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► **To cite this version:**

Redamy Perez Ramos. Inclusive hadronic distributions at small  $x$ . QCD and Hadronic Interactions, La Thuile, Italy, 18-25 Mar 2006., Mar 2006, La Thuile, Val d'Aosta, Italy. <hal-00068938>

**HAL Id: hal-00068938**

**<https://hal.science/hal-00068938v1>**

Submitted on 15 May 2006

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## Inclusive hadronic distributions at small $x$ inside jets

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**Abstract:** After giving their general expressions valid at all  $x$ , double differential 1-particle inclusive distribution inside a quark and a gluon jet produced in a hard process, together with the inclusive  $k_{\perp}$  distribution, are given at small  $x$  in the Modified Leading Logarithmic Approximation (MLLA), as functions of the transverse momentum  $k_{\perp}$  of the outgoing hadron.

### Introduction

This work concerns the production of two hadrons inside a high energy jet (quark or gluon); they hadronize out of two partons at the end of a cascading process that we calculate in pQCD[1]. Considering this transition as a “soft” process is the essence of the “Local Parton Hadron Duality” (LPHD) hypothesis[2]. We give indeed, in the MLLA scheme of resummation, the double differential inclusive 1-particle distribution and the inclusive  $k_{\perp}$  distribution as functions of the transverse momentum of the emitted hadrons in the limit that proved successful when describing energy spectra of particles in jets, that is  $Q_0 \approx \Lambda_{QCD}$ , the so called “limiting spectrum”[2].

### The process under consideration

In a hard collision ( $pp$ ,  $p\bar{p}$  collisions,  $e^+e^-$  annihilation) we consider a jet of half opening angle  $\Theta_0$  initiated by a parton  $A_0$ , which can be a quark or a gluon, see Fig.1.  $A_0$ , by a succession of partonic emissions (quarks, gluons), produces a jet of half opening angle  $\Theta_0$ , which, in particular, contains the parton  $A$ ;  $A$  splits into  $B$  and  $C$ , which hadronize respectively into the two hadrons  $h_1$  and  $h_2$ .  $\Theta$  is the angle between  $B$  and  $C$ . Because the virtualities of  $B$  and  $C$  are much smaller than that of  $A$ ,  $\Theta$  can be considered to be close to the angle between  $h_1$  and  $h_2$ ; angular ordering (AO) is also a necessary condition for this property to hold.  $A_0$  carries the energy  $E$  and gives rise to the (virtual) parton  $A$ , which carries the fraction  $u$  of the energy  $E$ .  $A \rightarrow B + C$  occurs with probability  $\propto \Phi$ ,  $\Phi$  is the corresponding DGLAP splitting function,  $B$  and  $C$  carry respectively the fractions  $uz$  and  $u(1-z)$  of  $E$ ;  $h_1$  carries the fraction  $x_1$  of  $E$ ;  $h_2$  carries the fraction  $x_2$  of  $E$ . One sets  $\Theta \leq \Theta_0$ .

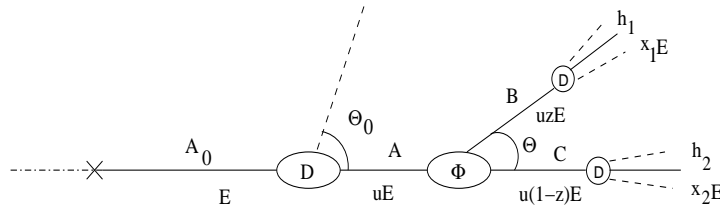


Fig. 1: process under consideration: two hadrons  $h_1$  and  $h_2$  inside one jet.

Since we are concerned with observables depending on the transverse momentum of outgoing hadrons,

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$k_{\perp}$  should be defined with respect to the jet axis which is identified with the direction of the energy flow[1].

### Double differential 1-particle inclusive distribution at all values of $x_1$

The energy conservation sum rule, considered together with DGLAP evolution equations and AO lead the general expression for the double differential 1-particle inclusive distribution

$$\frac{d^2 N}{dx_1 d \ln \Theta} = \frac{d}{d \ln \Theta} \sum_A \int_{x_1}^1 du D_{A_0}^A(u, E\Theta_0, uE\Theta) D_A^{h_1}\left(\frac{x_1}{u}, uE\Theta, Q_0\right) \quad (1)$$

which is valid at all  $x_1$ .  $\frac{d^2 N}{dx_1 d \ln \Theta}$  in (1) is described indeed by the convolution of two fragmentation functions,  $D_{A_0}^A$  and  $D_A^{h_1}$ , over the energy fraction  $u$  of  $A$ . Namely,  $D_{A_0}^A$  is purely partonic and describes the probability to emit parton  $A$  with energy  $uE$  off parton  $A_0$ , moreover, it takes into account the evolution of the jet between  $\Theta_0$  and  $\Theta$ ;  $D_A^{h_1}$  describes the probability to produce hadron  $h_1$  of energy fraction  $x_1/u$  at angle  $\Theta$  with respect to the direction of the energy flow inside the subjet  $A$ . On the other hand, the transverse momentum of  $h_1$  should be bigger than the collinear cut-off parameter  $Q_0$  ( $uE\Theta \geq Q_0$ ). Consequently, a lower bound for  $\Theta$  is obtained:  $\Theta \geq \Theta_{min} \approx Q_0/x_1E$ .

