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# ACTIVE CONTROL OF SYNCHRONOUS MACHINE VIBRATIONS: AN ADAPTIVE OPTIMAL MIMO CONTROLLER

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## ABSTRACT

This paper deals with a new multiple input multiple output active control algorithm applied to reduce synchronous machine vibrations. Usually, this kind of algorithm has good performance on linear time-invariant, or slowly time-varying systems. The present system has been developed taking into account the presence of a linear periodically time-varying transfer function. Signals are first shown periodic with the same fundamental frequency. Next, Fourier coefficients of the signals measured by sensors are linked to the inputs ones thanks to a linear matrix equation. Classical least mean square minimization can then be used to find the optimal input Fourier coefficients. Finally, a simple adaptive algorithm updating these coefficients is elaborated, in order to converge to the optimal solution and to follow the disturbances fluctuations. Results on real vibration signals show good attenuation performance with one or several sensors.

## 1. INTRODUCTION AND PROBLEM STATEMENT

For most of rotating machines, vibrations are noisy and damaging. Therefore strategies which lead to their attenuation are interesting. This paper deals with such an active control system applied to a synchronous alternator. It is supposed to be triphase with  $p$  pole pairs, driven with a constant rotating frequency  $\nu_0$  and run without load. Machine vibrations are measured by  $M$  accelerometers placed around the stator frame ( $\alpha_j$  denotes the angular position of the  $j$ th sensor).

This system acts as follows: the three stator coils are supplied with currents  $i_1(t)$ ,  $i_2(t)$  and  $i_3(t)$ , meant to create an additional magnetic field. It generates a force, and finally an additional vibration of the stator frame. The aim of the strategy set out in this paper is to elaborate input currents, whose engendered vibrations interact destructively with natural machine ones.

The transfer model between input currents and vibrations measured by the  $j$ th sensor is determined in [1], and given in Fig. 1. Input gains, function of  $\alpha_j$ , represent the effect of stator coils on input currents. The sum of these three scaled signals is next modulated by:

$$m_j(t) = X \cos(2\pi p \nu_0 t - p \alpha_j) \quad (1)$$

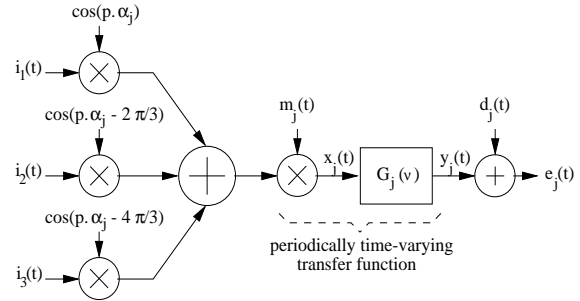


Figure 1: Transfer model between input currents and vibrations measured by the  $j$ th sensor

This linear periodically time-varying (LPTV) transfer can be determined in real-time, thanks to an angular sensor placed on the rotor shaft.  $x_j(t)$  is then proportional to the force applied on the stator frame at angular position  $\alpha_j$ . The stator frame reaction toward this force is modeled as a linear time-invariant (LTI) system whose transfer function is  $G_j(\nu)$ . It is supposed to have been previously identified. Its output  $y_j(t)$  is equivalent to the stator frame additional vibration engendered by  $i_1(t)$ ,  $i_2(t)$ , and  $i_3(t)$  at  $\alpha_j$ . This signal, summed with the natural vibration one  $d_j(t)$ , forms the total vibration  $e_j(t)$  measured by the  $j$ th accelerometer, also called error signal. This model is valid whatever  $j = 1, \dots, M$ .

