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Statistical Matrix Representation Of Time-Varying Electrical Signals. Reconstruction And Prediction Applications.

V. Ignatova, Z. Styczynski, *Senior Member, IEEE*, P. Granjon, S. Bacha

Abstract— Power system currents and voltages magnitudes are time variant due to continual changes in system configuration and load conditions. This paper deals with the statistical description of measured electrical signals. A matrix representation is chosen in order to preserve the information about the temporal evolution of the recorded signal. Two matrix forms are investigated: transitions probabilities (Markov) matrix and transitions number matrix. Their performance is further analyzed in the paper by investigating two of their applications – reconstruction and prediction. In deed, the availability of the information about the time evolution of the recorded data can be used to restore the original signal from its corresponding matrix form. Another possible application is the forecasting of the electrical signals behavior in the future. Both applications are illustrated on measurement data acquired from a real power network.

Index Terms—data storage, Markov processes, matrix methods, power quality, power system harmonics, signal prediction, signal reconstruction, statistics, time varying systems

I. INTRODUCTION

THE time-varying nature of currents and voltages is well known and ever present in power systems. It is mainly due to the variability of non-linear loads with varying operating point, or well to linear loads with fixed operating conditions, when switching on and off to the grid.

Many recent research interests are focused on the non stationary behavior of electrical signals and especially on the time-varying nature of the harmonics, an important aspect of the power quality. As the FFT algorithm is not accurate in the harmonic estimation in case of random variations, different techniques for time-varying harmonics assessment have been proposed in the literature: wavelet transform [1], neural network [2], Min norm method and Wigner-Ville distribution [9]. Other recent publications are focused on the probabilistic

harmonic analysis, the harmonic summation and propagation in systems with multiple non linear loads [3,4]. Revision of standards is proposed [10] or even made [13] including probabilistic limits for time-varying harmonic currents and voltages. The application of the actual steady-state harmonic distortion limits to non stationary harmonics is also investigated in [11].

Another subject of research interests is the representation in statistical terms of recorded data showing time-varying distortions. The simplest approach describes recorded data by statistical measures: minimum value, maximum value, mean value and standard deviation [5]. More appropriate methods for statistical representation of a set of measurements are the probability density function and the probability distribution function. The probability density function indicates the frequency of occurrence of the recorded signal values in a vector form. Its accuracy can be improved by considering the signal as a sum of deterministic and random component [6]. The probability distribution function is the integral of the probability density function. It provides the same information and has the same advantages and drawbacks as the probability density function.

The vector form representation is an easy and efficient way to describe random behavior of electrical signals. However, information about time evolution of the recorded data is completely lost. In order to take it into account, a matrix description of the recorded signal should be applied.

This paper deals with the statistical matrix representation of time-varying electrical signals. Two matrix forms are investigated. The first one is the transition probability matrix, which terms represent the probability that the signal passes from one value to another. This matrix is also known as Markov matrix and is already applied in case of non stationary harmonics [7]. The second one is the transitions number matrix, which represents the number of times that the signal has passed from one value to another. The main advantage of the matrix representation with respect the previous vector form is that it contains information about the temporal structure of the recorded signal, which can be exploited to reconstruct this signal and to forecast its future behavior.

This paper is organized as follows. Section II deals with the statistical matrix representation of recorded data. The derivation of the probability density function and the classical statistic measures from both matrices is also described.

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Sections III and IV present two applications of the statistical matrix representation: signal reconstruction and signal prediction. In section III a signal is stored and then reconstructed from the Markov matrix, the state transitions matrix and the probability density function. In the three cases, the results are presented and analyzed. In section IV two methods for signal prediction are applied and the results are discussed.

II. MATRIX REPRESENTATION OF MEASURED DATA

In this section recorded data are statistically described by Markov matrix and transitions number matrix. Both matrices are defined and their computation is given in details. Then, the probability density function and the most important statistical measures are derived from both matrices.