### Perturbative expansion parameter and kinematics

We conveniently define the variables  $\ell_1 = \ln \frac{1}{x_1}$ ,  $y_1 = \ln \frac{x_1 E \Theta}{Q_0}$ . In what follows, we use the anomalous dimension  $\gamma_0$  defined through the running coupling constant  $\alpha_s$  as

$$\gamma_0^2(k_{\perp}^2) = \frac{2N_c \alpha_s(k_{\perp}^2)}{\pi} = \frac{2}{\beta \ln \frac{k_{\perp}^2}{\Lambda_{QCD}^2}} \equiv \gamma_0^2(\ell_1 + y_1) = \frac{1}{\beta(\ell_1 + y_1 + \lambda)} = \frac{1}{\beta(Y_{\Theta} + \lambda)}, \quad \lambda = \ln \frac{Q_0}{\Lambda_{QCD}}. \quad (2)$$

It determines the rate of multiplicity growth with energy.  $N_c = 3$  for  $SU(3)$ ,  $\beta = \frac{1}{4N_c}(\frac{11}{3}N_c - \frac{4}{3}T_R)^{n_f=3} \approx 0.75$  with  $T_R = \frac{1}{2}n_f$  where  $n_f = 3$  is the number of light quarks,  $\Lambda_{QCD}$  is the QCD scale,  $Y_{\Theta} = \ell_1 + y_1 = \ln \frac{E\Theta}{Q_0}$  and  $Y_{\Theta_0} = \ln \frac{E\Theta_0}{Q_0}$ . For instance, in the LHC environment we can take the typical value  $Y_{\Theta_0} = 7.5 \Rightarrow \gamma_0 \simeq 0.4$  by setting  $\lambda = 0$  ( $Q_0 = \Lambda_{QCD}$ ),  $\Theta = \Theta_0$  and  $Q_0 = 250$  MeV in (2).  $\gamma_0$  can be therefore treated as the small parameter of the perturbative expansion at MLLA.

### Double differential 1-particle inclusive distribution at small $x_1$ , $x_1 \ll 1$

The convolution integral (1) is dominated by  $u \approx 1$ . In order to obtain an analytical expression of (1), since  $x_1/u \ll 1$ , we perform a perturbative expansion in  $\gamma_0$  such that (1) gets factorized and derived in the region of soft multi-particle production[1]

$$\frac{d^2 N}{d\ell_1 d \ln k_{\perp}} = \frac{\langle C \rangle_{q,g}}{N_c} \frac{d}{dy_1} \tilde{D}_g(\ell_1, y_1) + \frac{1}{N_c} \tilde{D}_g(\ell_1, y_1) \frac{d}{dy_1} \langle C \rangle_{q,g}, \quad (3)$$

$$\frac{d}{dy_1} \tilde{D}_g(\ell_1, y_1) = \mathcal{O}(\gamma_0) = \mathcal{O}(\sqrt{\alpha_s}), \quad \frac{d}{dy_1} \langle C \rangle_{q,g} = \mathcal{O}(\gamma_0^2) = \mathcal{O}(\alpha_s). \quad (4)$$

The first term in (3) is the main contribution to the double differential 1-particle inclusive distribution while the second one constitutes its MLLA correction of relative order  $\mathcal{O}(\gamma_0)$  (4).  $D_A^{h_1}$  in (1) has been replaced by the distribution  $D_g(\ell_1, y_1)$  in (3) that describes the MLLA ‘‘hump-backed plateau’’ in the limit where  $Q_0$  can be taken down to  $\Lambda_{QCD}$  (‘‘limiting spectrum’’)[2].  $\langle C \rangle_{q,g}$  is the total colour current that describes the evolution of the jet between  $\Theta_0$  and  $\Theta$ . It decomposes into its leading term  $\langle C \rangle_{q,g}^0$  and the MLLA correction  $\delta \langle C \rangle_{q,g} = \mathcal{O}(\gamma_0)$ [1]. In Fig.2 we represent  $\langle C \rangle_{q,g}^0$  (straight line) and the full expression  $\langle C \rangle_{q,g} = \langle C \rangle_{q,g}^0 + \delta \langle C \rangle_{q,g}$  (curve) for two values of  $\ell_1$ ;  $\ell_1 =$

