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A NEW PROCEDURE FOR MULTI-INPUT DYNAMIC SYSTEMS IDENTIFICATION

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Abstract

One of the major problem for the identification of industrial multi-input multi-output (MIMO) systems is the too small variance of some signals collected during several experiments. Usually, sufficient information exist but are often dispersed among many data-files. In this case, it is difficult to conciliate this information to build a single well adapted model. The ALS procedure (Alternate Least Squares) has been performed in order to solve this kind of problem. In the first part of this paper, the authors present the principle of the ALS procedure ; the details of the complete algorithm are given in part two. Then, based on some examples, a comparison of the Prediction Error Method (PEM) with our new method is presented.

Key words : identification, multi-inputs system.

1. Introduction

During the last twenty years, the identification of continuous and discrete MIMO systems has been the subject of many works of research. A brief description of the main methods as well as the contribution of the proposed method are presented below.

Let us consider a system with r inputs and s outputs. It can be decomposed into s multi-input/single-output (MISO) systems. Each MISO system comprises r single-input/single-output (SISO) subsystems ; the output of a subsystem, also called partial output or intermediate output, is linked to the corresponding MISO system input by the relation :

$$y_{ij}(t) = \frac{B_{ij}(q^{-1})}{A_{ij}(q^{-1})} u_j(t-k_j) \quad i = 1, \dots, s \quad j = 1, \dots, r \quad (1)$$

where k_j are the delays, $A_{ij}(q^{-1})$ and $B_{ij}(q^{-1})$ are constant polynomials and q^{-1} represents the backward shift operator ($q^{-1} f(t) = f(t-1)$) :

$$A_{ij}(q^{-1}) = 1 + a_{ij1} q^{-1} + \dots + a_{ijm_{ij}} q^{-m_{ij}} \quad (2a)$$

$$B_{ij}(q^{-1}) = b_{ij0} + b_{ij1} q^{-1} + \dots + b_{ijn_{ij}} q^{-n_{ij}} \quad (2b)$$

The outputs of the MIMO system are then described by the following equation :

$$y_i(t) = \sum_{j=1}^r \frac{B_{ij}(q^{-1})}{A_{ij}(q^{-1})} u_j(t-k_j) \quad i = 1, \dots, s \quad (3)$$

To identify the parameters of the SISO system, several methods have been developed. The main one is, without any doubt, the Least Squares (LS) method whose the basic principles have been established by Gauss in 1809 [8]. However, this method gives a biased estimator if the matrix observation composed by the measures are correlated with the residuals. Therefore, several methods have been proposed to solve this problem

among which we can cite the Generalised Least Squares (GLS) method presented by Clark [2]. The least squares estimation of the transfer function parameters of process and noise, using a single-stage estimator, was developed by Talmon and Der Boom [15]. The two least squares estimation and their recursive approximation were proposed by Pandya and Pagurek [13]. The approximation on-line algorithm for maximum likelihood identification of linear dynamic system, proposed by Furht and Carapic [7], based on the application of quasilinearization technique. The instrumental matrix methods whose one of them is proposed by Young [17] based on the substitution of the output measurement by the simulated outputs of an auxiliary model and those proposed by Banon [1] using the instrumental matrix delayed observation.

All of these methods enable to identify the mono-variable systems easily. Their extension to the MISO case is possible under the condition that we use the common denominator reduction principle of the transfer functions of the subsystems. Therefore, several methods give solutions to this problem. Among them, we mention :

- The DU method, proposed by Diekmann and Unbehauen [3,4], whose algorithm is composed by two identification phases. The first one consists in the reduction on the common denominator of the transfer functions of the subsystem and in the identification of the obtained parameters. The second phase consists in estimating the intermediate outputs using the parameters found in the first phase and identifying the actual parameters using the estimated intermediate outputs.

- The GDU method presented by Rao, Dieckmann and Unbehauen [14], based on the intermediate outputs estimation.

- The PEM (Prediction Error Method) which the presentation has been done by Ljung [11].