As shown in Fig. 1,  $m_j(t)$  followed by  $G_j(t)$  can be seen as a single LPTV transfer function. Unfortunately, classical active control methods are developed for LTI or slowly time-varying systems [2, 3, 4], and have middling performance in this strong time-varying case [5]. The multiple input multiple output (MIMO) control strategy proposed in this paper locally reduces the stator vibrations power, although the system is LPTV. In section 2, we directly compute optimal input signals expression  $i_1(t)$ ,  $i_2(t)$  and  $i_3(t)$  in the mean square sense. As disturbances  $d_j(t)$  due to the machine rotation are periodic, optimal input signals are also shown periodic, and are expanded in Fourier series. Optimal Fourier coefficients minimizing the squared error are calculated, and are shown stationary. Their time-invariance leads to increased attenuation performance, in comparison with classical active control methods. A prac-

tical adaptive algorithm allowing to reach these solutions, and simple enough to be implemented in a digital signal processor, is also provided. In section 3, the efficiency of this control system is investigated on real vibration signals. In this paper, the system characteristics (pole pair number  $p$ , rotating frequency  $\nu_0$ , sensors angular positions  $\alpha_j$ , and transfer functions  $G_j(\nu)$ ) are supposed to be known, or previously estimated.

## 2. THEORETICAL STUDY

### 2.1. Preliminary remarks

Disturbances  $d_j(t)$  being due to the machine rotation, their spectra mostly consist of spectral lines located at harmonic frequencies of  $\nu_0$  [5]. Now, to reach good attenuation rates, “antivibrations”  $y_j(t)$  have to be strongly correlated with signals to cancel [4]. Therefore, their spectra must have the same property and be also made up of spectral lines. Next, LPTV systems formed by the association of  $m_j(t)$  and  $G_j(\nu)$  imply only  $p\nu_0$  frequency shifts [5, 6]. Therefore, input signals also should be modeled as a set of sine waves with frequencies  $n\nu_0$  ( $n \in \mathbb{N}$ ). Finally, these signals are supposed to be band-limited, and approximated by their  $N$  first harmonics. All signals are then periodic with fundamental frequency  $\nu_0$ , their Fourier transforms consist of a sum of generalized Dirac functions located at  $n\nu_0$  ( $n \in \{-N, \dots, +N\}$ ), and scaled by complex numbers, called Fourier coefficients. Moreover, only the  $N + 1$  coefficients corresponding to non-negative frequency are necessary since all signals are real. Fourier coefficients of  $e_j(t)$ , respectively  $d_j(t)$  and  $i_j(t)$ , corresponding to the frequency  $n\nu_0$  are noted  $e_{j_n}$ , respectively  $d_{j_n}$  and  $i_{j_n}$ .

### 2.2. $j$ th sensor signal analysis

Organizing the Fourier coefficients of  $e_j(t)$  in vector form, the following expression is obtained :

$$\mathbf{e}_j = \mathbf{d}_j + \mathbf{G}_j \cdot \mathbf{M}_j \cdot \mathbf{C}_j \cdot \mathbf{i} \quad (2)$$

where:

$$\mathbf{e}_j = [ e_{j_0} \quad e_{j_1} \quad \dots \quad e_{j_N} ]^T$$

$$\mathbf{d}_j = [ d_{j_0} \quad d_{j_1} \quad \dots \quad d_{j_N} ]^T$$

$$\mathbf{i} = [ i_{1(-p)} \dots i_{1N+p} \quad i_{2(-p)} \dots i_{2N+p} \quad i_{3(-p)} \dots i_{3N+p} ]^T$$

The superscript  $T$  denotes the standard transpose.

$$\mathbf{G}_j = \text{diag}_{(N+1)} [ G_j(0) \quad G_j(\nu_0) \quad \dots \quad G_j(N\nu_0) ]$$

$\text{diag}_{(N+1)}$  denotes an  $[N + 1] \times [N + 1]$  diagonal matrix. It represents the filtering effect of  $G_j(\nu)$  on the Fourier coefficients, and is diagonal thanks to the time-invariance of the transfer function.

$$\mathbf{M}_j = \frac{X}{2} \begin{pmatrix} e^{-jp\alpha_j} & \overbrace{0 \dots 0}^{2p-1 \text{ times}} & e^{+jp\alpha_j} & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & e^{-jp\alpha_j} & \overbrace{0 \dots 0} & e^{+jp\alpha_j} \end{pmatrix}$$

This  $[N + 1] \times [N + 2p + 1]$  matrix represents the amplitude modulation effect due to  $m_j(t)$ , which is a linear time-varying process. This transfer involves  $p\nu_0$  frequency shifts in the signal spectrum, or identically in vector components, which explains the matrix shape.

$$\mathbf{C}_j = ( \cos(p\alpha_j)\mathbf{I} \quad \cos(p\alpha_j - \frac{2\pi}{3})\mathbf{I} \quad \cos(p\alpha_j - \frac{4\pi}{3})\mathbf{I} )$$

$\mathbf{I}$  denotes the identity matrix. The  $[N + 2p + 1] \times [3 \times (N + 2p + 1)]$  matrix  $\mathbf{C}_j$  is made up of three diagonal matrices, corresponding to the input gains shown in Fig. 1.

