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What can we learn about dark energy evolution?

Marian Douspis^{1,2}, Yves Zolnierowski³, Alain Blanchard⁴, and Alain Riazuelo⁵

¹ Laboratoire d'Astrophysique de Tarbes et Toulouse, OMP, UMR5572 CNRS, 14 avenue É. Belin, F-31400 Toulouse, France
e-mail: douspis@ast.obs-mip.fr

² Institut d'Astrophysique Spatiale (IAS), Bâtiment 121, F-91405 Orsay, France
e-mail: marian.douspis@ias.u-psud.fr

³ Laboratoire d'Annecy-le-Vieux de Physique des Particules, UMR5814 CNRS, 9 Chemin de Bellevue, BP 110, F-74941 Annecy-le-Vieux cédex, France
e-mail: zolnierowski@lapp.in2p3.fr

⁴ Laboratoire d'Astrophysique de Tarbes et Toulouse, OMP, UMR5572 CNRS, 14 avenue É. Belin, F-31400 Toulouse, France
e-mail: alain.blanchard@ast.obs-mip.fr

⁵ CNRS, UMR 7095, Institut d'Astrophysique de Paris, Paris, F-75014 France ; Université Pierre et Marie Curie-Paris6, UMR 7095, Paris, F-75014 France
e-mail: riazuelo@iap.fr

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Abstract. We examine some of the constraints that can be obtained on a two parameters model of dark energy in which the characteristic parameter of the equation of state $w(z) = P(z)/\rho(z)$ does not vary smoothly with time as usually assumed, but undergoes a transition between two values over a period that can be significantly shorter than the Hubble time. We find that the most recent SNIa survey allows a transition between $w \sim -0.2$ to $w \sim -1$ (the first value being somewhat arbitrary) at redshift as low as 0.1, despite the fact that data extend beyond $z \sim 1$. Surveys with precision anticipated for space experiments would allow to improve this constraint but not by much, as a transition occurring at redshift as low as ~ 0.17 could still remain undistinguishable from a standard cosmological constant. The addition of a prior on the matter density $\Omega_m = 0.27$ improves the constraints, although in a rather limited way. This suggests that Hubble diagram of distant SNIa might hardly reveal the actual nature of dark energy at redshift above 0.2. Even deep space experiments would fail to identify a rapid transition at redshift above 0.5. This suggests that only the local dynamics of the quintessence can be caught by SNIa Hubble diagram. The reason for this phenomenon is investigated. On the contrary without any prior, we found that existing constraints from combining CMB and SNIa already reject a transition at redshift below 1.5.

Key words. Cosmology – Cosmic microwave background – Supernovae – Cosmological parameters

1. Introduction

The nature of dark energy is at present one of the most puzzling mystery of modern cosmology. It is now widely accepted that our universe is experiencing a phase of accelerated expansion (Blanchard et al. 2005). The evidence was first found using type Ia supernovae luminosity vs. redshift diagram (Schmidt et al. 1998; Perlmutter et al. 1999), but a number of other observations also support this conclusion (Bennett et al. 2003; Spergel et al. 2003). In particular, estimations of the matter density of the universe generally lead to a low value, while the CMB anisotropies point toward a spatially flat Universe (Lineweaver et al. 1997; de Bernardis et al. 2000). WMAP data also require the presence of dark energy (Spergel et al. 2003) unless one considers an unexpectedly low value of the Hubble parameter $H_0 \leq 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Blanchard et al. 2003). The possible correlation of CMB fluctuation map with

surveys of extragalactic objects (Fosalba et al. 2003; Corasaniti et al. 2005; Pogosian 2005) also provide direct evidence, although with a limited significance level ($< 3\sigma$), for the existence of an unclustered dark energy component in the universe, the detected correlation being explainable through the integrated Sachs-Wolfe effect, i.e., a time variation of the gravitational potential, which is achieved only if the (baryonic + cold) matter density parameter satisfies $\Omega_m \neq 1$. Finally, the shape of the correlation function on scales up to $100h^{-1} \text{ Mpc}$ which has been recently measured accurately (Eisenstein et al. 2005) in combination with the CMB data requires the presence of dark energy (Blanchard et al. 2005). However the nature of this dark energy has been the subject of numerous speculations. The simplest model of all, which was originally proposed (in another context) by Einstein (1917) as a pure cosmological constant Λ , a term on the left hand side of Einstein's equations. However, a cosmological constant can also be regarded as the contribution of the vacuum to the right hand side of the equa-