A. Matrices definitions

The matrix of transition probabilities describes the behavior of Markov chains and for that reason is also called Markov matrix. Each element in this matrix represents the probability of transition from a particular state (the matrix row index) to the next state (the matrix column index). Being probabilities, the elements of the Markov matrix take values between 0 and 1. The sum of the probabilities in each row is exactly 1, because from anyone state the system either remains in this state or moves to one of the others:

$$M = [p_{ij}] \quad 0 \leq p_{ij} \leq 1 \quad \sum_i p_{ij} = 1 \quad (1)$$

An alternative of the Markov matrix is the transitions number matrix, which elements, as its name indicates, represent the numbers of transitions between the different states. The elements of the transitions number matrix are always positive or zero:

$$R = [r_{ij}] \quad r_{ij} \geq 0 \quad (2)$$

B. Matrices estimation

The two previous matrices are easy to compute from successive data. In this section their computation is described and illustrated with an example.

1) Transitions number matrix

The transitions number matrix can be derived from the recorded data by increasing in each state transition the corresponding matrix element with an increment. The computational process is shown for the three-states system presented in fig.1, where states are denoted by S and the number of transitions for state i to state j by r_{ij} . When the data vector is achieved, an additional increment is added to the term corresponding of the transition between the last state and the first one. In deed, it is experimentally proved that this operation increases the accuracy of the matrix in its reconstruction and prediction applications. The elements r_{ij} are arranged in a matrix form R ; the size of the matrix is determined by the number of signal states (values).

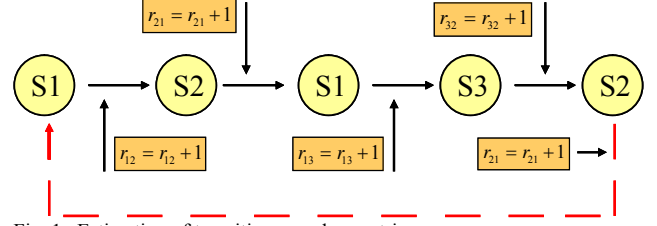


Fig. 1. Estimation of transitions number matrix

2) Markov matrix

The estimation technique applied for the Markov matrix is described in [8]. First the number of times r_{ij} that the signal has moved from state i to state j is calculated and arranged in a matrix form R as previously explained. Then, the probability of transition from state i to state j is estimated by dividing each term r_{ij} by the sum of the elements in the i -th row:

$$p_{ij} = \frac{r_{ij}}{\sum_{j=1}^n r_{ij}}, \quad (3)$$

where n is the states number.

3) Example of matrices computation

An example of the previous matrices computation is given in this paragraph. The recorded signal consists of the first voltage harmonic's magnitude acquired at one point of a real power network. The sampling period is 10 min and 144 samples are available, which corresponds to a duration of 24 hours (fig.2).

The recorded signal takes values from 227.536V to 237.346V during the 24 hours. Considering only its integer values, the signal is characterized by 10 states: $x = \{228 \ 229 \ 230 \ 231 \ 232 \ 233 \ 234 \ 235 \ 236 \ 237\}$, the non integer signal values being rounded. The size of the matrices is determined from the number of states, here 10x10. Better accuracy can be achieved if a more important number of states is considered, but the size of the matrices will increase and more memory will be required.

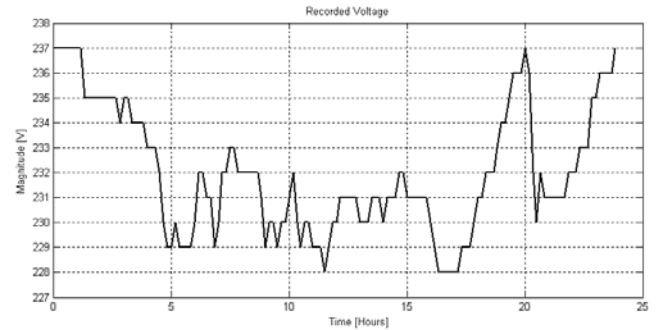


Fig.2 Recorded signal

The computation of the matrices is realized as previously explained. Their structures are graphically presented in fig.3.

