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(Dated: 3rd April 2006)

We propose a general mechanism of *disorder-induced order* by studying a particular case of classical ferromagnetic XY model in a random uniaxial field which breaks the continuous symmetry of the model. We prove rigorously that the system has spontaneous magnetization at temperature $T = 0$, and we present strong evidence that this is also the case for small $T > 0$. We discuss generalizations of this mechanism to various classical spin models and quantum systems. We propose a realization of this phenomenon with ultracold atoms in an optical lattice.

PACS numbers: 05.30.Jp, 64.60.Cn, 75.10.Nr, 75.10.Jm

One of the most appealing effects of disorder is that even extremely small randomness can have dramatic consequences. The paradigmatic example in classical physics is the Ising model for which an arbitrarily small random magnetic field destroys magnetization even at temperature T=0 in 2D [1] but not in $D > 2$ [2]. The quantum physics paradigm is Anderson localization which holds in 1D and 2D in arbitrarily small random potentials [3]. Even more intriguing is the opposite effect where disorder counter-intuitively favors ordering. In this Letter, we propose a general mechanism by which certain spin models magnetize at a higher temperature in the presence of arbitrarily small disorder than in the absence of disorder, provided that a continuous symmetry of the system is broken. We prove rigorously that a classical XY spin model in a uniaxial random field magnetizes spontaneously in the transverse direction at $T = 0$, and provide strong evidence that this is also the case for $T > 0$. We discuss generalizations of this mechanism to classical and quantum XY and Heisenberg models in 2D and 3D. Finally, we propose three possible quantum realizations of the phenomenon using ultracold atoms in optical lattices.

Consider a classical spin system on the 2D square lattice $\mathbf{Z}^2$. The spin variable $\sigma_i = (\cos\theta_i, \sin\theta_i)$ at a site $i \in \mathbf{Z}^2$ is a unit vector in the xy plane. The Hamiltonian (in units of the exchange term J) is given by

$$H/J = -\sum_{|i-j|=1} \sigma_i \cdot \sigma_j - \epsilon \sum_i \mathbf{h}_i \cdot \sigma_i. \qquad (1)$$

Here the first term is the standard nearest-neighbor interaction of the XY-model, and the second term represents a small random field perturbation; $\mathbf{h}_i$ are independent, identically distributed random two-dimensional vectors.

For $\epsilon = 0$, the model has no spontaneous magnetization at any positive T. This was first pointed out in Ref. [4], and later developed into a class of results known collectively as the Mermin-Wagner theorem [5] for various classical, as well as quantum two-dimensional spin systems with continuous symmetry. In higher dimensions the system does magnetize at low temperatures. This follows from the spin wave analysis [6], and has been given a rigorous proof in Ref. [7]. The impact of a random field term was first addressed in Ref. [1], where it was argued that if the distribution of the random variables $\mathbf{h}_i$ is symmetric with respect to rotations, there is no spontaneous magnetization at any positive T in any dimension $D \leq 4$. A rigorous proof of this statement was given in [8]. Both works use crucially the rotational invariance of the distribution of the random field variables.

Here we consider the case when $\mathbf{h}_i$ is directed along the y-axis: $\mathbf{h}_i = \eta_i \mathbf{e}_y$, where $\mathbf{e}_y$ is the y axis unit vector. Such a random field obviously breaks the continuous symmetry of the interaction and a question arises whether the model still has no spontaneous magnetization in 2D. This question has been discussed in Refs. [9, 10], and given contradicting answers: while Ref. [9] predicts that a small vertical random field does not change the behavior of the model, Ref. [10] argues that it leads to the presence of spontaneous magnetization in the direction perpendicular to the random field axis in low (but not arbitrarily low) temperatures. Both works use renormalization group analysis, with Ref. [10] starting from a version of the Imry-Ma scaling argument to prove that the model magnetizes at zero temperature.

We first present a complete proof that the system indeed magnetizes at $T = 0$, and argue that the magnetization is stable under inclusion of small thermal fluctuations. For this we use a version of the Peierls contour argument [11], eliminating first the possibility that Bloch walls or vortex configurations destroy the transition.

