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APPLICATION OF SPECTRAL KURTOSIS TO BEARING FAULT DETECTION IN INDUCTION MOTORS

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Abstract

This paper deals with the application of the Spectral Kurtosis (SK) to bearing fault detection in asynchronous machines. This one-dimensional spectral measure allows to study the nature of the harmonic components of the stator current of an induction motor running at a constant rotation speed. It provides additional information with respect to second order quantities given by the Power Spectrum Density (PSD). This information can be used to discriminate between constant amplitude harmonics, time-varying amplitude harmonics and noise. Since the harmonic components for a healthy machine can be considered as constant amplitude harmonics, the SK can provide a measure of the distance of the analyzed machine from a healthy one. For example, the amplitude fluctuations of the harmonic components produced by a mechanical fault can be highlighted by the Spectral Kurtosis.

Keywords

Asynchronous Machines, Stator Current, Harmonic Components, Spectral Measures, Higher Order Statistics, Spectral Kurtosis, Bearing Fault Detection.

I. Introduction

The higher order statistics have been an extensive field of research in the past few years [1,2]. These works lead to several analysis tools, complementary to classical second order methods. One useful tool is the fourth-order cumulant based kurtosis, providing a measure of the distance to gaussianity. In the frequency domain, the Spectral Kurtosis (SK) of a signal is defined as the kurtosis of its frequency components. It was initially defined and used to detect “randomly occurring signals” in [3,4]. In this work, the kurtosis of the real and imaginary parts of the frequency components of signals are estimated separately. This allows to deal with real variables, but leads to an incomplete definition and estimation. In [5,6], the SK approach is generalized by using the frequency components modulus. Unfortunately, the authors give neither theoretical definition nor properties and use only a moment based estimator instead of the cumulant based one. Moreover, they present the SK only in Gaussian ambient noise context. In discrete time, the correct theoretical definition of SK, using cumulants of complex random variables, was finally given in a source separation context [7,8]. In this work, the SK is used to measure the distance to gaussianity of different spectral components, but a biased estimator is provided. Finally, its continuous time definition is given in [9] and an unbiased estimator is provided and studied in detail in [10].

In this paper we propose to study the nature of the harmonic components of random “mixed” processes made up of a sum of complex harmonics buried in noise (boldface letters represent terms with a random nature):

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{n}(t) = \sum_{k=1}^K \mathbf{A}_k e^{j2\pi f_k t} + \mathbf{n}(t) \quad (1)$$

where the following assumptions are satisfied:

A1 The frequencies f_k are real deterministic constants, with $f_i \neq f_j$ when $i \neq j$.

A2 The amplitudes $\mathbf{A}_k = \mathbf{a}_k e^{j\phi_k}$ are complex random variables, with moduli \mathbf{a}_k and phases ϕ_k mutually independent real random variables. Moreover, the phases ϕ_k are supposed to be uniformly distributed over $[0, 2\pi)$. Thereby, we can infer that, whatever k , the \mathbf{A}_k 's are circular [11] and $\mathbf{s}(t)$ is a stationary random harmonic process.

A3 $\mathbf{n}(t)$ is a stationary mixing process, i.e. its multicolorrelations are absolutely summable [1, pp. 8]. Moreover, $\mathbf{n}(t)$ is independent of the harmonic process $\mathbf{s}(t)$.

Such processes have “mixed spectra” in the sense that they consist of infinite lines superimposed on a bounded continuous spectrum [1, pp. 173], [12].

The next section is devoted to theoretical definition and properties of the SK. Main results concerning its estimation are exposed in section III. These analytical results are illustrated in section IV.1 by applying this tool on a synthetic example. Results on current stator signals are presented in the last parts of section IV.