Finally, Eq. (2) gives the relation between the  $j$ th error signal, input signals, and the  $j$ th disturbance signal. This relation also depends on the system characteristics, *i.e.* the known matrices  $\mathbf{G}_j$ ,  $\mathbf{M}_j$  and  $\mathbf{C}_j$ .

### 2.3. Global analysis

$M$  accelerometers placed around the stator frame at angular positions  $\alpha_j$  are now considered. The optimal input signals minimizing the vibration power on these sensors are desired. First, we show that Fourier coefficients of the  $M$  error signals are linked to the inputs ones thanks to a linear matrix equation. This generalizes Eq. (2) and regroup the Fourier coefficients of all error signals. Using definitions of vectors and matrices given in section 2.2, the following elements are defined:

$$\mathbf{d} = [ \mathbf{d}_1^T \quad \dots \quad \mathbf{d}_j^T \quad \dots \quad \mathbf{d}_M^T ]^T$$

$$\mathbf{e} = [ \mathbf{e}_1^T \quad \dots \quad \mathbf{e}_j^T \quad \dots \quad \mathbf{e}_M^T ]^T$$

These two  $[M \times (N + 1)]$  vectors regroup the Fourier coefficients of all disturbance and error signals.

$$\mathbf{T} = \begin{pmatrix} \mathbf{G}_1 \cdot \mathbf{M}_1 \cdot \mathbf{C}_1 \\ \vdots \\ \mathbf{G}_M \cdot \mathbf{M}_M \cdot \mathbf{C}_M \end{pmatrix}$$

This  $[M \times (N + 1)][3 \times (N + 2p + 1)]$  matrix represents the transfer function of the global system between the three input signals and the vibrations they generate at the sensors.

These different elements verify the global equation:

$$\mathbf{e} = \mathbf{d} + \mathbf{T} \cdot \mathbf{i} \quad (3)$$

### 2.4. Cost function

Input signals are said optimal if they minimize the sum of errors power, thanks to a minimum amount of inputs power. A cost function corresponding to this aim is:

$$C = \frac{1}{2} \left[ \sum_{j=1}^M P_{e_j} + \gamma \sum_{j=1}^3 P_{i_j} \right] = \mathbf{e}^H \cdot \mathbf{e} + \gamma \mathbf{i}^H \cdot \mathbf{i} \quad (4)$$

where the superscript  $H$  denotes the Hermitian transpose, and  $P_{x_j}$  the power of  $x_j(t)$ . The real constant  $\gamma$  controls the balance between reducing the squared errors  $\mathbf{e}^H \cdot \mathbf{e}$ , and increasing the “control effort”  $\mathbf{i}^H \cdot \mathbf{i}$ . The same cost function is studied in [7] for LTI or slowly time-varying systems, and

for one harmonic only. Using Eq. (3),  $C$  is expanded out into a generalized matrix quadratic form in the variable  $\mathbf{i}$ :

$$C = \mathbf{i}^H \cdot [\mathbf{T}^H \cdot \mathbf{T} + \gamma \mathbf{I}] \cdot \mathbf{i} + \mathbf{d}^H \cdot \mathbf{d} + \mathbf{i}^H \cdot [\mathbf{T}^H \cdot \mathbf{d}] + [\mathbf{T}^H \cdot \mathbf{d}]^H \cdot \mathbf{i} \quad (5)$$

$C$  presents a unique global minimum with regard to  $\mathbf{i}$ , because the matrix  $[\mathbf{T}^H \cdot \mathbf{T} + \gamma \mathbf{I}]$  becomes positive definite taking  $\gamma > 0$ . The first line of (5) contains the vibrations power generated by inputs acting alone, summed with the disturbances power and a fractional part of the inputs power. These terms are never negative, and cannot minimize the function  $C$ . Nevertheless, the terms of the second line, *i.e.* the cross-power between antivibrations and disturbances, can be negative and used in order to achieve this minimization.

