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# Disorder contra Mermin-Wagner-Hohenberg Effect: From Classical Spin Systems to Ultracold Atomic Gases

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We propose a general mechanism of *random-field-induced order* (RFIO) by studying a case of classical ferromagnetic  $XY$  model in a random uniaxial field which breaks the continuous symmetry of the model. We prove rigorously that the system has spontaneous magnetization at temperature  $T = 0$ , and we present strong evidence that this is also the case for small  $T > 0$ . We discuss generalizations of this mechanism to various classical and quantum systems. Our results shed light on controversies in existing literature, and open the way to realize RFIO with ultracold atoms in an optical lattice.

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One of the most appealing effects of disorder is that even extremely small randomness can have dramatic consequences. The paradigmatic example in classical physics is the Ising model for which an arbitrarily small random magnetic field destroys magnetization even at temperature  $T=0$  in 2D [1, 2] but not in  $D > 2$  [3]. The quantum physics paradigm is Anderson localization which holds in 1D and 2D in arbitrarily small random potentials [4]. In this Letter, we propose an even more intriguing opposite effect where disorder counter-intuitively favors ordering: a general mechanism of *random-field-induced order* (RFIO) by which certain spin models magnetize at a higher temperature in the presence of arbitrarily small disorder than in the absence of disorder, provided that a continuous symmetry of the system is broken. We prove rigorously that a classical  $XY$  spin model in a uniaxial random field magnetizes spontaneously in the transverse direction at  $T = 0$ , and provide strong evidence that this is also the case for  $T > 0$ . We discuss generalizations of this mechanism to classical and quantum  $XY$  and Heisenberg models in 2D and 3D. Finally, we propose three possible realizations of the phenomenon using ultracold atoms in optical lattices.

Consider a classical spin system on the 2D square lattice  $\mathbf{Z}^2$ . The spin variable  $\sigma_i = (\cos \theta_i, \sin \theta_i)$  at a site  $i \in \mathbf{Z}^2$  is a unit vector in the  $xy$  plane. The Hamiltonian (in units of the exchange term  $J$ ) is given by

$$H/J = - \sum_{|i-j|=1} \sigma_i \cdot \sigma_j - \epsilon \sum_i \mathbf{h}_i \cdot \sigma_i. \quad (1)$$

Here the first term is the standard nearest-neighbor interaction of the  $XY$ -model, and the second term represents a small random field perturbation;  $\mathbf{h}_i$  are independent, identically distributed random two-dimensional vectors.

For  $\epsilon = 0$ , the model has no spontaneous magnetization  $m$  at any positive  $T$ . This was first pointed out in Ref. [5], and later developed into a class of results known as the Mermin-Wagner-Hohenberg theorem [6] for various classical, as well as quantum two-dimensional spin systems with continuous symmetry. In higher dimensions the system does magnetize

at low temperatures. This follows from the spin wave analysis [7], and has been given a rigorous proof in Ref. [8]. The impact of a random field term was first addressed in Ref. [1, 2], where it was argued that if the distribution of the random variables  $\mathbf{h}_i$  is symmetric with respect to rotations, there is no spontaneous  $m$  at any positive  $T$  in any dimension  $D \leq 4$ . A rigorous proof of this statement was given in [2]. Both works use crucially the rotational invariance of the distribution of the random field variables.

Here we consider the case when  $\mathbf{h}_i$  is directed along the  $y$ -axis:  $\mathbf{h}_i = \eta_i \mathbf{e}_y$ , where  $\mathbf{e}_y$  is the  $y$  axis unit vector. Such a random field obviously breaks the continuous symmetry of the interaction and a question arises whether the model still has no spontaneous  $m$  in 2D. This question has been given contradicting answers in Refs. [9, 10]: while Ref. [9] predicts that a small random field in the  $y$ -direction does not change the behavior of the model, Ref. [10] argues that it leads to the presence of spontaneous  $m$  in the direction perpendicular to the random field axis in low (but not arbitrarily low) temperatures. Both works use renormalization group analysis, with Ref. [10] starting from a version of the Imry-Ma scaling argument to prove that the model magnetizes at zero temperature.

We first present a complete proof that the system indeed magnetizes at  $T = 0$ , and argue that the magnetization is stable under inclusion of small thermal fluctuations. For this we use a version of the Peierls contour argument [11], eliminating first the possibility that Bloch walls or vortex configurations destroy the transition.