II. Definition and properties

II.1. Spectral Kurtosis (SK) definition

In order to study the nature of the harmonic components of the random mixed process defined by Eq. (1), the complex amplitudes \mathbf{A}_k have to be characterized. These amplitudes being random, their statistical properties are given by their cumulants of order $p + q$ defined by [13,14]:

$$C_{A_k(p)}^{(q)} = \text{Cum}[\underbrace{A_k, \dots, A_k}_p, \underbrace{A_k^*, \dots, A_k^*}_q] \quad (2)$$

where $*$ denotes the complex conjugate.

Moreover, A_k being circular (assumption **A2**), its cumulants verifying $p \neq q$ vanish [11]. Then, until the 5th order, the two possibly non-null cumulants of A_k are the variance¹ $C_{A_k(1)}^{(1)} = E[|A_k|^2]$ and the fourth order cumulant $C_{A_k(2)}^{(2)} = E[|A_k|^4] - 2E^2[|A_k|^2]$. Therefore, the random variable A_k can be completely characterized until the 5th order by its variance $C_{A_k(1)}^{(1)}$ and its kurtosis \mathcal{K}_{A_k} defined as:

$$\mathcal{K}_{A_k} = C_{A_k(2)}^{(2)} / \left(C_{A_k(1)}^{(1)} \right)^2 \quad (3)$$

The main advantage of this last quantity is due to its normalization. Actually, \mathcal{K}_{A_k} is only dependent on the stochastic nature of the random variable A_k .

It is well known that $C_{A_k(1)}^{(1)}$ can be obtained through the classical Power Spectrum Density (PSD). We will see in the following that \mathcal{K}_{A_k} can be obtained thanks to the SK of $\mathbf{x}(t)$, defined at each frequency ν as [9]:

$$\mathcal{K}_x(\nu) = \lim_{T \rightarrow +\infty} \frac{\text{Cum}[X_T(\nu), X_T(\nu), X_T^*(\nu), X_T^*(\nu)]}{\left(\text{Cum}[X_T(\nu), X_T^*(\nu)] \right)^2} \quad (4)$$

where $X_T(\nu)$ is a complex random variable given by the Fourier transform of $\mathbf{x}(t)$ observed on a finite duration T :

$$X_T(\nu) = \int_{-\frac{T}{2}}^{\frac{T}{2}} \mathbf{x}(t) e^{-j2\pi\nu t} dt \quad (5)$$

This truncation ensures the existence of the Fourier transform of stationary random mixed processes.

II.2. Theoretical properties

It has been shown in [9] that for processes $\mathbf{x}(t)$ given by Eq. (1), the cumulant-based SK defined in Eq. (4) verifies:

$$\mathcal{K}_x(\nu) = \begin{cases} \mathcal{K}_{A_k} & \nu = f_k, k = 1, \dots, K \\ 0 & \text{elsewhere} \end{cases} \quad (6)$$

On the one hand, the SK gives the kurtosis \mathcal{K}_{A_k} of the complex amplitude A_k when the frequency parameter ν equals the frequency f_k of a harmonic component. On the other hand, when $\nu \neq f_k$, the SK vanishes completely. Thereby, the SK is theoretically insensitive to any stationary mixing process, since its value is completely independent of the noise characteristics.

Information accessible through the SK can be further studied by assuming that A_k takes the following form:

$$A_k = a_k e^{j\phi_k} = (C_k + V_k) e^{j\phi_k} \quad (7)$$

¹ $E[\cdot]$ denotes mathematical expectation

where C_k is a deterministic real constant and V_k a zero mean real random variable. In this case, it is well known that the PSD contains only second order information, more precisely related to $C_k^2 + E[V_k^2]$. As for the SK, it becomes [9]:

$$\mathcal{K}_x(f_k) = \frac{\mathcal{K}_{V_k} + 4S_{V_k}\sqrt{\mathcal{R}_k} + 2 - (1 - \mathcal{R}_k)^2}{(1 + \mathcal{R}_k)^2} \quad (8)$$

where \mathcal{K}_{V_k} , S_{V_k} are the kurtosis, respectively, the skewness of V_k , and \mathcal{R}_k represents the amplitude Signal to Noise Ratio (SNR) given by:

$$\mathcal{R}_k = \frac{C_k^2}{E[V_k^2]} \quad (9)$$

Fig. 1 represents $\mathcal{K}_x(f_k)$ as a function of \mathcal{R}_k , for three different random variables V_k .