tion with a specific equation of state, i.e., a component with negative pressure P_Λ related to the energy density ρ_Λ by the relation $P_\Lambda = -\rho_\Lambda$. Indeed, quantum field theory predicts that the lowest energy state of any mode contributes to a vacuum energy density which behaves exactly as a cosmological constant (see, e.g. Binétruy 2000). A number of problems arise with this possibility, in particular the so-called hierarchy problem: the expected contribution is usually enormous, naive calculation give $\rho_{\text{vac}} \sim 10^{76} \text{ GeV}^4$, around 122 orders of magnitude larger than the present critical density of the universe. There exist however mechanisms, such as supersymmetry which allow to reduce considerably the vacuum energy density, but since supersymmetry is broken at a scale larger than 100 GeV one is still plagued with a an enormous vacuum energy density of order of 10^8 GeV^4 . The usual loophole argument is then to say that there exists a yet unknown mechanism which ensures that the contribution of the vacuum energy density is zero. One is therefore left with the less annoying problem to explain the nature of dark energy, which differs from a cosmological constant, allowing to escape the extreme fine tuning requested to obtain the observed dark energy density (Binétruy 2000). Most models which have been addressed so far (quintessence models) therefore relies on the idea that some scalar field (Caldwell et al. 1998) behave today similarly as a cosmological constant, exactly as another scalar field did during inflation. There is no fundamental principle that give the potential of such scalar field, so that one is led to propose a framework which is sufficiently general to handle most of the interesting cases, and in the same time which allows to predict a set of observable quantities. The most remarkable feature of quintessence models is that both the scalar field pressure P_Q and energy density ρ_Q evolve according to dynamical equations, so that the so-called equation of state parameter, $w_Q \equiv P_Q/\rho_Q$, usually varies with time between -1 and 1 , as the field evolves along the potential. In some extreme (and possibly ill-defined) models, this parameter can even take any arbitrary values, for example if one allows the density ρ_Q to take negative values or a change in the sign of the kinetic term (Caldwell 2002). Other models involving scalar tensor theories also allow for such a transient behaviour (Elizalde et al. 2004).

This paper is organized as follows : in Sec. 2, we recall a few basics about simple quintessence models, and motivates a new parametrization of the equation of state parameter $w_Q(z)$ which we think very convenient and somewhat more general than many proposed parametrizations (Doran et al. 2005; Pogosian et al. 2005; Johri 2004). One crucial issue we examine is the impact of the epoch of observation on the parameter estimation. We find that dark energy models are better constrained when observed before the epoch of dark energy domination. This is done in Sec. 3. In Sec. 4, we describe the various types of data analysis we perform, and we state our main results. We draw the main conclusions of our work in sec. 5.

2. Dark energy parametrization

Historical quintessence model rely on the idea of tracking solution (Wetterich 1988; Ratra & Peebles 1988), which involve a scalar field evolving in an inverse power law poten-

tial, $V(Q) \propto Q^{-\alpha}$, the proportionality constant being tuned so as to obtain the desired value of dark energy density parameter $\Omega_Q \sim 0.7$ today. The main feature of these models is that the pressure to energy density ratio, w_Q remains constant both in the radiation era and in the matter era (with different values during each epoch), and that it tends toward -1 once the quintessence energy density dominates. The value for the parameter w_Q depends on the power law index of its potential. From existing data, it seems that only values close to $w_Q \sim -1$ today, or even possibly lower, are acceptable (Caldwell 2002; Melchiorri et al. 2003). Note however that in the latter case, value of w_Q below -1 cannot be obtained naturally through a standard scalar field. Single power law potentials suffer from the fact that the w_Q parameter goes slowly toward -1 once quintessence dominates, so that today, if the quintessence density parameter Ω_Q is close to 0.7, then w_Q is still far from the asymptotic value -1 , contrary to what most analysis suggest. In order to escape this problem, one has to add extra features in the potential, such as a break in the slope of the potential or a local minimum in the potential, such as in the SUGRA model proposed by Brax & Martin (1999). A large number of other possibilities has been proposed since then (see for exemple references in Brax et al. 2000; Peebles & Ratra 2003).

On the other hand, without precise ideas about the correct quintessence model to use, it has become natural to take a more phenomenological approach in which one parametrizes the functional form of $w_Q(z)$ which possibly exhibits the main features described above.