Therefore, considerable attention has been paid in the literature to the parameter estimation problem of MIMO system. Various methods have been applied towards the solution of the problem. Gauthier and Landau [9] discussed the recursive identification of MIMO system using a normalized input-output description in terms of polynomial matrices straight for wardly related to a canonical state space realization. A system reference adaptive model approach has also been suggested by Unbehauen and Rao [16]. For more details, a brief survey is presented by Mukhopadhyay and al [12].

However, these methods give insufficient results in the case where the collected information is disseminated on several measurement campaigns that cannot be directly assembled. With the object to bring a solution to this problem, we have developed the present method. In order to avoid the over parametrisation of the form which uses the reduction to the common denominator of the transfer functions, we start from Fkirin's idea [6] and those of Kamoun and Titli [10], which consists to decompose the MIMO system in MISO subsystems and then to estimate the partial outputs of the MISO models after decomposing them in single-

input, single-output (SISO) models. The knowledge of these partial outputs enable to identify the corresponding transfer function SISO models.

The paper is organized as follows. In the first part, we present the general principle of the method and its algorithm ; the details of the complete algorithm are given in part two. Then, based on some examples, a comparison of the Prediction Error Method (PEM) with our method is presented.

2. Presentation of the method

The ALS technique enables MIMO systems, which may be split into MISO systems, to be identified. Such systems can be represented in block diagrams (figure 1).

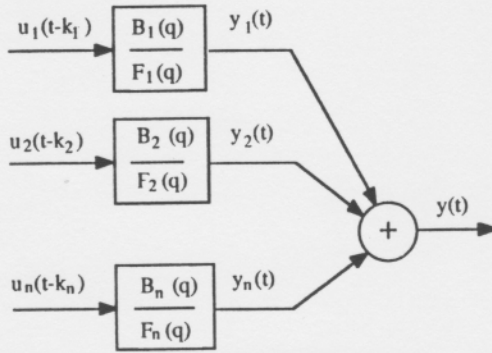


figure 1 : model structure

The transfer function of each subsystem is described by the following polynomials :

$$B_i(q^{-1}) = \sum_{j=0}^{m_i} b_{ij} q^{-j} \quad (4a)$$

$$F_i(q^{-1}) = 1 + \sum_{j=1}^{n_i} f_{ij} q^{-j} \quad (4b)$$

The relation between the n inputs $u_i(t)$ and the global output $y(t)$ is :

$$y(t) = \frac{B_1(q)}{F_1(q)} u_1(t - k_1) + \frac{B_2(q)}{F_2(q)} u_2(t - k_2) + \dots + \frac{B_n(q)}{F_n(q)} u_n(t - k_n) \quad (5)$$

where the k_n are the different transfer-function delays. The partial output are defined by :

$$y_i(t) = \frac{B_i(q)}{F_i(q)} u_i(t - k_i) \quad i = 1, \dots, n \quad (6)$$

Using the expression of the partial outputs, the global output is written as well by :

$$y(t) = y_1(t) + y_2(t) + \dots + y_n(t) \quad (7)$$

The expression (5) of the system output $y(t)$ with respect to the different parameters f_{ij} and b_{ij} is non-linear. To be able to apply a simple method of estimation, we have to remove the coupling that exist between the different subsystems due to the fact that the sum of the intermediate outputs is known and equal to $y(t)$. This remark is the origin of the method that we propose. The principle is based on the following condition : if the intermediate output $y_i(t)$ of the subsystem "i" is known, the estimation of the parameters f_{ij} and b_{ij} of the transfer function of this subsystem is easily achieved. As the intermediate outputs are not measured, they must be estimated from the inputs ; besides, the sum of all these outputs has to be the same as the complete model output $y(t)$. A two levels relaxation algorithm enables to estimate gradually the intermediate outputs : indeed, if (n-1) intermediate outputs are known then the last may be estimated from the constraint (7).

3. The identification algorithm

Based on the preceding two levels relaxation scheme, the whole algorithm is now detailed. The scheme of the algorithm is shown on figure 2. So that we can dispose of a certain number of degrees of freedom, the constraint is changed into the following form :

$$y(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t) + \dots + \alpha_n y_n(t) \quad (8)$$

In this relaxation, the parameters α_i play the role of intervention parameters that enable to modify the local identification of each subsystem guaranteeing the respect of the constraint (4).