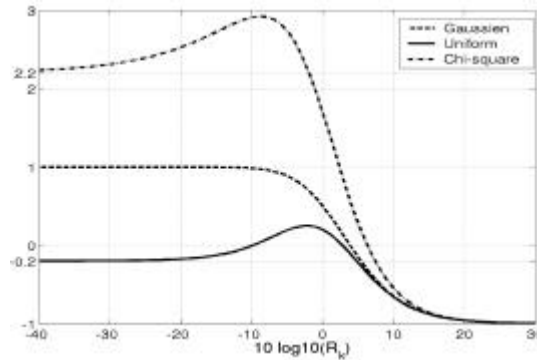


Figure 1: $\mathcal{K}_x(f_k)$ for 3 different random variables V_k

The two interesting cases are the following ones:

$\mathcal{R}_k \rightarrow +\infty$: The amplitude modulus of the harmonic component becomes nearly constant since $a_k \rightarrow C_k$. Eq. (8) simplifies to $\mathcal{K}_x(f_k) \simeq -1$, which is verified in Fig. 1. Actually, the three curves tend toward -1 whatever V_k .

$\mathcal{R}_k \rightarrow 0$: In this case $a_k \rightarrow V_k$, therefore the amplitude modulus of the harmonic component has a completely random nature. Eq. (8) reduces to $\mathcal{K}_x(f_k) \simeq \mathcal{K}_{V_k} + 1$ and the value of the SK becomes dependent on the kurtosis of V_k , and hence on its probability density function. This is verified in Fig. 1 as well.

The previous properties of the SK lead to the following conclusions for random mixed processes:

- if $\mathcal{K}_x(\nu) = -1$, $x(t)$ contains a harmonic component with a constant amplitude modulus at this frequency,
- if $\mathcal{K}_x(\nu) \neq 0$ or $\neq -1$, $x(t)$ contains a harmonic component of which the amplitude modulus has a random part,
- if $\mathcal{K}_x(\nu) = 0$, the spectral content of $x(t)$ at frequency ν is given by either a mixing process (see Eq. (6)), or a harmonic component with a \mathcal{R}_k for which the relation (8) vanishes. In this last case, the discrimination between a mixing process and a harmonic component is made by looking at the PSD of the random mixed process.

Thanks to these additional information, the SK can be viewed as a complementary tool with respect to the classical PSD.

III. Estimation

In order to estimate the SK, we firstly consider that the random mixed process defined by Eq. (1) is discretized, giving $\mathbf{x}(n)$. Moreover, we suppose that this discrete random process is known over N samples, denoted $\mathbf{x}_N(n)$. In this case of finite length random process, the SK defined in Eq. (4) becomes:

$$\mathcal{K}_{\mathbf{x}}^N(m) = \frac{\text{Cum}[\mathbf{X}_N(m), \mathbf{X}_N(m), \mathbf{X}_N^*(m), \mathbf{X}_N^*(m)]}{(\text{Cum}[\mathbf{X}_N(m), \mathbf{X}_N^*(m)])^2} \quad (10)$$

where $\mathbf{X}_N(m)$ is a complex random variable given by the N -points Discrete Fourier Transform (DFT) of $\mathbf{x}_N(n)$.

In the following, we suppose that the discretization was made in such a way that one can consider the frequency bins m_k approximatively equal to f_k/F_s , where $k = 1, \dots, K$ and F_s is the sampling frequency.