Let us start by a rigorous analysis of the ground state. Consider the system in a square  $\Lambda$  with the ‘right’ boundary conditions,  $\sigma_i = (1, 0)$  for the sites  $i$  on the outer boundary of  $\Lambda$ . The energy of any spin configuration decreases if we replace the  $x$ -components of the spins by their absolute values and leave the  $y$ -components unchanged. It follows that in the ground state,  $x$ -components of all the spins are nonnegative. *A priori* this ground state could coincide (in the infinite volume limit) with the ground state of the Random Field Ising Model, in which all spins have zero  $x$ -component. The following

argument shows that this is actually *not* the case. Suppose that the spin  $\sigma_i$  in a given site  $i$  is aligned along the  $y$ -axis, *i.e.*  $\cos\theta_i = 0$ . Since the derivative of the energy function with respect to  $\theta_i$  vanishes at the minimum, we obtain

$$\sum_{j:|i-j|=1} \sin(\theta_i - \theta_j) = 0. \quad (2)$$

Since  $\cos\theta_i = 0$ , this implies  $\sum_{j:|i-j|=1} \cos\theta_j = 0$ . Because in the ‘right’ ground state all spins lie in the (closed) (right)  $x \geq 0$  half-plane, all terms in the above expression are nonnegative and hence have to vanish. This means that at all the nearest neighbors  $j$  of the site  $i$ , the ground state spins are directed along the  $y$ -axis as well. Repeating this argument, we conclude that the same holds for all spins, except possibly those at the inner boundary of  $\Lambda$ , *i.e.* the ground state is the (unique) Random Field Ising Model ground state. This, however, leads to a contradiction, since assuming this, one can construct a field configuration, occurring with a positive probability, which forces the ground state spins to have nonzero  $x$ -components. To achieve this we put strong positive ( $\eta_i > 0$ ) fields on the boundary of a square and strong negative fields on the boundary of a concentric smaller square. If the fields are very weak inside the box, the spins will form a Bloch wall, rotating gradually from  $\theta = \pi/2$  to  $\theta = -\pi/2$ . Since such a local field configuration occurs with a positive probability, the ground state *cannot have* zero  $x$ -components everywhere. Note, that this argument applies to weak, as well as to strong random field, so that the ground state is never, strictly speaking, field-dominated and always exhibits  $m$  in the  $x$ -direction. We argue below that this effect still holds at small positive temperatures, the critical temperature depending on the strength of the random field (and presumably going to zero as the strength of the field increases).

To study the system at low  $T$ , we need to ask what are the typical low energy excitations from the ground state. For  $\epsilon = 0$ , continuous symmetry allows Bloch walls, *i.e.* configurations in which the spins rotate gradually over a large region, for instance from left to right. The total excitation energy of a Bloch wall in 2D is of order one, and it is the presence of such walls that underlies absence of continuous symmetry breaking. However, for  $\epsilon > 0$ , a Bloch wall carries additional energy, coming from changing the direction of the  $y$ -component of the spin, which is proportional to the volume of the wall (which is of the order  $L^2$  for a wall of linear size  $L$  in two dimensions), since the ground state spins are adapted to the field configuration, and hence overturning them will increase the energy per site. Similarly, vortex configurations, which are important low-energy excitations in the nonrandom XY model, are no longer energetically favored in the presence of a uniaxial random field.

We are thus left, as possible excitations, with sharp domain walls, where the  $x$ -component of the spin changes sign rapidly. To first approximation we consider excited configurations, in which spins take their ground state values, or their reflections in the  $y$ -axis. As in the standard Peierls argument [11], in the presence of the right boundary conditions, such

configurations can be described in terms of contours  $\gamma$  (domain walls), separating spins with positive and negative  $x$ -components. If  $m_i$  is the value of the  $x$ -component of the spin  $\sigma_i$  in the right ground state, the energy of a domain wall is the sum of  $m_i m_j$  over the bonds  $(ij)$  crossing the boundary of the contour. The Peierls estimate shows that in our approximation probability of such contour is bounded above by  $\exp(-2\beta \sum_{(ij)} m_i m_j)$ , with  $\beta = J/k_B T$ .

We want to show that for a typical realization of the field  $\mathbf{h}$  (*i.e.* with probability one), the sum of these probabilities over all contours containing the origin in their interior are summable. It then follows that in a still lower  $T$ , this sum is small, and the Peierls estimate proves that the system magnetizes (alternatively, a simple argument shows that summability of the contour probabilities already proves existence of spontaneous  $m$ ). To show that a series of random variables is summable with probability one, it suffices to prove summability of the series of the expected values. We present two arguments for the last statement to hold.