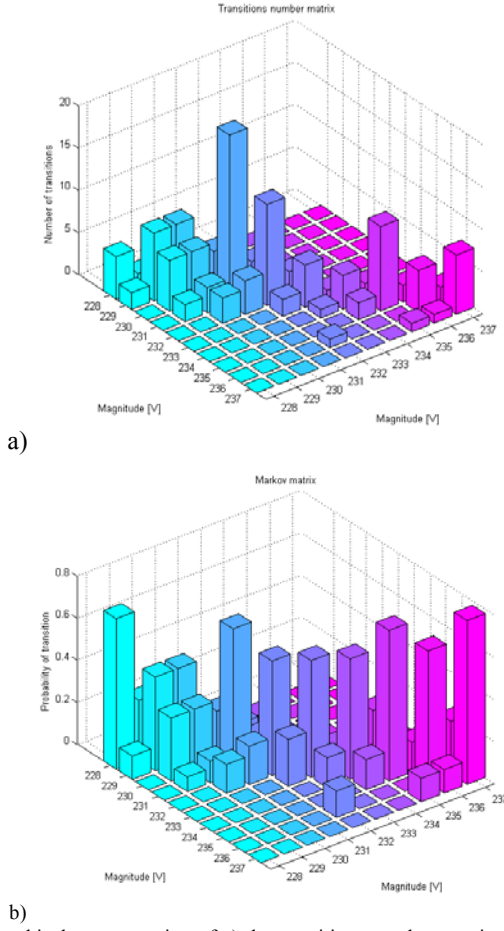


Fig.3 Graphical representation of a) the transitions number matrix and b) the Markov matrix

C. Available information from the statistical matrices

The universal methods for data storage can be derived from the transition matrices. The determination of the probability density function p_i from both matrix forms is presented in (4).

$$p_i = \frac{\sum_{j=1}^n M(i, j)}{n} \quad p_i = \frac{\sum_{j=1}^n R(i, j)}{\sum_{i=1}^n \sum_{j=1}^n R(i, j)} \quad (4)$$

By knowing the vector of signal states x_i and the probability density function p_i , the statistical measures: mean $E(x)$ and standard deviation σ can be calculated:

$$E(x) = \sum_{i=1}^n p_i x_i \quad (5)$$

$$\sigma = \sqrt{\sum_{i=1}^n p_i (x_i - E(x))^2} \quad (6)$$

The information about the probability or the frequency of occurrence of the transitions between the states can be used to reconstruct the signal and to forecast its future evaluation, as described in the next two sections.

In addition to the information provided by classical methods of statistical data storage, the statistical matrices take into

account the temporal evolution of the signal. Their structure is relevant for the signal variations: if most part of the matrix elements are situated on or near the main diagonal, the signal is characterized by slow variations. On the contrary, if the main matrix elements are not localized close to the main diagonal, the signal magnitude is characterized by sudden and strong variations. Concerning the signal presented in fig. 2, the corresponding matrices (fig.3) have almost a diagonal structure, which shows that the signal varies slowly.

The statistical matrices represent an efficient and interpretable way to store recorded data without loss of important information. The information about the probability or the frequency of occurrence of the transitions between the states can be used to reconstruct the signal and to forecast its future evaluation, as described in the next two sections.

III SIGNAL RECONSTRUCTION

In this section, recorded data are first described by probability density function, Markov matrix and transitions number matrix. Secondly, these three statistical quantities are used to reconstruct the original signal and their performance is compared and discussed.

A Algorithms

As the probability density function does not contain information about the time distribution of the recorded data, the reconstruction of the signal using this quantity is realized by generation of random numbers having the corresponding probability distribution.

The signal reconstruction using the transitions number matrix begins from an arbitrary-chosen matrix term. Every following signal state is derived from the last one and the matrix element on the corresponded row containing the highest transitions number. For every reconstructed point, the matrix term used for its determination decreases by an increment equal to 1. The described algorithm is the opposite of the one used for the transition numbers matrix estimation shown in fig.1.