2.5(left), 3.5(right) in function of  $y_1$ . Two types of MLLA corrections are displayed in this figure. Namely,  $\delta \langle C \rangle_{q,g} < 0$  is given by the vertical difference between the straight and curved lines. The other correction is given by the slope of the curve, that is  $\frac{d}{dy_1} \langle C \rangle_{q,g}$  in (4), this one is large and positive for  $y_1 \geq 1.5$ . For  $\ell_1 = 2.5$  and  $y_1 \approx 1.5$ ,  $\delta \langle C \rangle_{q,g}$  represents 50% of  $\langle C \rangle_{q,g}$  while for  $\ell_1 = 3.5 > 2.5$  it gets under control. We are thus allowed to set the range of applicability of our soft approximation to  $\ell_1 \geq \ell_{1,min} \approx 2.5$  ( $x_1 \lesssim 0.08$ )  $\Rightarrow y_1 \leq y_{1,max} = Y_{\Theta_0} - \ell_{1,min} = 5.0$ .

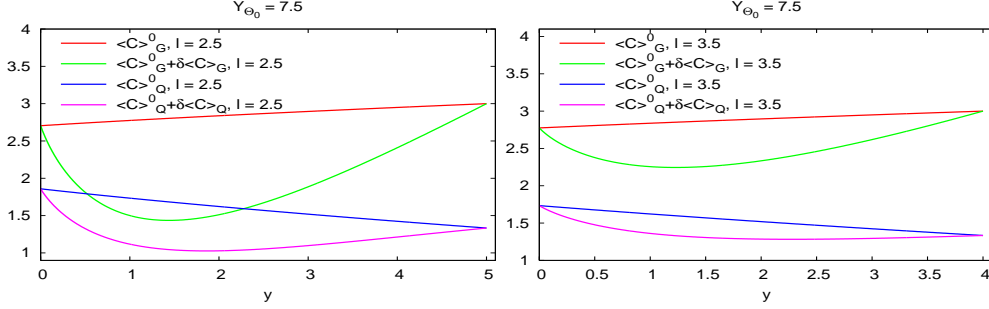


Fig. 2:  $\langle C \rangle_{A_0}^0$  and  $\langle C \rangle_{A_0}^0 + \delta \langle C \rangle_{A_0}$  for quark and gluon jets, as functions of  $y$ , for  $Y_{\Theta_0} = 7.5$ ,  $\ell = 2.5$  on the left and  $\ell = 3.5$  on the right.

Furthermore, one should stay in the perturbative regime, which needs  $y_1 \geq 1$  ( $k_\perp > 2.72\Lambda_{QCD} \approx 0.7$  GeV). We finally get an estimate of the range of applicability of the MLLA scheme of resummation to be  $1.0 \leq y_1 \leq 5.0$  in the LHC environment. In Fig.3 below, we represent the double differential 1-particle inclusive distribution (3) for  $\ell_1 = 3.5$  in function of  $y_1$ . We compare our results with a naive DLA-inspired case where one does not take into account the evolution of the jet between  $\Theta_0$  and  $\Theta$  but sets  $\langle C \rangle_q = C_F$  (quark jet) and  $\langle C \rangle_g = N_c$  (gluon jet).

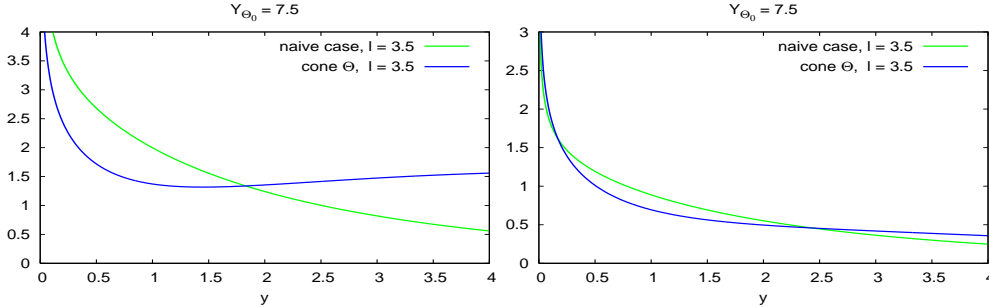


Fig. 3:  $\frac{d^2 N}{d\ell_1 d\ln k_\perp}$  for a gluon jet (left) and for a quark jet (right) at fixed  $\ell_1 = 3.5$ , MLLA and naive approach.