If the random variables  $m_i$  are bounded away from zero, *i.e.*  $m_i > \sqrt{c}$ , for some  $c > 0$ , the moment generating function of the random variable  $\sum_{(ij)} m_i m_j$  satisfies

$$\mathbf{E}[\exp(-\beta \sum_{(ij)} m_i m_j)] \leq \exp[-c\beta L(\gamma)], \quad (3)$$

with  $L(\gamma)$  denoting the length of the contour  $\gamma$ . The sum of the probabilities of the contours enclosing the origin is bounded thus by  $\sum_{\gamma} \exp[-c\beta L(\gamma)]$ . The standard Peierls-Griffiths bound proves the desired summability.

The above argument does not apply if the distribution of the ground state  $m$  contains zero in its support. For unbounded distribution of the random field this may very well be the case, and then another argument is needed. If we assume that the terms in the sum  $\sum_{(ij)} m_i m_j$  are independent and identically distributed, then  $\mathbf{E}[\exp(-2\beta \sum_{(ij)} m_i m_j)] = \mathbf{E}[\exp(-2\beta m_i m_j)]^{L(\gamma)} = \exp\{L(\gamma) \log \mathbf{E}[\exp(-2\beta m_i m_j)]\}$  and we just need to observe that  $\mathbf{E}[\exp(-2\beta m_i m_j)] \rightarrow 0$  as  $\beta \rightarrow \infty$  (since the expression under the expectation sign goes pointwise to zero and lies between 0 and 1) to conclude that  $\mathbf{E}[\exp(-2\beta \sum_{(ij)} m_i m_j)]$  behaves as  $\exp[-g(\beta)L(\gamma)]$  for a positive function  $g(\beta)$  with  $g(\beta) \rightarrow \infty$  as  $\beta \rightarrow \infty$ . While  $m_i m_j$  are not, strictly speaking, independent, it is natural to assume that their dependence is weak, *i.e.* their correlation decays fast with the distance of the corresponding bonds  $(ij)$ . The behavior of the moment generating function of their sum is then qualitatively the same, with a renormalized rate function  $g(\beta)$ , still diverging as  $\beta \rightarrow \infty$ . As before, this is enough to carry out the Peierls-Griffiths estimate which implies spontaneous  $m$  in the  $x$ -direction [12].

It is thus expected that the RFIO predicted here will lead to  $m$  of order 1 at low temperatures in systems much larger than the correlation length of typical excitations. However, the effect may be obscured by finite size effects, which, due to long-range power law decay of correlations, are particu-

larly strong in the  $XY$  model in 2D. In particular, the 2D- $XY$  model shows finite  $m$  in small systems [13] so that RFIO would result in an increase of the magnetization. Our Monte-Carlo simulations [14] in lattices up to  $200 \times 200$  confirm that this is indeed the case. For example, at  $T = 0.7J/k_B$ ,  $m$  increases by 1.6% in presence of uniaxial disorder.

The effect may be generalized to other spin models, in particular those that have finite correlation length. Here we list the most spectacular generalizations:

*i) 2D Heisenberg ferromagnet (HF) in random fields of various symmetries.* Here the interaction has the same form as in the  $XY$  case, but spins take values on a unit sphere. As for the  $XY$  Hamiltonian, if the random field distribution has the same symmetry as the interaction part, *i.e.* if it is symmetric under rotations in three dimensions, the model has no spontaneous  $m$  up to 4D [1, 2]. If the random field is uniaxial, *e.g.* oriented along the  $z$  axis, the system still has a continuous symmetry (rotations in the  $xy$  plane), and thus cannot have a spontaneous  $m$  in this plane. It cannot magnetize in the  $z$  direction either, by the results of [2]. Curiously enough, a field distribution with an intermediate symmetry may lead to symmetry breaking. Namely, arguments fully analogous to the previous ones imply that if the random field takes values in the  $yz$  plane with a distribution invariant under rotations, the system will magnetize in the  $x$  direction. We are thus faced with the possibility that planar field distribution breaks the symmetry, which is broken neither by a field with a spherically symmetric distribution nor by a uniaxial one.

*ii) 3D  $XY$  and HF in a random field of various symmetries.* We have argued that the 2D  $XY$  model with a small uniaxial random field orders at low  $T$ . Since in the absence of the random field spontaneous  $m$  occurs only at  $T = 0$ , this can be equivalently stated by saying that a small uniaxial random field raises the critical temperature  $T_c$  of the system. By analogy, one can expect that the (nonzero)  $T_c$  of the  $XY$  model in 3D becomes higher and comparable to that of the 3D Ising model, in the presence of a small uniaxial field. A simple mean field estimate suggests that  $T_c$  might increase by factor 2. The analogous estimates for the Heisenberg model in 3D suggest increase of  $T_c$  by factor 3/2 (3) in a small uniaxial (planar rotationally symmetric) field.