Considering the complex amplitudes of harmonic components modeled by Eq. (7), we can define a local Signal to Noise Ratio (SNR_l), at each frequency bin m_k , as:

$$SNR_l(m_k) = N \frac{C_k^2 + E[V_k^2]}{\gamma_m(m_k)} \quad (11)$$

where $\gamma_m(m)$ is the PSD of the noise part. This local signal to noise ratio depends on the observation length N , and can be measured on the PSD of $\mathbf{x}_N(n)$, since for finite length random mixed processes there are no infinite lines associated with harmonic components.

At frequency bins m_k where harmonic components are present, it can be easily shown that the SK given by Eq. (10) becomes:

$$\mathcal{K}_{\mathbf{x}}^N(m_k) = \frac{\mathcal{K}_{V_k} + 4S_{V_k} \sqrt{\mathcal{R}_k} + 2 - (1 - \mathcal{R}_k)^2}{(1 + \mathcal{R}_k)^2} \left(\frac{SNR_l(m_k)}{1 + SNR_l(m_k)} \right)^2 \quad (12)$$

The first term of this equation is the same as in Eq. (8). The second one provides a lower bound of the SK value. Thereby, having the $SNR_l(m_k)$ information given by the PSD of $\mathbf{x}_N(n)$, we can determine if the harmonic component at the frequency bin m_k has a constant amplitude modulus or not (see the first example detailed in section IV.1).

For all frequency bins where no harmonic components are present, i.e. $m \neq m_k$ with $k = 1, \dots, K$, $\mathbf{X}_N(m)$ tends toward a zero mean *Gaussian* circular complex random variable [1, pp. 94-98] as $N \rightarrow \infty$. Hence, the SK vanishes at these frequency bins [10]. That is, $\mathcal{K}_{\mathbf{x}}^N(m) = 0$, for all bins $m \neq m_k$.

Let $x_N^1(n), \dots, x_N^M(n)$ be M realizations of the random process $\mathbf{x}_N(n)$ known over N samples, and $X_N^1(m), \dots, X_N^M(m)$ their N -points DFT. These M complex quantities $X_N^i(m)$ represent M realizations of the complex random variable $\mathbf{X}_N(m)$.

In this case, an unbiased estimator of the SK is given by [10]:

$$\hat{\mathcal{K}}_{\mathbf{x}}^N(m) = \frac{M}{M-1} \left[\frac{(M+1) \sum_{i=1}^M |X_N^i(m)|^4}{\left(\sum_{i=1}^M |X_N^i(m)|^2 \right)^2} - 2 \right] \quad (13)$$

The variance of this estimator, at each frequency bin m where only the noise part is involved, is [10]:

$$\text{Var}\{\hat{\mathcal{K}}_{\mathbf{x}}^N(m)\} = \frac{4M^2}{(M-1)(M+2)(M+3)} < \frac{4}{M} \quad (14)$$

IV. Applications

IV.1. Synthetic signal

We consider first a synthetic random process composed of:

- two pure sine waves of frequencies 0.07 and 0.18 ,
- a sine wave of frequency 0.33 with random amplitude modulus following a zero mean Gaussian law,
- a white Gaussian noise filtered by a resonant system, with an important resonance at frequency 0.24 and a small low-pass one.

The PSD (shown in Fig. 2) and the SK (shown in Fig. 3) has been estimated over $M = 2000$ realizations of this random process, each of them known over $N = 512$ samples. In this case, the PSD has been estimated by using the averaged periodogram method and the SK by Eq. (13).