## 2.5. Optimal solution and adaptive algorithm

The global minimum of  $C$  is reached with the optimal value of  $\mathbf{i}$ :

$$\mathbf{i}_{\text{opt}} = - [\mathbf{T}^H \cdot \mathbf{T} + \gamma \mathbf{I}]^{-1} \cdot \mathbf{T}^H \cdot \mathbf{d} \quad (6)$$

This equation gives the vector of the optimal Fourier coefficients of input signals as a function of system characteristics  $\mathbf{T}$ , the constant  $\gamma$ , and the frequency components of the disturbances  $\mathbf{d}$ . It is the so-called ‘‘Wiener solution’’. Unfortunately, it is difficult to use in practice because disturbances  $d_j(t)$  are not directly measured. In order to obtain a solution as a function of the measurable vector  $\mathbf{e}$ , a gradient descent algorithm with a convergence rate  $\mu$  [8, 9] is derived from Eq. (5):

$$\mathbf{i}_{l+1} = (1 - 2\mu\gamma)\mathbf{i}_l - 2\mu\mathbf{T}^H \cdot \mathbf{e}_l \quad (7)$$

Vector  $\mathbf{i}$  represents the frequency components of  $\frac{1}{\nu_0}$  periodic signals. Therefore it needs to be updated following Eq. (7) every  $\frac{1}{\nu_0}$  seconds (subscript  $l$  denotes a variable valid during  $l$ th period). Moreover, the innovation term of (7) depends on known matrix  $\mathbf{T}$ , and vector  $\mathbf{e}_l$  including Fourier coefficients of signals measured by sensors. Only  $\mathbf{e}_l$  then has to be processed each period, thanks to a Fourier transform. This algorithm converges to the optimal solution (6) because the cost function has no local minima.

In the next section, the performance of this MIMO active control system is investigated through computer experiments on real signals.

## 3. SIMULATION RESULTS

### 3.1. Simulation conditions

Real vibration signals have been measured on an alternator with  $p = 1$  pole pair and a rotating frequency  $\nu_0 = 23.8\text{Hz}$ . Moreover, they have been filtered to limit their frequency range between 1 and 3kHz ( $N$  was chosen big enough to take these frequencies into account). The following results were obtained using these signals, with a simulated machine. Mechanicals LTI transfer functions  $G_j(\nu)$  were modeled with different order 4 infinite impulse response filters, and modulation signals  $m_j(t)$  were supplied by an angular sensor placed on the rotor shaft.

Moreover, algorithm parameters were set constant ( $\mu = 0.1$ ,  $\gamma = 0.01$ ) in order to compare the different results.

Finally, an attenuation performance criterion is necessary to quantify the system efficiency. The *Normalized Cost Function*, defined in (8), is chosen:

$$C_n = 100 \times \frac{C}{\mathbf{d}^H \cdot \mathbf{d}} \quad (8)$$

where  $C$  is the cost function defined in (4). This quantity is represented in percentage, and should be as small as possible if the control system is efficient.

### 3.2. Single output results

In this section, the number of sensor is limited to  $M = 1$ , so as to analyse the system behavior at a single point of the stator frame.

Firstly, the effect of the number of inputs on the attenuation performance is investigated. One sensor was shifted all around the machine, and  $C_n$  was calculated for each sensor position  $\alpha_1$ , using one and then three input currents. It is represented in polar coordinates in Fig. 2. For the one

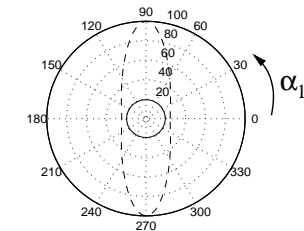
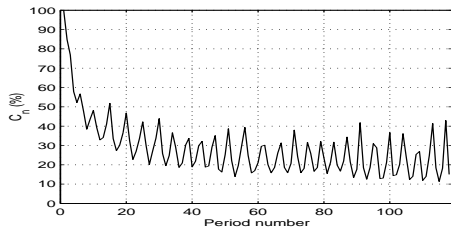