*iii) Antiferromagnetic systems.* By flipping every second spin, the classical ferromagnetic models are equivalent to antiferromagnetic ones (on bipartite lattices). This equivalence persists in the presence of a random field with a distribution symmetric with respect to the origin. Thus the above discussion of the impact of random fields on continuous symmetry breaking translates case by case to the antiferromagnetic case.

*iv) Quantum systems.* All of the above predicted effects should, in principle have quantum analogues. Quantum fluctuations might, however, destroy the long-range order, so each of the discussed models should be carefully reconsidered in the quantum case. Some models, such as the quantum spin  $S = 1/2$  Heisenberg model, for instance, have been widely studied in literature [15]. The Mermin-Wagner theorem [6] implies that the model has no spontaneous  $m$  at positive tem-

peratures in 2D. For  $D > 2$  spin wave analysis shows existence of spontaneous  $m$  (though a rigorous mathematical proof of this fact is still lacking). In general, one does not expect major differences between the behaviors of the two models at  $T \neq 0$ . It thus seems plausible that presence of a random field in the quantum case is going to have effects similar to those in the classical Heisenberg model. Similarly, one can consider the quantum Heisenberg antiferromagnet (HAF) and expect phenomena analogous to the classical case, despite the fact that unlike their classical counterparts, the quantum HF and HAF systems are no longer equivalent. We expect to observe spontaneous staggered magnetization in a random uniaxial, or planar field HF. A possibility that a random field in the  $z$ -direction can enhance the antiferromagnetic order in the  $xy$  plane has been pointed out in [16].

Further understanding of the phenomena described in this Letter may benefit from experimental investigations. Below, we discuss possibilities to design quantum simulators for these quantum spin systems using ultracold atoms in optical lattices (OL). Consider a two-component Bose gas confined in an OL with on-site inhomogeneities. The low- $T$  physics is captured by the Bose-Bose Hubbard model (BBH) [17]:

$$\begin{aligned}
 H_{\text{BBH}} = & \sum_j \left[ \frac{U_b}{2} \mathbf{n}_j (\mathbf{n}_j - 1) + \frac{U_B}{2} \mathbf{N}_j (\mathbf{N}_j - 1) \right. \\
 & \left. + U_{\text{bb}} \mathbf{n}_j \mathbf{N}_j \right] + \sum_j (v_j \mathbf{n}_j + V_j \mathbf{N}_j) \quad (4) \\
 & - \sum_{\langle j,l \rangle} \left[ (J_b \mathbf{b}_j^\dagger \mathbf{b}_l + J_B \mathbf{B}_j^\dagger \mathbf{B}_l) + \text{h.c.} \right] \\
 & - \sum_j \left( \frac{\Omega_j}{2} \mathbf{b}_j^\dagger \mathbf{B}_j + \text{h.c.} \right)
 \end{aligned}$$

where  $\mathbf{b}_j$  and  $\mathbf{B}_j$  are the annihilation operators for both types of Bosons in the lattice site  $j$ ,  $\mathbf{n}_j = \mathbf{b}_j^\dagger \mathbf{b}_j$  and  $\mathbf{N}_j = \mathbf{B}_j^\dagger \mathbf{B}_j$  are the number operators, and  $\langle j, l \rangle$  denote a pair of adjacent sites. In Hamiltonian (4), (i) the first term describes on-site interactions between different types of Bosons; (ii) the second accounts for on-site energies; (iii) the third describes quantum tunneling between adjacent sites and (iv) the fourth transforms one Boson type into the other with a probability amplitude  $|\Omega|/\hbar$ . The last term can be implemented with an optical two-photon Raman process if the two Bosonic ‘species’ correspond to two internal states of the same atom. Possibly, both on-site energies  $v_j, V_j$  and the Raman complex amplitude  $\Omega_j$  can be made site-dependent using speckle laser light [18].

Consider the limit of strong repulsive interactions ( $0 < J_b, J_B, |\Omega_j| \ll U_b, U_B, U_{\text{bb}}$ ) and a total filling factor of 1 (*i.e.* the total number of particles equals the number of lattice sites). Proceeding as in the case of Fermi-Bose mixtures, recently analyzed by two of the authors in [19], we derive an effective Hamiltonian  $H_{\text{eff}}$  for the Bose-Bose mixture. In brief, we restrict the Hilbert space to a subspace  $\mathcal{E}_0$  generated by  $\{ \prod_j |n_j, N_j\rangle \}$  with  $n_j + N_j = 1$  at each lattice site, and we incorporate the tunneling terms via perturbation theory as in [19]. Hamiltonian  $H_{\text{eff}}$  describes the dynamics of

composite particles whose annihilation operator at site  $j$  is  $\mathcal{B}_j = \mathbf{b}_j^\dagger \mathbf{B}_j \mathcal{P}$ , where  $\mathcal{P}$  the projector onto  $\mathcal{E}_0$ .