Before the first iteration 1, the algorithm begins with a first static regression between the inputs and the output :

$$y(t) = \alpha_1^0 u_1(t - k_1) + \alpha_2^0 u_2(t - k_2) + \dots + \alpha_n^0 u_n(t - k_n) \quad (9)$$

The T-values (ratio of each coefficient regression and its standard deviation) of this regression are computed and stored. Then, to improve the static model (9), the method proposes to replace each parameter α_i^0 by a transfer function, the parameters of which are estimated as follows : each iteration consists first in building a temporary variable $y^0(t)$ regarded as an estimation of the current partial output. We have now to choose an order for the variables to be processed ; this can be done for example by using the decreasing T-values order as a criterion. At iteration 1, suppose that $y_1(t)$ is the first output to be calculated (because the t-value for α_1^0 is the most expressive :

$$y_1^0(t) = y(t) - \alpha_2^0 u_2(t - k_2) - \dots - \alpha_n^0 u_n(t - k_n) \quad (10)$$

Considering that $y_1^0(t)$ and $y_1(t)$ are similar, the knowledge of $y_1^0(t)$ and $u_1(t - k_1)$ allows us to identify $B_1(q)$ and $F_1(q)$ by using the SISO linear model (6). $B_1^1(q)$ and $F_1^1(q)$ are the first estimates of $B_1(q)$ and $F_1(q)$. Then we can compute $y_1^1(t)$, the first estimate of $y_1(t)$:

$$y_1^1(t) = \frac{B_1^1(q)}{F_1^1(q)} u_1(t - k_1) \quad (11)$$

Since initial values of partial outputs are unknown at this point, $y_1^1(t)$ is computed with null initial values.

The second part of iteration 1 is a static regression using the general output $y(t)$ and the estimated partial outputs $y_i(t)$; at this time, only $y_1(t)$ has been computed :

$$y(t) = a_1^1 y_1^1(t) + a_2^1 u_2(t - k_2) + \dots + a_n^1 u_n(t - k_n) \quad (12)$$

This regression ends with iteration 1. At iteration 2, ALS reestimates $y_1(t)$ using the parameters α_i^1 , which are better estimations of static gains than the parameters α_i^0 . ALS algorithm computes $y_1(t)$ until that the correlation coefficient stops increasing. Then ALS computes $y_2(t)$. For instance, if the correlation coefficient doesn't change after iteration 1, and of the second partial output which must be computed is $y_2(t)$, then the temporary variable $y_2^0(t)$, regarded as an estimation of this partial output, is defined by :

$$y_2^0(t) = y(t) - \alpha_1^1 y_1^1(t) - \alpha_3^1 u_3(t - k_3) - \dots - \alpha_n^1 u_n(t - k_n) \quad (13)$$

Since $B_2^2(q)$ and $F_2^2(q)$ are computed from equation (2) using $y_2^0(t)$ and $u_2(t - k_2)$; it is now possible to generate $y_2^2(t)$:

$$y_2^2(t) = \frac{B_2^2(q)}{F_2^2(q)} u_2(t - k_2) \quad (14)$$

At each iteration, only one partial output is reestimate while the others keep their preceding value. thus we have :

$$y_1^2(t) = y_1^1(t) \quad (15)$$

At the end of procedure, the last correlation coefficient computed from regression (9) is stored ; then, after each partial output has been reestimated, this correlation coefficient is compared to the current one. When no more substantial change occurs, the processing is stopped.