The simplest model of quintessence (in the sense that it introduces only one new parameter as compared to a Λ CDM model) is to assume a constant w_Q . However there is little motivation for constant w_Q beyond the economical argument and it is more and more recognized that evolving w_Q should be investigated with a minimal level of priors. In the absence of well motivated theoretical considerations one is left with the empirical option to examine constraints on analytical form for $w_Q(z)$. Most investigations were based on expressions with one or two parameters. However, it should be realized that such expressions often varies with time in a relatively slow way and that rapidly varying expressions have to be examined as well. In other words, if one considers the typical time scale:

$$\tau_Q \sim \frac{w}{\dot{w}}, \quad (1)$$

constant w correspond to $\tau_Q \gg t_H$ where $t_H = 1/H$ is the Hubble time, smoothly varying expression such as the inverse power law potential corresponds to $\tau_Q \sim t_H$ and more rapidly varying w correspond to $\tau_Q \ll t_H$, such as in the SUGRA model. Our aim is primary to investigate constraints on models for which $\tau_Q \ll t_H$. This has conducted us to use the following model which allows arbitrarily rapid transitions and in which the dark energy w_Q parameter evolves as a function of the scale factor a according to

$$w_Q(a) = \frac{1}{2}(w_i + w_\infty) - \frac{1}{2}(w_i - w_\infty) \tanh\left(\Gamma \log\left(\frac{a}{a_t}\right)\right). \quad (2)$$

The w parameter goes from w_i at early times to w_∞ at late times, the transition occurring at a_t which may even be in the future.

The transition occurs at redshift $z_t = a_0/a_t - 1$ (a negative value of which corresponding to a transition in the future), where a_0 is the scale factor today, and lasts of order of Γ^{-1} Hubble times. There are several advantages to this parametrization. First, the quintessence conservation equation $D_\mu T_Q^{\mu\nu} = 0$ can be integrated exactly to give

$$\rho_Q(a) = \rho_Q^0 \left(\frac{a}{a_0} \right)^{-3(1+w_{av})} \left(\frac{(a/a_t)^\Gamma + (a/a_t)^{-\Gamma}}{(a_0/a_t)^\Gamma + (a_0/a_t)^{-\Gamma}} \right)^{\frac{3\Delta w}{2\Gamma}}, \quad (3)$$

where we have set

$$w_{av} \equiv \frac{1}{2}(w_i + w_\infty), \quad \Delta w \equiv (w_i - w_\infty), \quad (4)$$

and where ρ_Q^0 and a_0 correspond to the values of the quintessence energy density and scale factor at some given epoch. Therefore, the model can be implemented without much modification in existing cosmological codes such as CAMB (Lewis et al. 2000). Second, this parametrization decouples at best the late time behavior of the parameter $w_Q(z)$ to that of the transition it may have experienced earlier. This model has four parameters. As we said earlier, simple quintessence model have less: constant w model correspond to $w_i = w_\infty$, with undefined a_t and Γ , whereas of inverse power law potential, one has $w_\infty = -1$, $w_i = -2/(\alpha + 2)$, whereas the epoch of transition is fixed approximatively by the constraint $\Omega_Q(z_t) \sim \Omega_m(z_t) \sim 0.5$ and the duration of the transition is somewhat larger than the Hubble time (it depends in fact on how steep the potential is, that is, on α). For the SUGRA potential the first two above constraints on w_i and w_∞ remain, whereas the latter are modified: the epoch of transition for w_Q does no longer necessarily correspond to the scalar field domination (but corresponds to the epoch where the field reaches a local minimum in its potential), and the transition duration is usually much shorter. Various other models have different predictions concerning these parameters, but we think that our parametrization is sufficiently general to encompass a large number of already proposed models.

Let us note that one of these parameters is not as relevant as the others. The parameter w_i does not play a crucial role, since the epoch when $w_Q(z)$ has this value usually corresponds to that when $\Omega_Q(z)$ is negligible as compared to Ω_m , so that quintessence does not play a crucial role then, as least with regard to supernovae and CMB data. This is all the more true that since as we shall see data favor a low value of w_Q today together with the usual $\Omega_Q^0 \sim 0.7$, there are in practice no model which pass the observational constraint with $w(z) \sim w_i$ when Ω_Q has already become large. The only subtle point here is that it wise to impose a sufficiently low value of w_i in order to insure that at early times $\Omega_Q \ll \Omega_m$. The reason is that with $w_i = 0$ and a low z transition Ω_Q would be close to its present value at recombination epoch (or even nucleosynthesis, see references in Peebles & Ratra 2003). At low redshift, this would lead to a dramatic suppression of the cosmological perturbation growth rate (Douspis et al. 2003) and we found that this introduces additional changes in the C_l curve at high l (i.e., other than changes due to the modification of the angular distance). This is the reason why we fix for definiteness $w_i = -0.2$ for which

such effect does not appear. Putting a constraint on w_∞ is more objectionable since it implicitly selects a limited class of models, which do not seem excluded by the data. For definiteness, we have chosen the value $w_\infty = -1$, which seems in agreement with the present data, and we focus on the two remaining parameters, Γ and a_t which describe the transition experienced by $w_Q(z)$ between its early and late behaviour. We are therefore studying a two parameter model.