The algorithm of signal reconstruction using Markov matrix is analogous to the one applied in the case of transitions number matrix. The reconstruction of the stored signal starts from the term with the highest probability. After each point determination, the matrix term employed for the reconstruction decreases by an increment value $\frac{N_{st}}{N_{sa}}$, where N_{st} is the states number and N_{sa} is the samples number of the stored signal.

B Results

The wave-forms of real and reconstructed signals are compared in fig.4 and their corresponding probability density functions are shown in fig.5. The deviations between real and reconstructed signals in the three cases are presented in Table 1 by relative errors in the wave forms, in the probability density functions and in the statistical measures (mean value and variance). In order to compare the dynamics of the

different signals, another important parameter is introduced in Table 1: the number of state changes.

The reconstructed signal from the probability density function is random and does not have the same dynamics as the real signal. The deviation between the two wave forms is important. However, the reconstructed signal has very similar probability density function and statistical measures than the real signal.

The reconstructed signal from the Markov matrix has a wave form similar to the wave form of the real signal, but it doesn't have the same probability density function. It is due to the fact that the terms of Markov matrix represent the probability that the system passes from one state to another, but they do not provide information about the frequency of occurrence for the different signal states. The deviation between the statistical measures of real and reconstructed signals is also important.

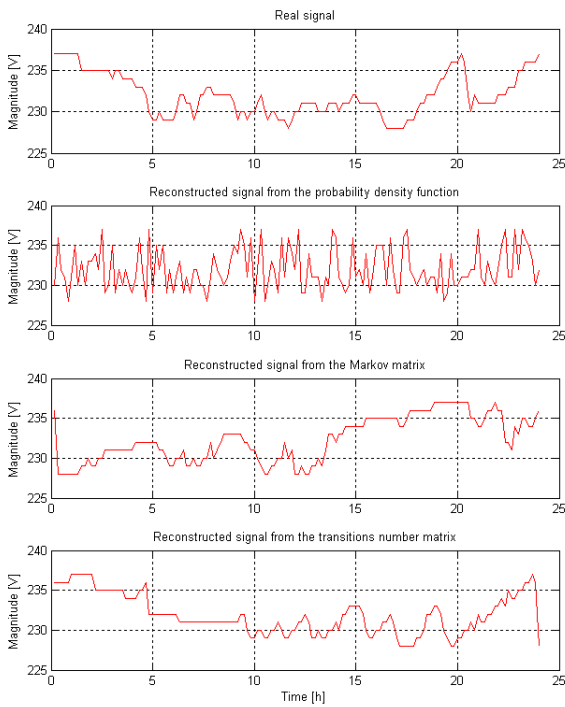


Fig. 4 Real and reconstructed signals using the probability density function, the Markov matrix and the transitions number matrix

In the signal reconstruction the transitions number matrix combines the advantages of Markov matrix and probability density function. The restored signal has the same dynamics as the real signal and very similar probability density function and statistical measures.

The performance of the transitions number matrix can also be analyzed thanks to Table 1, where the results from the three signal reconstruction methods are compared. The signal reconstructed from the transitions number matrix has minimal errors in the wave-form as well as in the probability density function and almost the same dynamics as the real signal.

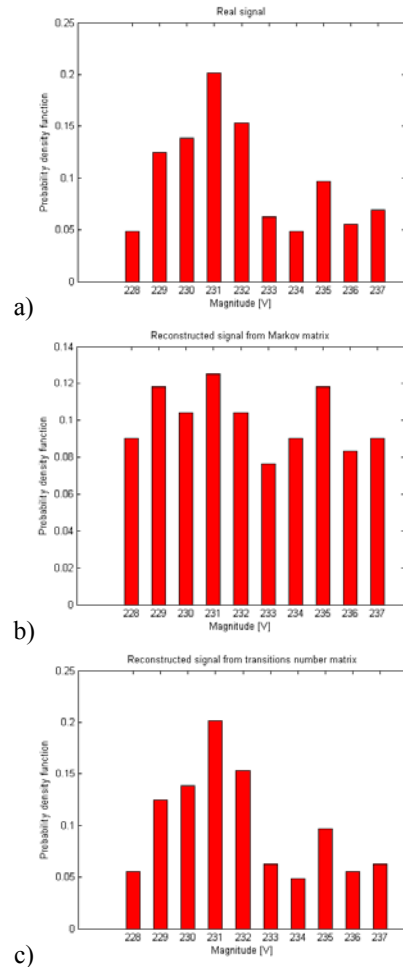


Fig. 5 Probability density function for: a) the real signal b) the reconstructed signal from Markov matrix c) the reconstructed signal from transitions number matrix