Let us start by a rigorous analysis of the ground state. Consider the system in a square Λ with the 'right' boundary conditions, $\sigma_i = (1, 0)$ for the sites i on the outer boundary of Λ. The energy of any spin configuration decreases if we replace the horizontal components of the spins by their absolute values and leave the vertical components unchanged. It follows that in the ground state, all the spins have nonnegative horizontal components. *A*

priori this ground state could coincide (in the infinite volume limit) with the ground state of the Random Field Ising Model, in which all spins have zero x-component. The following argument shows that this is actually *not* the case. Suppose that the spin σ_i in a particular site i is vertical in the 'right' ground state, *i.e.* $\cos\theta_i = 0$. Since the derivative of the energy function with respect to θ_i vanishes at the minimum, we obtain

$$\sum_{j:|i-j|=1} \sin(\theta_i - \theta_j) = 0. \tag{2}$$

Since $\cos\theta_i = 0$, this implies $\sum_{j:|i-j|=1} \cos\theta_j = 0$. Because in the 'right' ground state all spins lie in the (closed) right half-plane, all terms in the above expression are nonnegative and hence have to vanish. This means that at all the nearest neighbors j of the site i, the ground state spins are vertical as well. Repeating this argument, we conclude that all spins, except possibly those at the inner boundary of Λ are vertical, *i.e.* the ground state is the (unique) Random Field Ising Model ground state. This, however, leads to a contradiction, since assuming this, one can construct a field configuration, occurring with a positive probability, which forces the ground state spins to assume nonvertical values. To achieve this we put strong positive ($\eta_i > 0$) fields on the boundary of a square and strong negative fields on the boundary of a concentric smaller square. If the fields are very weak inside the box, the spins will form a Bloch wall, rotating gradually from $\theta = \pi/2$ to $\theta = -\pi/2$ (note that with the vertical boundary conditions the ground state spins no longer have to lie in the right half-plane). Since such a local field configuration occurs with a positive probability, the ground state cannot be uniformly vertical. Note, that this argument applies to a weak or strong as well as to a vanishing random field, so that the ground state is never, strictly speaking, field-dominated and always exhibits magnetization in the horizontal direction. However, we expect the magnetization to vanish at $T > 0$, unless the field is small.

To study the system at low T, we need to ask what are the typical low energy excitations from the ground state. For $\epsilon = 0$, continuous symmetry allows Bloch walls, *i.e.* configurations in which the spins rotate gradually over a large region, for instance from left to right. The total excitation energy of a Bloch wall in 2D is of order one, and it is the presence of such walls that underlies absence of continuous symmetry breaking. However, for $\epsilon > 0$, a Bloch wall carries additional energy, coming from changing the direction of the vertical component of the spin, which is proportional to the volume of the wall (which is of the order L^2 for a wall of linear size L in two dimensions), since the ground state spins are adapted to the field configuration, and hence overturning them will increase the energy per site. Similarly, vortex configurations, which are important low-energy excitations in the nonrandom XY model, are no longer energetically favored in the presence of a uniaxial random field.

We are thus left, as possible excitations, with sharp domain walls, where the horizontal component of the spin changes sign rapidly. To first approximation we consider excited configurations, in which spins take their ground state values, or their reflections in the y-axis. As in the standard Peierls argument [11], in the presence of the right boundary conditions, such configurations can be described in terms of contours γ (domain walls), separating right and left spins. If m_i is the value of the horizontal component of the spin at σ_i in the right ground state, the energy of a domain wall is the sum of $m_i m_j$ over the bonds (ij) crossing the boundary of the contour. The Peierls estimate shows that in our approximation probability of such contour is bounded above by $\exp(-2\beta \sum_{(ij)} m_i m_j)$, with $\beta = J/k_B T$.

We want to show that for a typical realization of the field h (*i.e.* with probability equal one), the sum of these probabilities over all contours containing the origin in their interior are summable. It then follows that in a still lower T, this sum is small, and the Peierls estimate proves that the system magnetizes (alternatively, a simple argument shows that summability of the contour probabilities already proves existence of spontaneous magnetization). To show that a series of random variables is summable with probability one, it suffices to prove summability of the series of the expected values. We present two arguments for the last statement to hold.

If the random variables m_i are bounded away from zero, the moment generating function of the random variable $\sum_{(ij)} m_i m_j$ satisfies

$$\mathbf{E}[\exp(-t \sum_{(ij)} m_i m_j)] \leq \exp[-ctL(\gamma)], \tag{3}$$

for some $c > 0$, with $L(\gamma)$ denoting the length of γ. The sum of the probabilities of the contours enclosing the origin is bounded thus by $\sum_\gamma \exp[-c\beta L(\gamma)]$. The standard Peierls-Griffiths bound proves the desired summability.

