

How to decode the NP-complex problem $Y = HX + B$?

> State of the art: Zero forcing, MMSE (simple but low performances), PIC, SDP, Sphere Decoding (good performance but iterative and complex => not well suited for a VLSI implementation).

New proposition HIS Decoder:

- Geometrical approach to resolve problem (1).
- Nearly optimum performance.
- Low computational complexity.
- An inherent parallelism structure.

Application case: Multi-user detection for MC-CDMA system.

Problem formulation:

Let's find:
$$\hat{x} = \min_{x \in \xi \subset Z^n} \|y - Hx\|_2 \quad (1)$$

Where:
$$y \in \mathbb{R}^m, \quad H \in \mathbb{R}^{m \times n}, \quad x \in \{-1, 1\}^n$$

- Search space is discrete $\{-1, 1\}^n$
- Given a channel real matrix H
- Given a received vector defined as (2) where v is an M -dimensional Additive Gaussian Noise vector with variance σ^2 :

$$y = Hx + v \quad (2)$$

- Find a vector \hat{x} which minimize the following cost function over the discrete space.

$$f(x) = \|y - Hx\|_2^2 \quad (3)$$

Known to be a NP-hard problem

Hyperplane Intersection Selection Algorithm

- ✓ **Input** : The real channel matrix H , the real received vector Y and σ^2 .
- ✓ **Output** : Quasi-optimal solution x_{hisd} for problem (1).
- ✓ **Pre-processing** : Extract the D right singular vectors v_1, \dots, v_D associated to the D smallest singular values of the channel matrix H .

✓ **Step 1** : Calculate an initial sub-optimal solution ρ . $\rho = \rho_{zf}$ or $\rho = \rho_{mmse}$

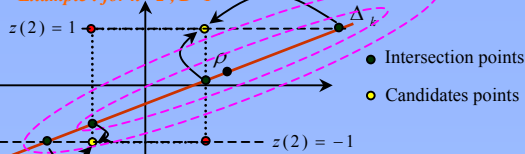
✓ **Step 2** : Generate the D references lines $\Delta_1, \Delta_2, \dots, \Delta_D$ defined by the offset ρ and the D right singular vectors v_1, \dots, v_D .

$$\Delta_k = \left\{ z \in \mathbb{R}^n / z = \rho + \alpha \cdot v_k, \quad \alpha \in \mathbb{R} \right\} \quad (4)$$

✓ **Step 3** : for each lines Δ_k , find all intersection points between this line and all hyper-plane P defined as (5). There are $2n$ points to be calculated. Finally project all intersection point on $\{-1, 1\}^n$, suppress redundant points and generate candidates points Ip_k .

$$P = \bigcup_{i=1}^n \left[\{z \in \mathbb{R}^n / z(i) = 1\} \cup \{z \in \mathbb{R}^n / z(i) = -1\} \right] \quad (5)$$

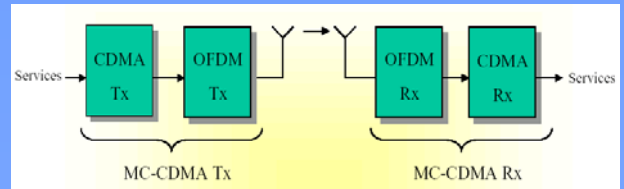
Example : for $n=2, D=1$



✓ **Step 4** : For each direction k ($k=1 \dots D$), evaluate the cost function for all Ip_k .

- ✓ **Step 5** : For each Ip_k , extract the M candidates points having the lowest cost function.
- ✓ **Step 6** : use each candidate point $b_{k,t}$ ($k=1 \dots D$ and $t=1 \dots M$) as a starting point of a greedy function (local search function)
- ✓ **Step 7** : Extract Quasi-optimal solution x_{hisd} for problem (1).

Multi-user detection MC-CDMA model



> The received signal can be written as :

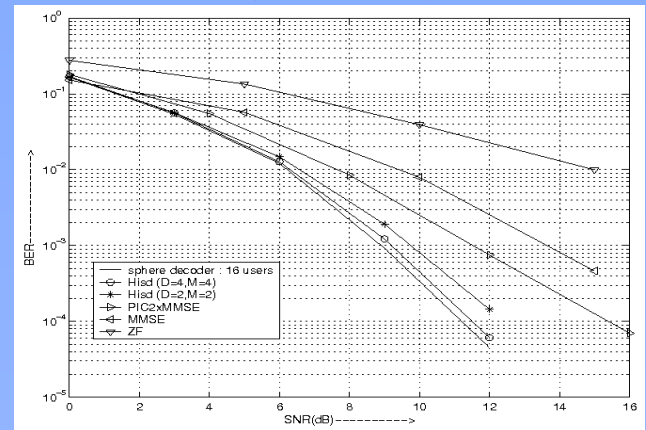
$$y = ACx + v = Hx + v$$

Where :

- A : is a $Np \times Np$ diagonal matrix. The channel effect on i^{th} is given by a_{ii} and $a_{ij}=0$ if $i \neq j$.
- C : is a $Lc \times Nu$ spreading matrix. We suppose that $Nu = Np = Lc$
- x : is the transmitted vector. The i^{th} coordinates correspond to the i^{th} user.
- v : is an AWGN vector.

Note : Nu : number of users, Np number of sub-carriers and Lc is the length of spreading code.

Simulation result: Fully loaded MC-CDMA system, 4-QAM and 16 users



Conclusion and futur work:

> Low computational complexity :

- Mult : $\left(\frac{3}{2} + l + 2M\theta \right) Dn^2 + \left(\frac{1}{2} + l + 2M\theta \right) Dn$
- Add/sub: $2Dn^3 + (4 + l + 4M\theta) Dn^2 + (5 + 2l + 2M\theta) Dn$

where l is the number of distinct points (step3) and $\theta=2$ number of iterations in the greedy function (Step6).

> HISD is suitable to an hardware implementation : inherent parallelism structure.

> Can be generalized to different systems : BLAST, CDMA, MIMO and also image treatment.