TABLE I
ERRORS IN THE RECONSTRUCTION FROM PROBABILITY DENSITY FUNCTION,
MARKOV MATRIX AND TRANSITIONS NUMBER MATRIX

	Real signal	Reconstructed signal		
		Probability density function	Markov matrix	Probability matrix
Minimal value	228	228	228	228
Maximal value	237	237	237	237
Average value	231.972	232.2222	232.4236	231.9097
Mean relative error for the wave form [%]	-	1.17	1.09	0.63
Mean relative error for the prob. distribution [%]	-	2.73	39.4	3.44
Number of states changes (dynamics)	62	144	61	63

IV SIGNAL PREDICTION

Classical signal prediction methods give usually good results, but only for few time steps in the future. They are usually based on the correlation function of the signal (linear prediction, Kalman filter) and give worse results after certain number of time steps, when the correlation disappears. Markov probabilities are also applied in time series prediction [12], but only for real time forecasting, where the originally

forecast values are updated or modified as measured data become available.

Power system harmonics prediction is a subject of interest only if an important number of samples are predicted. In this section, transitions matrices are applied to forecast the harmonics future behavior in a long term.

The prediction of a large number of samples from the presented in this paper transitions matrices is investigated. A stochastic and a deterministic approaches based on the transitions matrices are proposed. Both methods are applied in the case of Markov matrix, the prediction from transitions number matrix being analogous.

The deterministic approach is similar to the method used for signal reconstruction. The prediction of the signal begins from the last measured point of the real signal. Every following signal state is determined from the last state and the term with high probability on the corresponded row. After each signal point prediction, a new matrix is computed, decreasing by an increment the matrix term used for the last signal point generation. The value of the increment may vary in order to obtain better results.

In the stochastic approach the signal prediction is effectuated by a generation of random variables with Gaussian probability distribution. Every next state is found by a generation of a random number with Gaussian probability distribution corresponding of the previous state. In other terms, by supposing that the signal is in the state i , the next state j is determined by:

$$x_j = \sigma_i \text{randn}(1) + \mu_i, \text{ where} \quad (8)$$

randn : function generating random numbers with normal distribution with mean zero, variance 1 and standard deviation 1

μ_i mean value for the state i

σ_i standard deviation for the state i

One of the advantages of the stochastic approach is that it does not need a new matrix computation after each point determination. Although, an important error may be induced due to the fact that the signal is supposed to have a Gaussian distribution in anyone of its states, which is not always valid.

The results obtained from the deterministic and the stochastic approach are presented in fig.6 and fig.7 respectively. In the chosen example, 1 hour worth data are used, the sampling time is 6 seconds. The signal behavior is predicted for 1 hour (600 points).

As it can be seen from table II, the deterministic method gives better results than the stochastic one. In both cases the predicted signal is closer to the real signal for small periods of time.

The methods applied for signal prediction from Markov matrix can be used in the case of transitions number matrix. In fact, the transitions number matrix may be reduced to a Markov matrix using equation (3), which allows the application of the same prediction techniques as in the case of Markov matrix.

