduction below the standard quantum limit (equal here to 1) is achieved for any super-mode characterized by a non-zero Λ_k value and that the smallest fluctuations are obtained close to threshold and at zero Fourier frequency:

$$\left(V_k\right)_{\min} = \left(\frac{\Lambda_0 - |\Lambda_k|}{\Lambda_0 + |\Lambda_k|}\right)^2 \quad (31)$$

In particular, if one uses as the local oscillator the critical mode $k = 0$, identical to the one oscillating just above the threshold $r = 1$, one then gets perfect squeezing just below threshold and at zero noise frequency, just like in the c.w. single mode case. But modes of $k \neq 0$ may be also significantly squeezed, provided that $|\Lambda_k/\Lambda_0|$ is not much different from 1. In conclusion, we have studied the quantum behaviour of a degenerate synchronously-pumped OPO, which seems at first sight a highly multimode system, since it involves roughly 10^5 different usual single frequency modes for a $100fs$ pulse. We have shown that its properties are more easily understood if one considers the "super-modes", linear combinations of all these modes that are eigen-modes of the SPOPO set of evolution equations and describe in a global way the frequency comb -or, equivalently, the train of pulses- generated by the SPOPO. The super-mode of minimum threshold plays a particular role, as it is the one which turns out to be perfectly squeezed at threshold and will oscillate above threshold. The present paper gives a first example of the high interest of studying frequency combs at the quantum level, as they merge the advantages of two already well-known non-classical states of light: the c.w. light beams, with their high degree of coherence and reproducibility and the single pulses of light, with their high peak power enhancing the non-linear effects necessary to produce pure quantum effects.

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[16] Should $\Lambda_0 < 0$ then the null eigenvalue at threshold would be $\lambda_0^{(-)}$ instead of $\lambda_0^{(+)}$ and the following analysis should be accordingly modified.