For both, the quark and gluon jets, new MLLA corrections arising from (4) push up the distribution when  $y_1$  increases as compared with the naive case. On the other hand, it is enhanced when  $y_1 \rightarrow 0$  by the running of  $\alpha_s(k_\perp)$  at  $k_\perp \rightarrow \Lambda_{QCD}$ .

### Transverse momentum inclusive $k_\perp$ distribution

Integrating (3) over the whole phase space in the logarithmic sense ( $\ell_1 = \ln(1/x_1)$ ) we obtain the inclusive  $k_\perp$  distribution inside a quark and a gluon jet[1]

$$\left( \frac{dN}{d\ln k_\perp} \right)_{q,g} = \int_0^{Y_{\Theta_0} - y_1} d\ell_1 \left( \frac{d^2 N}{d\ell_1 d\ln k_\perp} \right)_{q,g}. \quad (5)$$

We give results for  $Y_{\Theta_0} = 7.5$  in Fig.4 below and compare with the naive case.

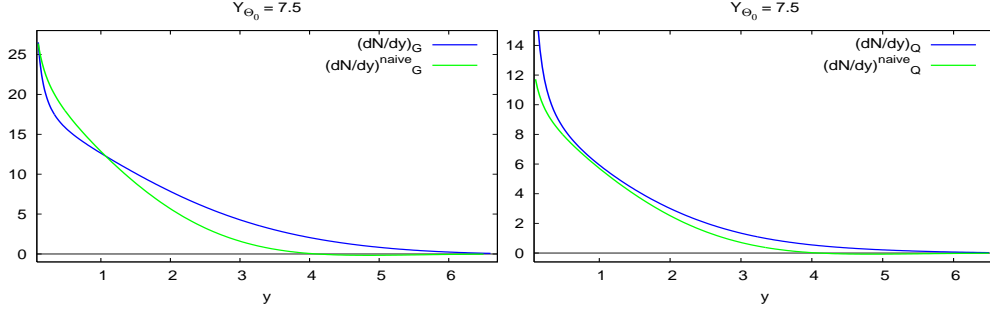


Fig. 4:  $\frac{dN}{d \ln k_{\perp}}$  for a gluon jet, MLLA and naive approach with enlargement

We observe in particular that the positivity of the distribution when  $y_1$  increases is restored as compared with the naive case. This stems from the global role of MLLA corrections in the range.

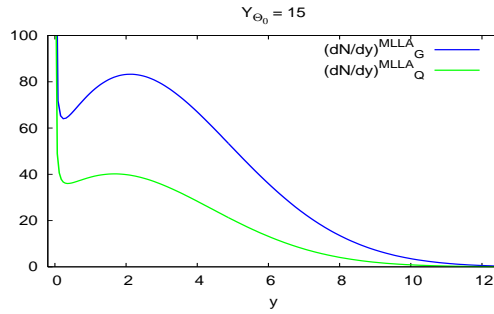


Fig. 5:  $\frac{dN}{d \ln k_{\perp}}$  for a gluon jet (blue) and for a quark jet (green) at  $Y_{\Theta_0} = 15.0$

In Fig.5 above, we represent the evidence of two competing effects at the unrealistic value  $Y_{\Theta_0} = 15.0$ . One observes that as  $y_1 \rightarrow 0$  the distribution is depleted by QCD coherence effects. Indeed, in this region of the phase space ( $k_{\perp} \rightarrow Q_0 \approx \Lambda_{QCD}$ ) gluons are pushed at larger angles, they are thus emitted independently from the rest of the partonic ensemble. At the opposite, when the value  $k_{\perp} \approx Q_0 \approx \Lambda_{QCD}$  is reached, the distribution is enhanced by the running of  $\alpha_s$ .

## Conclusion

Results for the double 1-particle inclusive distribution and the inclusive  $k_{\perp}$  distribution at small  $x$  have been discussed and displayed. Sizable differences with the naive approach in which one forgets the evolution of the jet between its half opening angle  $\Theta_0$  and the emission angle  $\Theta$  have been found. The global role of new MLLA corrections is emphasized to recover the positivity of the inclusive  $k_{\perp}$  distribution. On the other hand, at realistic energy scales (LHC, Tevatron, LEP), QCD coherence effects are screened by the running of  $\alpha_s$  (see Fig.4) which, furthermore, forbids extending the confidence domain for  $y_1 \leq 1$ . The range of applicability of the soft approximation has been discussed from the analysis of corrections not to exceed  $y_{max} = Y_{\Theta_0} - \ell_{min} = 5.0$ ; it is indeed smaller at LEP and Tevatron energies (smaller  $Y_{\Theta_0}$ ). Our results will be compared with forthcoming data from CDF.

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