Since the commutation relations of  $\mathcal{B}_j, \mathcal{B}_j^\dagger$  are those of Schwinger Bosons [20], we may directly turn to the spin representation [20] by defining  $\mathbf{S}_j^x + i\mathbf{S}_j^y = \mathcal{B}_j$  and  $\mathbf{S}_j^z = 1/2 - \mathcal{N}_j$ , where  $\mathcal{N}_j = \mathcal{B}_j^\dagger \mathcal{B}_j$ . It is important to note that since Raman processes can convert  $b$  Bosons into  $B$  Bosons (and respectively),  $\sum_j \langle \mathcal{N}_j \rangle$  is not fixed by the total number of Bosons of each species, *i.e.* the  $z$  component of  $m$ ,  $\sum_j \langle \mathbf{S}_j^z \rangle$  is not constrained. For small inhomogeneities ( $\delta_{j,l} = v_j - v_l, \Delta_{j,l} = V_j - V_l \ll U_b, U_B, U_{bB}$ ), Hamiltonian  $H_{\text{eff}}$  is then equivalent to the anisotropic Heisenberg  $XXZ$  model [20] in a random field:

$$H_{\text{eff}} = -J_\perp \sum_{\langle j,l \rangle} (\mathbf{S}_j^x \mathbf{S}_l^x + \mathbf{S}_j^y \mathbf{S}_l^y) - J_z \sum_{\langle j,l \rangle} \mathbf{S}_j^z \mathbf{S}_l^z - \sum_j (h_j^x \mathbf{S}_j^x + h_j^y \mathbf{S}_j^y + h_j^z \mathbf{S}_j^z) \quad (5)$$

where  $J_\perp = 4J_b J_B / U_{bB}$ ,  $J_z = 2[2J_b^2 / U_b + 2J_B^2 / U_B - (J_b^2 + J_B^2) / U_{bB}]$ ,  $h_j^x = \Omega_j^R$ ,  $h_j^y = -\Omega_j^I$ ,  $h_j^z = \mathcal{V}_j - \zeta J_z / 2$ , with  $\zeta$  the lattice coordination number,  $\mathcal{V}_j = V_j - v_j + \zeta[4J_b^2 / U_b + 4J_B^2 / U_B - (J_b^2 + J_B^2) / U_{bB}]$  and  $\Omega_j = \Omega_j^R + i\Omega_j^I$ . In atomic systems, all these terms can be controlled almost at will [19, 21]. In particular, by employing various possible control tools one may reach the HF ( $J_\perp = J_z$ ) and XY ( $J_z = 0$ ) cases.

The quantum ferromagnetic XY model in random field may be alternatively obtained using the same BBH model, but with strong state dependence of the optical dipole forces. One can imagine a situation in which one component (say  $b$ ) is in the strong interaction limit, so that only one  $b$  atom at a site is possible, whereas the other ( $B$ ) component is Bose condensed and provides only a coherent ‘background’ for the  $b$ -atoms. Mathematically, this situation is described by Eq. (4), in which  $n_i$ ’s can be equal to 0 or 1 only, whereas  $B_i$ ’s can be replaced by a classical complex field (condensate wave function). In this limit the spin  $S = 1/2$  states can be associated with the presence, or absence of a  $b$ -atom in a given site. In this way, setting  $v_j = 0$  and  $\Omega_j^I = 0$ , one obtains the quantum version of the XY model (1) with  $J = J_b$  and a uniaxial random field in the  $x$  direction with the strength determined by  $\Omega_j^R$ .

Finally, the  $S = 1/2$  HAF model may be realized with a Fermi-Fermi mixture at half filling for each component. This implementation might be important for future experiments with Lithium atoms. As recently calculated [22], the critical temperature for the Néel state in 3D is of order of 30nK. Mean field estimates give  $\simeq 45$  (90)nK by placing the system in a uniaxial (planar) random field, created using the same methods as discussed above.

Similar proposals have been formulated before [23], but none of them treat simultaneously essential aspects for the present schemes: i) disordered fields, but not bonds; ii) arbitrary directions of the fields; iii) possibility of exploring Ising, XY or Heisenberg symmetries; iv) realizing coherent source

of atoms; and v) avoiding constraints on the magnetization along the  $z$  axis.

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