The above argument does not apply if the distribution of the ground state magnetization contains zero in its support. For unbounded distribution of the random field this may very well be the case, and then another argument is needed. Let us make the very natural assumption that the random variables m_i are weakly dependent, so that $\mathbf{E}[\exp(-2\beta \sum_{(ij)} m_i m_j)]$ behaves as $\exp[-g(\beta)L(\gamma)]$ for a positive function $g(\beta)$ with $g(\beta) \to \infty$ as $\beta \to \infty$. As before, this is enough to carry out the Peierls-Griffiths estimate which implies spontaneous magnetization in the horizontal direction [12].

It is thus expected that the disorder-induced order predicted here will lead to magnetization of order 1 at low tempratures in systems much larger than the correlation length of typical excitations. However, the effect may be obscured by finite size effects, which, due to long-range

power law decay of correlations, are particularly strong in the XY model in 2D. In particular, the 2D-XY model shows finite magnetization in small systems [13] so that the mechanism described here would result in an increase of the magnetization. Our Monte-Carlo simulations [14] in lattices up to 200×200 confirm that this is indeed the case. For example, at $T = 0.7J/k_{\rm B}$, magnetization increases by 1.6% in presence of uniaxial disorder.

The effect may be generalized to other spin models, in particular those that have finite correlation length. Here we list the most spectacular generalizations:

i) 2D Heisenberg ferromagnet in a random field of various symmetry properties. Here the interaction has the same form as in the XY case, but spins take values on a unit sphere. As for the XY model, if the random field distribution has the same symmetry as the interaction part of the Hamiltonian, *i.e.* if it is symmetric under rotations of the three-dimensional space, the model has no spontaneous magnetization up to 4D [1, 8]. If the random field is uniaxial, e.g. oriented along the z axis, the system still has a continuous symmetry (rotations in the xy plane), and thus cannot have a spontaneous magnetization in this plane. It cannot magnetize in the z direction either, by the results of [8]. Curiously enough, a field distribution with an intermediate symmetry may lead to symmetry breaking. Namely, arguments fully analogous to those presented above imply that if the random field takes values in the yz plane with a distribution invariant under rotations in this plane, the system will magnetize in the x direction. We are thus faced with the possibility that planar field distribution breaks the symmetry, which is broken neither by a field with a spherically symmetric distribution nor by a uniaxial one.

ii) 3D XY and Heisenberg model in a random field of various symmetry properties. We have argued that the 2D XY model with a small uniaxial random field orders at low T. Since in the absence of the random field spontaneous magnetization occurs only at $T = 0$, this can be equivalently stated by saying that a small uniaxial random field raises the critical temperature T_c of the system. By analogy, one can expect that the (nonzero) T_c of the XY model in 3D becomes higher and comparable to that of the 3D Ising model, if a small random uniaxial field is turned on. A simple mean field estimate suggests that T_c might increase by factor 2. The analogous estimates for the Heisenberg model in 3D suggest increase of T_c by factor 3/2 (3) in a small uniaxial (planar rotationally symmetric) field.

iii) Antiferromagnetic systems. By flipping every second spin, the classical ferromagnetic models are equivalent to antiferromagnetic ones (on bipartite lattices). This equivalence persists in the presence of a random field with a distribution symmetric with respect to the origin. Thus the above discussion of the impact of random fields on continuous symmetry breaking translates case by case to the antiferromagnetic case.

iv) Quantum systems. All of the above predicted effects should, in principle have quantum analogues. Quantum fluctuations might, however, destroy the long large order, so each of the discussed models should be carefully reconsidered in the quantum case. Some models, such as the quantum spin $S = 1/2$ Heisenberg model, for instance, have been widely studied in literature [15]. The Mermin-Wagner theorem [5] implies that the model has no spontaneous magnetization at positive temperatures in 2D. For $D > 2$ spin wave analysis shows existence of spontaneous magnetization (though a rigorous mathematical proof of this fact is still lacking). In general, one does not expect major differences between the behaviors of the two models at nonzero temperatures. It thus seems plausible that presence of a random field in the quantum case is going to have effects similar to those in the classical Heisenberg model. Similarly, one can consider the quantum Heisenberg antiferromagnet and expect phenomena analogous to the classical case. We note, however that, unlike their classical counterparts, the quantum Heisenberg ferromagnetic and antiferromagnetic systems are no longer equivalent. In fact, the antiferromagnetic system has a stronger tendency to order. It may thus be easier to observe experimentally the effects predicted by us in the antiferromagnetic case.