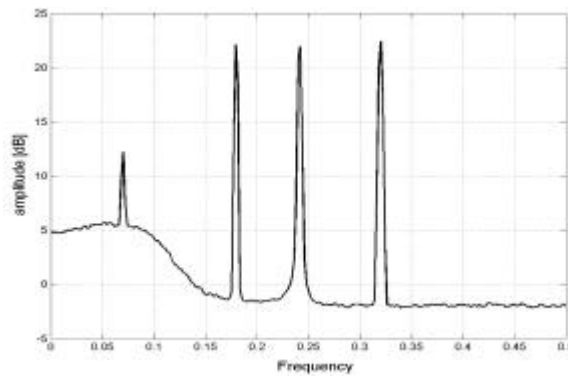


Figure 2: PSD of the synthetic random process

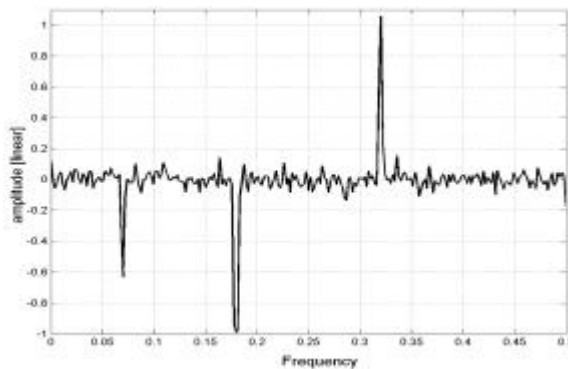


Figure 3: SK of the synthetic random process

As expected, the SK estimator is a function of the nature of the spectral components. The values obtained for the harmonic components of frequencies 0.07 , 0.18 and 0.33 verify Eq. (12). For example, the estimated PSD gives a SNR_l of about 6.5 dB for the component of frequency 0.07 . This component having a constant amplitude modulus, $\mathcal{R}_k \rightarrow +\infty$ and its SK takes the theoretical value -0.66 , which corresponds to the estimated one in Fig. 3. Hence, using the SNR_l information given by the PSD jointly with the SK, we infer that the sine waves of frequencies 0.07 and 0.18 has constant amplitude moduli, contrary to the harmonic component of frequency 0.33 . At the other frequency bins, the SK tends to zero with a variance verifying Eq. (14), even for the important resonance at frequency 0.24 . This property can be used to discriminate spectral components induced by resonances from the harmonic components contained in the process.

IV.2. Real signal – artificial bearing fault

Bearing faults are the most frequent faults in induction machine according to the IEEE motor reliability study [15]. In the case of large motors, this kind of fault are usually detected by analyzing vibration signals, but this technique is not economically realistic in the case of small motors. A solution to this problem can be the use of quantities that are already measured in a drive system, e.g. the machine's stator currents. Recently, Blödt et al. have proposed a new model for bearing fault detection using stator currents, which shows that such faults produce very small amplitude and frequency modulations of the stator current harmonics [16]. In this case, stator currents can be viewed as a sum of several harmonics with time-varying complex amplitudes, buried in noise. In section II.2, the SK has been shown to be dependant on the nature of the complex amplitudes of the analyzed harmonics. Therefore, it can be used to detect the modulations induced in the stator currents by the bearing faults.

This second example deals with the stator currents of a small induction machine (1.1 kW) running at a constant rotation speed. Two cases are considered here: in the first case, the motor was healthy, and in the second case, the front bearing was replaced with a bearing having an artificial inner raceway defect (a hole).

The stator current signals were sampled during 75 seconds, with a sampling frequency of 16 kHz. Choosing $N = 1500$ samples, $M = 800$ unoverlapped blocks was obtained. The value of N was chosen small in order to include all information into the alimentation harmonics, and to obtain a small variance of the SK estimates. Only one stator current signal for each case is considered here.