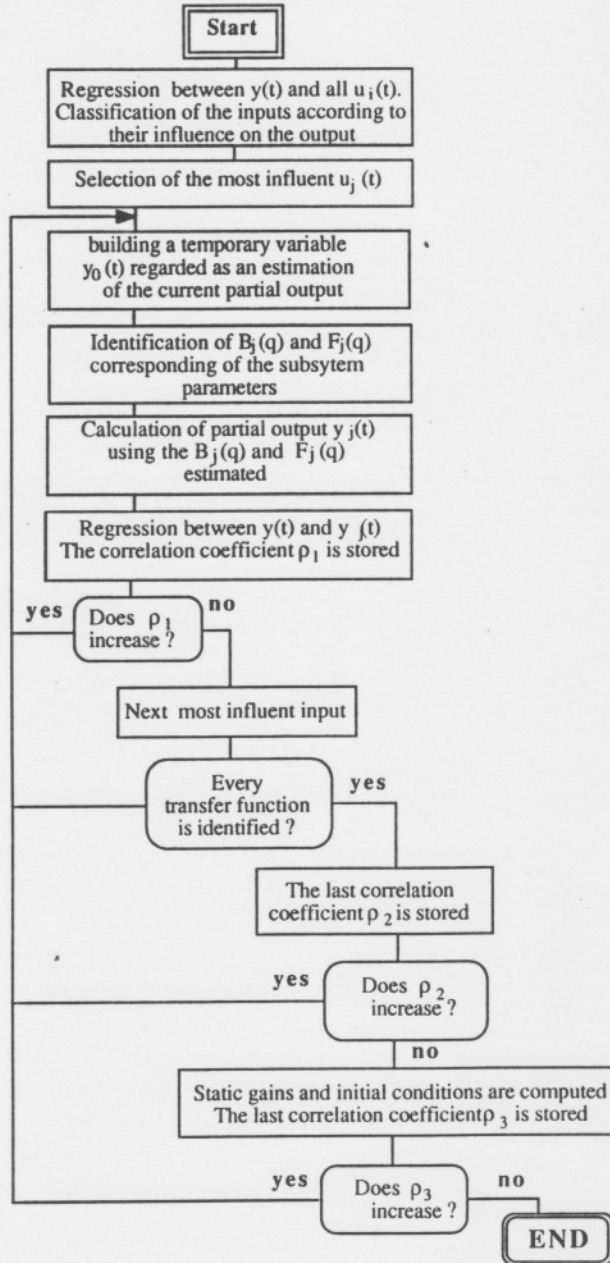


Figure 2 : organigram of the ALS

One of the advantage of the ALS technique gives the possibility of using information from many campaigns. The concatenation of these ones in a single file of measurements let some signal discontinuities appear ; indeed, the value of a variable at the beginning of a measurement campaign is not, usually, equal to the value of this variable at the end of the preceding campaign. To take account easily for these discontinuities, we use a recursive identification procedure of the parameters. Let's assume the algorithm treats the intermediate output number "i". At first, we identify b_{ij} and f_{ij} using the RLS method and the observations of the first measurement campaign.

Then, we identify once again these parameters on the whole of the other measurement campaign. The identification algorithm used in the i^{th} measurement campaign will be initialised by the vector composed by the parameters obtained at the end of the $(i-1)^{\text{th}}$ measurement campaigns. The final result which will be kept is the one obtained at the end of the last campaign when a vector parameters estimation is made. We can build an estimation of $y_i(t)$ on the whole of the campaign (equation (6)). The unknown initial conditions of $y_i(t)$ lay down a problem during its estimation. A way to rise above this difficulty consists to work on the measurement campaigns whose the first observations are constant values. In this case, the relation that links the input $u(t-k_i)$ to the output $y_i(t)$ is a proportionality relation, then the initial value of the $y_i(t)$ can be calculated by multiplying the initial values of the inputs by static gains. Static gains are computed after a first step (identification phases) using null initial values. A second step using these initial values contributes improving the precision of the parameters.

4. First example

This is a simulation example where the model structure is known :

$$y(t) = \frac{b_1}{1 + f_1 q} u_1(t) + \frac{b_2}{1 + f_2 q + f_3 q^2} u_2(t) + \frac{b_3}{1 + f_4 q} u_3(t) \quad (16)$$

and where the parameter values are :

$$\begin{aligned} b_1 &= 1 & f_1 &= -0.9 \\ b_2 &= 1 & f_2 &= -1.5 \\ b_3 &= 0.5 & f_3 &= 0.7 \\ & & f_4 &= -0.95 \end{aligned}$$

The output $y(t)$ and the three corresponding inputs $u_i(t)$ are shown on figure 3. As this is a simulation example, the partial outputs $y_i(t)$ are also known and they are represented on figure 4. In this example, the output is free-noise.