An important question that immediately arise is: how many of the dark energy parameters can accurately be measured by the (present or future) data, provided of course they describe accurately dark energy evolution? We are indeed in a situation where it is clearly hard to advocate for a specific parametrization of dark energy, a situation most convincingly illustrated by the wealth of different parametrizations that have been proposed for dark energy so far. One way to circumvent this difficulty is to use a method which is mostly insensitive to dark energy parametrization and which allows to see which part of the $w_Q(z)$ is most easy to constrain.

This task is very naturally performed by the Principal Component Analysis (PCA). Principal component analysis were introduced in cosmology rather recently (Francis & Wills 1999). They deal with situations where one is confronted with a large set of observable data which depend on a large set of parameters. This precisely corresponds to the present status of cosmology. PCA has already been used to constraint the initial power spectrum of cosmological perturbations (Crittenden & Pogosian 2005) or to address the degeneracies related to the tensor modes (Efstathiou 2002). One of the most important issue of PCA is that it allows to understand which region of the parameter space is best constrained by the data. This is particularly useful when the underlying parametrization is not known a priori. Dark energy corresponds to this case. Indeed, the nature of dark energy is not known, neither the evolution of dark energy equation of state unless a specific model is assumed. However the impact of the implicit assumptions of the underlying model are difficult to compute, therefore PCA brings an interesting contribution to dark energy phenomenology (Huterer & Starkman 2003; Linder & Huterer 2005; Leach 2005).

In these cases, PCA tries to reconstruct (or, in fact, says what can be reconstructed in) the time evolution of the dark energy equation of state through its impact on the luminosity distance vs. redshift relation. A very similar approach can also be made in weak lensing statistics, where it is mainly the impact of the dark energy equation of state parameter on the growth rate of cosmological perturbations that is studied (Hu 2002). Studies of constraints on $w_Q(z)$ can naturally be performed in this framework. They result in assuming that $w_Q(z)$ can take constant (possibly different) values in a series of N redshift intervals $\Delta z_i = z_{i+1} - z_i$, with $z_1 < z_2 < \dots < z_N$.

The results obtained in Huterer & Starkman (2003); Linder & Huterer (2005); Crittenden & Pogosian (2005), point to the conclusion (sometimes expressed in a rather humorous way) that only two parameters describing the evolution of $w_Q(z)$ can be reconstructed from existing and even future data, namely $w_Q(0)$ and $dw_Q/dz|_{z=0}$. Indeed, the authors of these papers focus on the well-known parametrization of the dark energy equation-of-state that is often confronted to supernovae data,

where one assumes that the dark energy equation of state evolve as

$$w(z) = w_0 + w_1 z. \quad (5)$$

These authors then conclude that these two parameters can possibly be extracted from the data and that no more parameters can be extracted from the data.

In the case where w_Q experiences a rapid transition, the parameter $w_1 \sim dw_Q/dz|_{z=0}$ is not very interesting, since in the limit of an instantaneous transition, this parameter is strictly zero by definition. Therefore it does not describe a situation like that of Eq. 2 in the limit $\Gamma \rightarrow \infty$. The aim of this paper is to show that is the case of a sudden transition the situation is from the observational point of view worse than in the case of a smooth transition.

3. Living at $z = 0$ and at $z = 0.3$

In order to distinguish quintessence models from a pure cosmological constant, it is crucial to be able to track the dark energy evolution as early as possible. This comes with some difficulties. The main impact of dark energy comes from its influence of the expansion rate of the universe, both when one looks at the distance luminosity vs. redshift relation using type Ia supernovae or when one looks at the growth rate of cosmological perturbation, which can be studied through cluster abundance or weak lensing measurements. Today, it seems likely that dark energy represents the most important contribution to the total energy density of the universe. Since its equation of state parameter is negative, dark energy density decays more slowly than ordinary matter ($(1+z)^{3(1+w_Q)}$ vs. $(1+z)^3$, where z is the redshift). Therefore in the past standard (baryonic + cold dark) matter was dominant. An important question is therefore to know till which epoch the dark energy density could play some role in the evolution of the universe, and as a corollary till which epoch one can hope to reconstruct either its energy density or its equation of state parameter.