Further understanding of the phenomena described in this Letter may beneficiate from experimental realizations of the above-mentioned models. Below, we discuss possibilities to design quantum simulators for the quantum spin systems discussed above using ultracold atoms in optical lattices (OL). Consider a two-component Bose gas confined in an OL with on-site inhomogeneities. The two components correspond here to two internal states of the same atom. The low-T physics is captured by the Bose-Bose Hubbard model (BBH) [16]:

$$\begin{aligned} H_{\rm BBH} = \sum_j \Big[\frac{U_{\rm b}}{2}\mathbf{n}_j(\mathbf{n}_j-1) + \frac{U_{\rm B}}{2}\mathbf{N}_j(\mathbf{N}_j-1) \\ + U_{\rm bB}\mathbf{n}_j\mathbf{N}_j \Big] + \sum_j (v_j\mathbf{n}_j + V_j\mathbf{N}_j) \\ - \sum_{\langle j,l\rangle} \Big[\Big(J_{\rm b}\mathbf{b}_j^\dagger\mathbf{b}_l + J_{\rm B}\mathbf{B}_j^\dagger\mathbf{B}_l \Big) + \text{h.c.} \Big] \\ - \sum_j \left(\frac{\Omega_j}{2}\mathbf{b}_j^\dagger\mathbf{B}_j + \text{h.c.} \right) \end{aligned} \tag{4}$$

where $\mathbf{b}_j$ and $\mathbf{B}_j$ are the annihilation operators for both types of bosons in the lattice site j, $\mathbf{n}_j = \mathbf{b}_j^\dagger\mathbf{b}_j^\dagger$ and $\mathbf{N}_j = \mathbf{B}_j^\dagger\mathbf{B}_j^\dagger$ are the corresponding number operators, and $\langle j,l\rangle$ denote a pair of adjacent sites in the OL. In Hamiltonian (4), (i) the first term describes on-site interactions between different types of Bosons with energies $U_{\rm b}$, $U_{\rm B}$ and $U_{\rm bB}$; (ii) the second accounts for on-site inhomogeneities; (iii) the third describes quantum tunneling between adjacent lattice sites and (iv) the fourth

transforms one Boson type into the other with a probability amplitude $|\Omega|/\hbar$. The last term can be implemented with a two-photon Raman process, and can be made site-dependent using inhomogeneous (speckle) laser light [17].

Consider the limit of strong repulsive interactions ($0 < J_{\rm b}, J_{\rm B}, |\Omega_j| \ll U_{\rm b}, U_{\rm B}, U_{\rm Bb}$) and a total filling factor of 1 (*i.e.* the total number of particles equals the number of lattice sites). For vanishing $J_{\rm b}$, $J_{\rm B}$ and Ω, the eigenstates of $H_{\rm BBH}$ are Fock states of the form $\prod_j |n_j, N_j\rangle$. The low-energy spectrum in the absence of tunneling is very similar to that of Fermi-Bose mixtures, analyzed recently by two of the authors in [18], and we only briefly summarize here the situation focusing on differences between the Fermi-Bose and Bose-Bose cases. The similarity is due to the large value of $U_{\rm B}$ that makes double occupancy of B bosons in a single site energetically unfavourable and this mimics the Pauli principe. The low-energy states correspond to $n_j + N_j = 1$ at each lattice site j. These form a manifold of 'ground states' $\mathcal{E}_0$ of energy width $|V_j - v_j|$ separated from a manifold of first excitations $\mathcal{E}_1$ by a typical energy $(U_{\rm b}, U_{\rm B}, U_{\rm bB})$. Assuming $|V_j - v_j|, k_{\rm B}T < U_{\rm b}, U_{\rm B}, U_{\rm bB}$, we can restrict the Hilbert space to $\mathcal{E}_0$. The tunneling terms are then incorprated via perturbation theory as in the Fermi-Bose case [18]. We thus derive an effective Hamiltonian, $H_{\rm eff}$, that describes the dynamics of *composite Schwinger bosons*, annihilated by the operators $\mathcal{B}_j = \mathbf{b}_j^\dagger \mathbf{B}_j \mathcal{P}$, made of one B boson and one b hole, with $\mathcal{P}$ the projector onto $\mathcal{E}_0$.