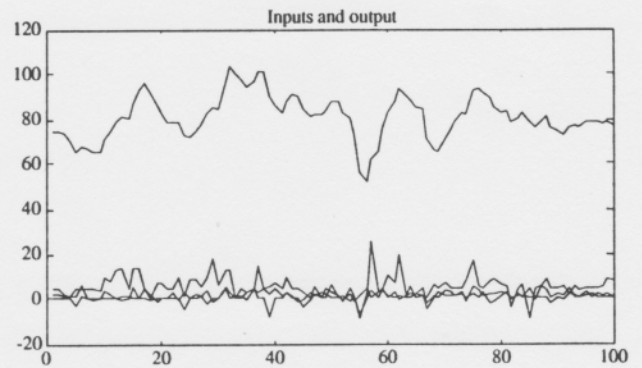


Figure 3

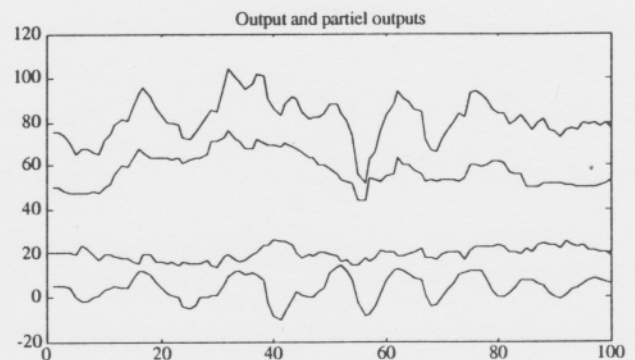


Figure 4

Figure 5 shows the comparison between the output and the modeled output after the different steps, and the corresponding residues. At the first step, the model is calculated using null initial conditions. steps 2, 3, 4 and 5 use at each time the initial conditions computed with the parameters given by the preceding step ; there is a perfect agreement between the measured output and the model output as the last step.

The Prediction Error Method proposed by MATLAB software (in Identification-Toolbox) has been tested on the same data and in the same conditions, and the simulation of the model, which was been poorly identified is also shown on figure 5.