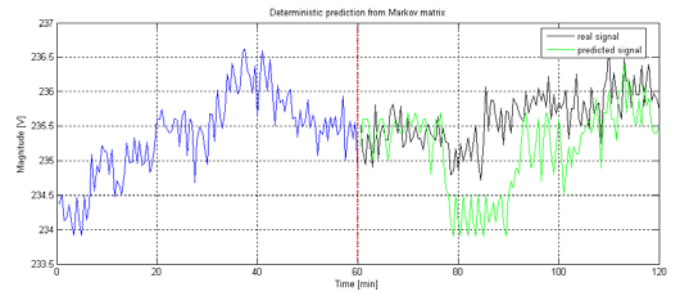


Fig.6 Signal prediction by deterministic approach

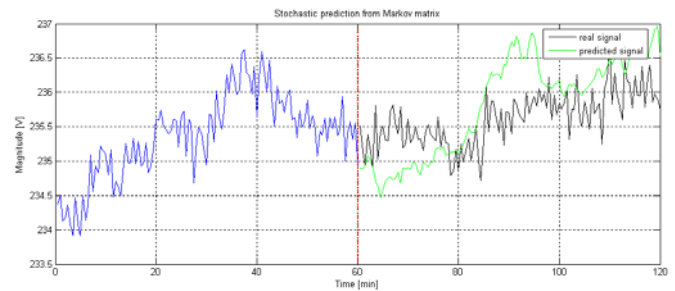


Fig.7 Signal prediction by stochastic approach

TABLE II
MEAN RELATIVE ERROR IN THE SIGNAL PREDICTED FROM THE DETERMINISTIC AND THE STOCHASTIC APPROACH

Approach	Mean relative error for	
	300 time steps	600 time steps
Deterministic	0.16 %	0.18 %
Stochastic	0.23 %	0.27 %

V CONCLUSION

The objective of the statistical description is to compress a large volume of data and to present it into a compact and easy to exploit form without losing important information.

The Markov matrix and the transitions number matrix present an efficient way to store the recorded data. In addition to the usual methods for data storage, these statistical matrices take into account the temporal evolution of the signal, which allows the restitution of the stored signal and the prediction of its future behaviour. They can be successfully applied for the statistical description of power quality disturbances like power system harmonics, voltage variations and voltage dips.

The use of transitions number matrix is recommended, because it gives better results in the signal reconstruction. Moreover, it can be easily reduced to a Markov matrix, the inverse process is not realizable.

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VII BIOGRAPHIES

Vanya Ignatova was born in Sofia, Bulgaria in 1979. She received her Engineer degree from the Technical University in Sofia in 2002 and her Master degree from the National Polytechnic Institute of Grenoble in 2003. Currently, she is a Ph. D. student in the Laboratory of Electrical Engineering of Grenoble, France. Her main research interests are focused on power quality and especially on voltage sags, power system harmonics and flickers.

Zbigniew Styczynski (M’1994, SM’2001) was born in Wroclaw, Poland. He studied at the Technical University of Wroclaw and got his PhD there in 1977. He finished his professorial dissertation in 1985 at the TH Wroclaw for which he received a special award from the Polish Ministry of Higher Education. From 1991 until 1999 he worked at the Technical University of Stuttgart, Germany. Since 1999 he holds the chair Electric Power Networks and renewable Energy Sources of the Faculty of Electrical Engineering and Information Technology at the Otto von Guericke University of Magdeburg, Germany. His special field of interest includes electric power network and systems, expert systems and optimization problems.

Pierre Granjon was born in Issoire, France, in 1971. He received the M.S. in electrical engineering from the Centre Universitaire des Sciences et Techniques (CUST), Clermont-Ferrand, France, in 1994 and the Ph.D. degree from the Institut National Polytechnique de Grenoble (INPG), France in 2000. He joined the Laboratory of Images and Signals (LIS) at INPG in 2001, where he holds a position as assistant professor. His general interests cover signal processing theory such as nonlinear signals and filters (higher order statistics, volterra filters), nonstationary signals and filters (cyclostationarity, LPTV filters) and active control. His current research is mainly focused on signal processing applications in electrical engineering such as fault diagnosis in electrical machines and power networks.

Seddik Bacha received his Engineer and Magister from Ecole Nationale Polytechnique of Algiers respectively in 1982 and 1990. He joined the Laboratory of Electrical Engineering of Grenoble (LEG) and received his PhD and HDR respectively in 1993 and 1998. He is presently manager of Power System Group of LEG and Professor at the University Joseph Fourier of Grenoble. His main fields of interest are power electronics systems, modeling and control, power quality, renewable energy integration.