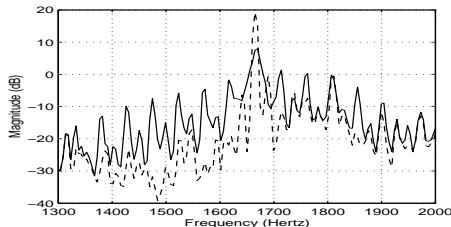
Figure 2:  $C_n$  versus  $\alpha_1$  in polar coordinates, obtained for one (dashed) and three (solid) input currents

input case,  $C_n$  varies from 20% to 100% as a function of  $\alpha_1$ . Therefore, the system presents spatial limitations since two stator frame points can not be reached ( $C_n \simeq 100\%$  for  $\alpha_1 = 90^\circ$  and  $270^\circ$ ). Nevertheless, when three inputs are used, they work in relays to generate an attenuation force at each point and performance remains constant and very good whatever  $\alpha_1$  ( $C_n \simeq 20\%$ ). In next simulations, all inputs then have been used.

Secondly, the convergence behavior and the attenuation effect on signals are represented in Fig. 3. The learning curve of the adaptive algorithm, representing  $C_n$  versus period numbers, is shown in Fig. 3(a). This criterion converges as expected toward an optimal value, which is approximately 25% of the disturbance power. The parameter  $\mu$  controls the convergence speed of the system, while  $\gamma$  controls the final value of  $C_n$  [7]. However, non-synchronous sampling involves oscillations of the curve after convergence. Fig. 3(b) represents the disturbance and error spectra after convergence. The disturbance spectral line located at 1666Hz is well attenuated (more than 10dB reduction), but some small interference spectral lines are created here and there, due to the sinusoidal amplitude modulation. In spite of these interferences, the system attenuated more than 75% of the disturbance power.



(a) Learning curve



(b) Disturbance (dashed) and error (solid) spectra after convergence

Figure 3: Single output attenuation performance

### 3.3. Multiple output results

In this last section, three sensors are used in order to minimize vibrations at several points at the same time. They are placed on the stator frame, with angular positions  $\alpha_1 = 0$ ,  $\alpha_2 = 45^\circ$  and  $\alpha_3 = 90^\circ$ . Only the time evolution of  $C_n$  is represented in Fig. 4, because the spectra obtained on each accelerometer are similar to those of the Fig. 3(b). After convergence,  $C_n$  represents approximately 50% of the

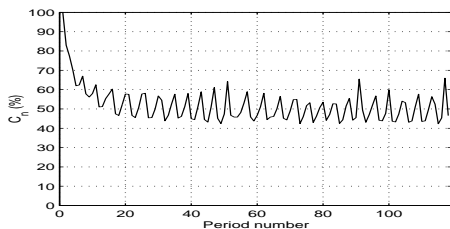


Figure 4: Multiple output attenuation performance: learning curve

disturbances power. Moreover, for sensor 1, respectively 2 and 3, the error power represents 49%, respectively 45% and 57% of the disturbance one. Therefore, this control system may be used to create a “zone of quiet” on the stator frame. This is interesting, for example, to have motionless fixing point on the stator frame, and isolate the direct machine environment from its vibrations.

## 4. CONCLUSION

A new MIMO active control algorithm applied to synchronous machine vibrations has been developed. Its

originality is due to the fact that it takes into account the linear and periodically time-varying transfer function, through which inputs act on disturbances. First, we have demonstrated that all signals are periodic with the same fundamental frequency. Next, the relation between their Fourier coefficients has been shown linear. Thereby, it is easy to find the optimal Fourier coefficients of the input signals minimizing a quadratic cost function. Finally, simulations carried out on real vibration signals lead to the following conclusions :

- attenuation performance becomes independent of the sensor location if three inputs are used,
- results obtained for one or several sensors are encouraging with a view to implement this algorithm in a digital signal processor.

Nevertheless, several points still have to be studied, like the optimal choice of the algorithm parameter  $\gamma$  in order to adjust the input signals power. Another improvement would consist in having good performance in case of fast variation of the rotating frequency  $\nu_0$ .

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