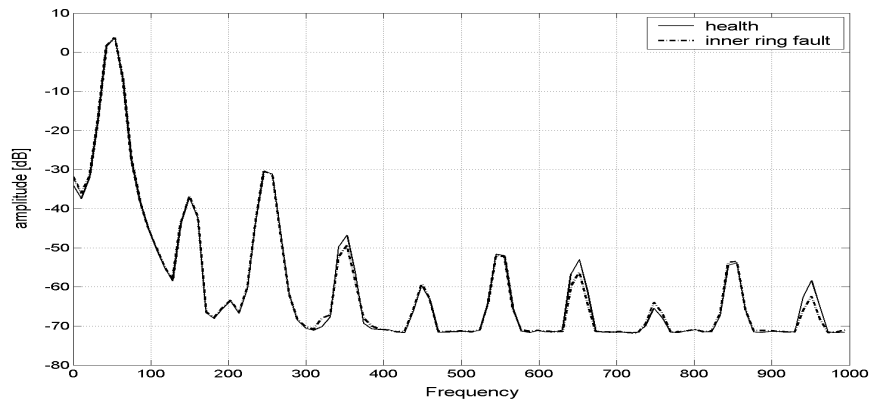


Figure 4: PSD of a healthy machine respectively a machine with an inner raceway defect

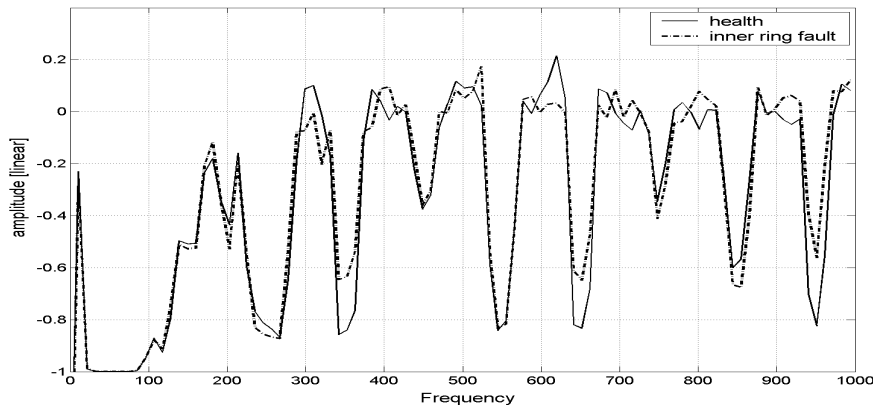


Figure 5: SK of a healthy machine respectively a machine with an inner raceway defect

The Power Spectral Densities (Fig. 4) were estimated using the Welch's modified periodogram. For the sake of simplicity, the studied frequency range is limited below 1000 Hz. No important differences between the two considered cases appear in these PSD.

In the case of the SK, the M unoverlapped blocks are considered as M realizations of a finite length discrete random mixed process $\mathbf{x}_N(n)$, as defined in section III. Following this idea, the two SK (Fig. 5) were estimated with the same parameters as for the PSD. The variance of these estimates is $5 \cdot 10^{-3}$ (see Eq. (14)). These SK show significant differences, especially at the 7th, 13th and 19th harmonics of the alimentation (350 Hz, 650 Hz, respectively 950 Hz). These differences are not due to the weakening of the SNR_i . For example, at 350 Hz, a difference of 2 dB can be observed in the PSD (Fig. 4), which means a variation of about $5 \cdot 10^{-3}$ for the SK supposing that the amplitude of this harmonic is constant (see Eq. (12)). At 950 Hz, the difference of 4 dB in the PSD means a variation of about 0.12 for the SK. As we can see in Fig. 5, the differences between the two SK are greater than those caused by the SNR_i variation. Hence, the SK is capable of highlighting the amplitude fluctuations of these harmonic components induced by the bearing fault.

IV.2. Real signal – realistic bearing fault

In the third example, a healthy motor was compared to a motor for which the front bearing was replaced by industrially used bearings. As in the previous example, only one stator current is analyzed thanks to its PSD and its SK. The results obtained with the same parameters as in section IV.1 are shown in Fig. 6 and 7.