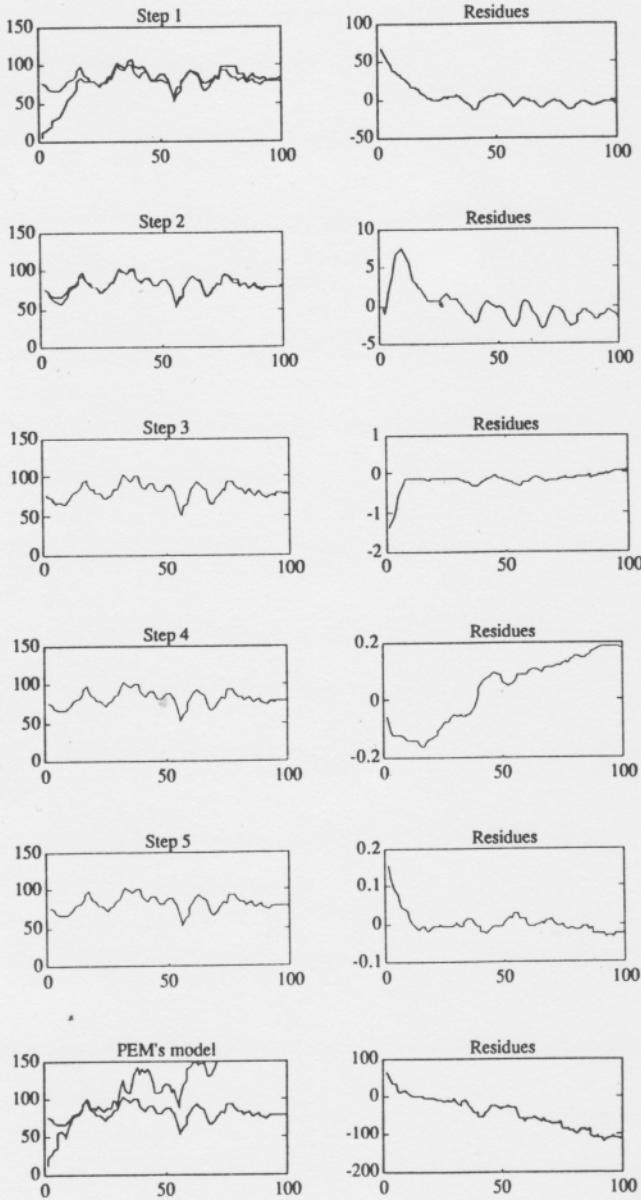


Figure 5

Table 1 represents the values of the parameters. At each step (the first column), this table gives the corresponding values of the parameters (column 2 to 8). The last column contains the correlation coefficient between the measured output and the model output. The line 6 contains the exact values and line 7 contains the parameters identified by PEM.

step	b1	b2	b3	f1	f2	f3	f4	ρ
1	0.918	0.917	0.574	-0.908	-1.527	0.589	-0.931	0.643
2	0.975	0.992	0.506	-0.902	-1.501	0.679	-0.950	0.974
3	0.996	1.005	0.499	-0.895	-1.499	0.698	-0.951	0.999
4	0.999	1.003	0.499	-0.899	-1.499	0.699	-0.950	0.999
5	1.000	1.000	0.500	-0.900	-1.500	0.700	-0.950	1.000
6	1.000	1.000	0.500	-0.900	-1.500	0.700	-0.950	
7	3.465	1.258	0.775	-0.879	-1.207	0.207	-0.780	0.246

Table 1 : Parameter values (ρ is the correlation coefficient)

5. Second example

The model and the parameters are the same as the preceding example. The only difference is that the output is now added up with a white noise (standard deviation $\sigma=1$). Figure 6 shows that in spite of difficult conditions (output with noise and very few observations) the procedure does still not diverge. The parameter values are stored in table 2.

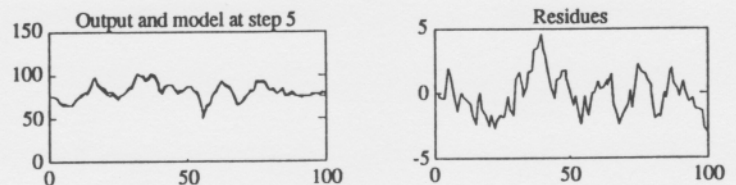


Figure 6

step	b ₁	b ₂	b ₃	f ₁	f ₂	f ₃	f ₄	ρ
1	0.414	0.821	0.577	-0.916	-1.438	0.502	-0.935	0.640
2	0.463	0.914	0.524	-0.935	-1.465	0.614	-0.950	0.954
3	0.471	0.899	0.514	-0.961	-1.458	0.652	-0.945	0.998
4	0.478	0.922	0.516	-0.926	-1.462	0.647	-0.953	0.989
5	0.471	0.903	0.515	-0.960	-1.457	0.651	-0.944	0.989
6	1.000	1.000	0.500	-0.900	-1.500	0.700	-0.950	

Table 2

6. Third example

This is still a simulation example and the model structure is the same :

$$y(t) = \frac{b_1}{1 + f_1 q} u_1(t) + \frac{b_2}{1 + f_2 q + f_3 q^2} u_2(t) + \frac{b_3}{1 + f_4 q} u_3(t) \quad (17)$$

The parameter values are now :

$$\begin{aligned} b_1 &= 1 & f_1 &= -0.9 \\ b_2 &= 0.5 & f_2 &= -1.5 \\ b_3 &= 2 & f_3 &= 0.7 \\ & & f_4 &= -0.95 \end{aligned}$$

This file is composed of a first recording from observation 1 to 50, and a second from observation 51 to 100. The model and the coefficients are the same for the two recordings, but the data are different ; the initial conditions of the second recording are different of the final conditions of the first one. The algorithm is informed that there is a discontinuity at the observation 50, because it is known that the data have been obtained by the concatenation of two records. The output is free-noise. The data are represented in figure 7 (due to the scale factor, only the output clearly appears on this figure).