Since the commutation relations of $\mathcal{B}_j, \mathcal{B}_j^\dagger$ are those of Schwinger bosons [19], we may directly turn to the spin representation [19] by defining $\mathbf{S}_j^x + i\mathbf{S}_j^y = \mathcal{B}_j$ and $\mathbf{S}_j^z = 1/2 - \mathcal{N}_j$, where $\mathcal{N}_j = \mathcal{B}_j^\dagger \mathcal{B}_j$. For small inhomogeneities ($\delta_{j,l} = v_j - v_l, \Delta_{j,l} = V_j - V_l \ll U_{\rm b}, U_{\rm b}, U_{\rm bB}$), Hamiltonian $H_{\rm eff}$ is then equivalent to the anisotropic Heisenberg XXZ model [19] in a random field:

$$H_{\rm eff} = -J_\perp \sum_{\langle j,l\rangle} \left(\mathbf{S}_j^x \mathbf{S}_l^x + \mathbf{S}_j^y \mathbf{S}_l^y\right) - J_z \sum_{\langle j,l\rangle} \mathbf{S}_j^z \mathbf{S}_l^z - \sum_j \left(h_j^x \mathbf{S}_j^x + h_j^y \mathbf{S}_j^y + h_j^z \mathbf{S}_j^z\right) \quad (5)$$

where $J_\perp = 4J_{\rm b}J_{\rm B}/U_{\rm bB}$, $J_z = 2[2J_{\rm b}^2/U_{\rm b} + 2J_{\rm B}^2/U_{\rm B} - (J_{\rm b}^2 + J_{\rm B}^2)/U_{\rm bB}]$, $h_j^x = \Omega_j^{\rm R}$, $h_j^y = -\Omega_j^{\rm I}$, $h_j^z = \mathcal{V}_j - \zeta J_z/2$, with ζ the lattice coordination number, $\mathcal{V}_j = V_j - v_j + \zeta[4J_{\rm b}^2/U_{\rm b} + 4J_{\rm B}^2/U_{\rm B} - (J_{\rm b}^2 + J_{\rm B}^2)/U_{\rm bB}]$ and $\Omega_j = \Omega_j^{\rm R} + i\Omega_j^{\rm I}$. In atomic systems, all these terms can be controlled almost at will [18, 20]. In particular, by employing various possible control tools one may reach the limiting ferromagnetic Heisenberg ($J_\perp = J_z$), or XY ($J_z = 0$) cases.

The quantum ferromagnetic XY model in random field may be alternatively obtained using the same BBH model, but with strong state dependence of the optical dipole forces. One can imagine a situation in which one component (say b) is in the strong interaction limit, so that only one b atom at a site is possible, whereas the other (B) component is Bose condensed and provides only a coherent 'background' for the b-atoms. Mathematically speaking this situation is described by Eq. (4), in which n_i's can be equal to 0 or 1 only, whereas B_i's can be replaced by a classical complex field (condensate wave function). In this limit the spin $S = 1/2$ states can be associated with the presence, or absence of a b-atom in a given site. In this way, setting $v_j = 0$ and $\Omega_j^{\rm I} = 0$, one obtains the quantum version of the XY model (1) with $J = J_{\rm b}$ and a uniaxial random field in the x direction with the strength determined by $\Omega_j^{\rm R}$.

Finally, the $S = 1/2$ antiferromagnetic Heisenberg model may be realized with a Fermi-Fermi mixture at half filling for each component. This implementation might be important for future experiments with Lithium atoms. As recently calculated [21], the critical temperature for the Néel in 3D is of order of 30nK. According to our (classical) estimates, it can be increased to $\simeq$ 45 (90)nK by placing the system in a uniaxial (planar) random field.

We acknowledge T. Roscilde for discussions and the support of DFG (SFB 407, SPP 1116), ESF Programme QUDEDIS, and Spanish MEC Grant FIS2005-04627. J.W. thanks ICFO for hospitality and support in the summer of 2004.

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