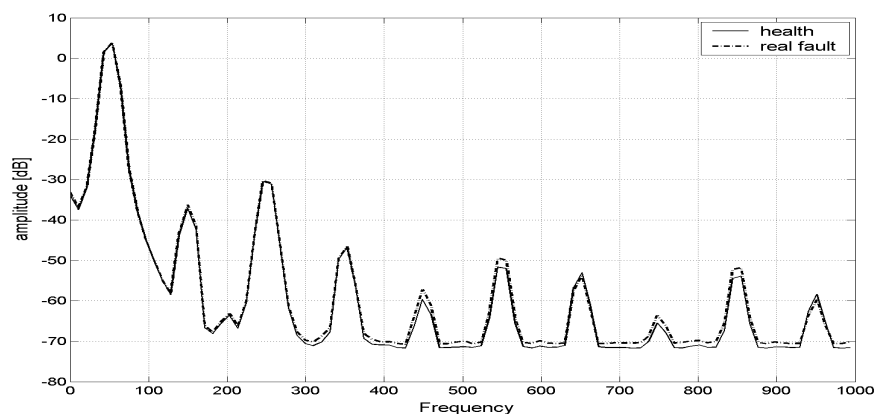


Figure 6: PSD of a healthy machine respectively a machine with a realistic bearing fault

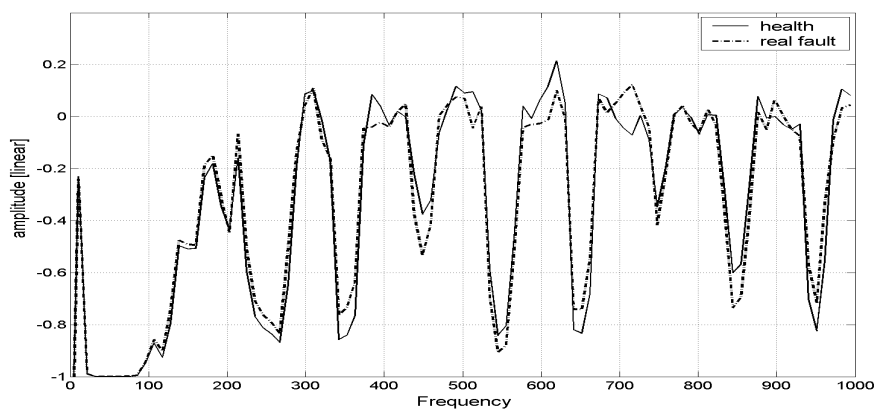


Figure 7: SK of a healthy machine respectively a machine with a realistic bearing fault

The results are quite similar to the previous ones. Indeed, the PSD of the healthy and the faulty motor show only small differences. On the contrary, the SK shows significant differences at the 7th, 13th and 19th harmonics of the stator current fundamental frequency, and easily highlights the small modulations caused by the bearing fault in this signal.

V. Conclusion

This paper was devoted to the use of the Spectral Kurtosis (SK) with the aim of detecting bearing faults in small induction motors.

Its theoretical definition and properties have been explained, as well as the bias and variance of its estimator in the case of random mixed processes. Thanks to its normalization, this fourth order spectral tool takes bounded and meaningful values, and its estimation variance is independent of the analyzed signal power. Moreover, this one dimensional spectral measure provides additional informations with respect to second order classical quantities. Applied on mixed processes, these informations can be used to discriminate between constant or non-constant amplitude harmonics, and mixing noise.

As shown in section IV, these different properties make the SK a useful tool in order to detect small modulations produced by bearing faults in the stator currents of small induction machines.

The results obtained for this application are encouraging, but some improvements are still to be done. Indeed, the variance of the SK estimator could be used to elaborate an optimal detector. Moreover, when bearing faults occur, they cause modulations in stator currents which become nonstationary. Therefore, the SK performance could be compared to other more classical methods (time-frequency representations, cyclostationary analysis), or with less classical approaches such as a nonstationary version of the Spectral Kurtosis recently proposed in [17].

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