The results of identification are represented in figure 8. We remark that the discontinuity is progressively taken into account. At the step 3, the discontinuity between the two records is perfectly appreciated. In table 3 are stored the values of the parameters at each step. For step 1, the change at time 50 has not been taken into account ; this will be done in the following steps.

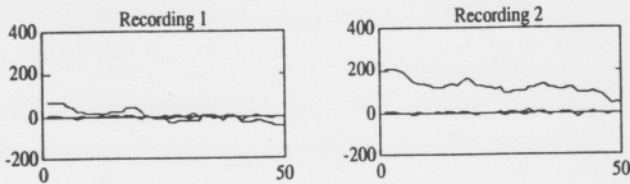


Figure 7

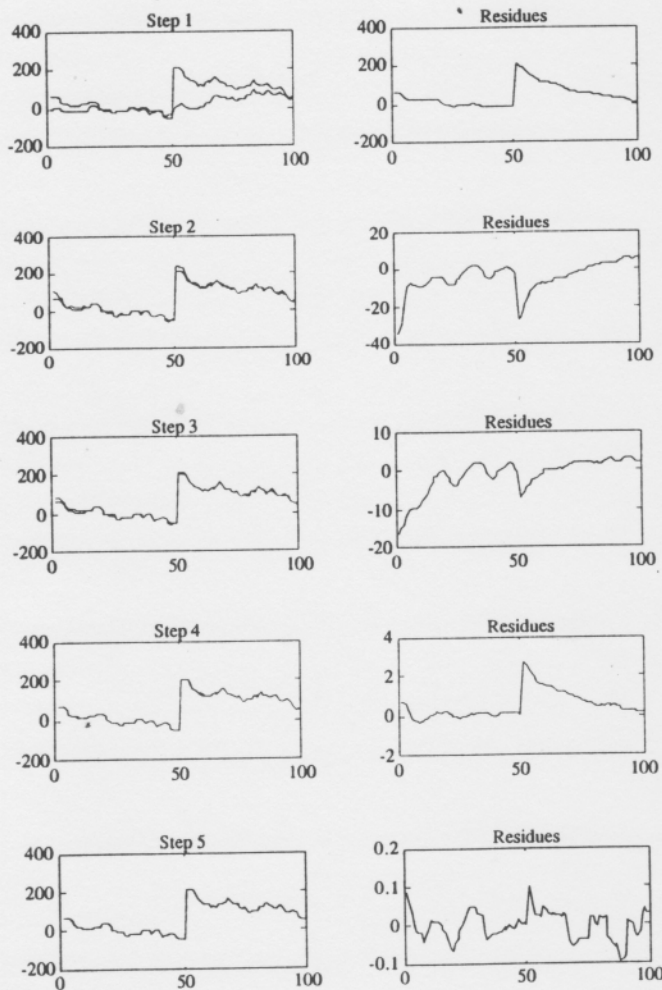


Figure 8

step	b ₁	b ₂	b ₃	f ₁	f ₂	f ₃	f ₄	ρ
1	0.806	0.430	1.687	-0.933	-1.152	0.207	-0.960	0.680
2	1.001	0.516	2.074	-0.947	-1.337	0.559	-0.948	0.998
3	0.969	0.513	2.063	-0.907	-1.418	0.642	-0.948	1.000
4	0.987	0.502	2.004	-0.900	-1.497	0.698	-0.949	1.000
5	1.000	0.500	1.999	-0.900	-1.500	0.699	-0.950	1.000
6	1.000	0.500	2.000	-0.900	-1.500	0.700	-0.950	

Table 3

7. Conclusion

This paper introduces a new identification procedure that is well adapted to take into account information arising out of different files ; but even when working on a single file, the results obtained are comparable to those of the classical methods. This procedure has been performed on many simulation cases and on industrial processes, and we noticed only a few cases of divergence occurrences compared to classical methods.

However, some problems probably require further research. For example, the model is only deterministic and does not include the influence of the noises acting on the process.

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