When confronted to this problem, one notices that the epoch at which one looks at the cosmological data becomes surprisingly crucial. Let us assume a model with $\Omega_Q = 0.7$, $\Omega_m = 0.3$ today, with w_Q presently constant and equal to -1 . The matter to dark energy transition, defined when $\Omega_Q(z) = \Omega_m(z) = 0.5$ occurs at redshift $z_t = (\Omega_Q/\Omega_m)^{1/2} - 1 \sim 0.33$. Let us now consider two alternatives. First, we can consider a pure cosmological constant model, with $w_Q = -1$ also at early times. Second, we can consider a model where $w_Q \sim 0$ for $z > z_t$. In the first case, one has an usual Λ CDM model, whereas in the second case, one has a model close to a flat Einstein-de Sitter model at early times. An observer who would live at $z = z_t$ should easily be able to distinguish between the two models, just as we are able to distinguish between a Λ CDM and a flat Einstein-de Sitter model today. Now, are we able to distinguish today between these two models which differ only for $z > z_t$? Surprisingly, the answer is no if one considers supernovae data only. The explanation of this paradox comes as follows. Present data favor dark energy because high redshift supernovae are dimmer than expected in a flat Einstein-de Sitter universe. This

is usually expressed as a difference of magnitude between the two models one considers for some standard candle at some redshift, the exact value of which depending on the quality of the data. The magnitude is essentially the logarithm of the luminosity distance as a function of the redshift. Let us define $d_L^\Lambda(z)$ and $d_L^{\text{EdS}}(z)$ the luminosity distance as a function of the redshift in a Λ CDM model with $\Omega_\Lambda = \Omega_m = 0.5$ today, and in a flat Einstein-de Sitter, respectively. Let us assume these two models can be distinguished. Let us now consider $\tilde{d}_L^\Lambda(z)$ and $d_Q(z)$ the luminosity distance vs. redshift relation in a Λ CDM model with $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$ today, and a dark energy model with $\Omega_Q = 0.7$, $\Omega_m = 0.3$ today, with w_Q experiencing a sudden transition from 0 to -1 at $z = z_t$. An observer living at $z = z_t$ would therefore measure either $d_L^\Lambda(z')$ or $d_L^{\text{EdS}}(z')$. The epoch corresponding to a redshift of z' measured by an observer living at z_t corresponds to a redshift z given by

$$z = (1 + z_t)(1 + z') - 1 = z_t + z' + z_t z', \quad (6)$$

measured by an observer living today. Let us define d_t as the luminosity distance of the observer living at $z = z_t$ as seen from today. The exact value of d_t does not matter here, but it can be obviously be computed as

$$d_t = -d_L^\Lambda \left(\frac{1}{1 + z_t} - 1 \right). \quad (7)$$

Luminosity distance do not add but are proportional to comoving distances. The luminosity distance at redshifts above z_t is therefore given by

$$\tilde{d}_L^\Lambda(z) = \frac{1 + z}{1 + z_t} d_t + \frac{1 + z}{1 + z'(z)} d_L^\Lambda(z'(z)), \quad (8)$$

$$d_L^Q(z) = \frac{1 + z}{1 + z_t} d_t + \frac{1 + z}{1 + z'(z)} d_L^{\text{EdS}}(z'(z)), \quad (9)$$

in the two models respectively. Now, in term of difference of magnitude between the two models, this is translated into

$$\Delta \tilde{m}(z) \propto \log \frac{d_t + \frac{1+z_t}{1+z'(z)} d_L^\Lambda(z'(z))}{d_t + \frac{1+z_t}{1+z'(z)} d_L^{\text{EdS}}(z'(z))}, \quad (10)$$

whereas for the observer living at z_t , this simply gives

$$\Delta m(z') \propto \log \frac{d_L^\Lambda(z')}{d_L^{\text{EdS}}(z')}. \quad (11)$$

Two differences arise here. First, the redshift range which today's observer must use in order to distinguish between the two models is larger than that of the observer living at z_t . If the latter must collect data between $z' = 0$ and $z' = z_*$, say, then the former must observe supernovae between $z(0) = z_t$ and $z(z_*) = z_t + z_* + z_t z_* > z_t + z_*$. Second, the magnitude difference between the two models is strongly attenuated for the observer living today because of the presence of the extra term d_t both in the numerator and the denominator of Eq. (10). The net result is that whereas both models are easy to distinguish at $z = 0.3$, this is no longer the case at $z = 0$ as is clearly seen on Fig. 1.

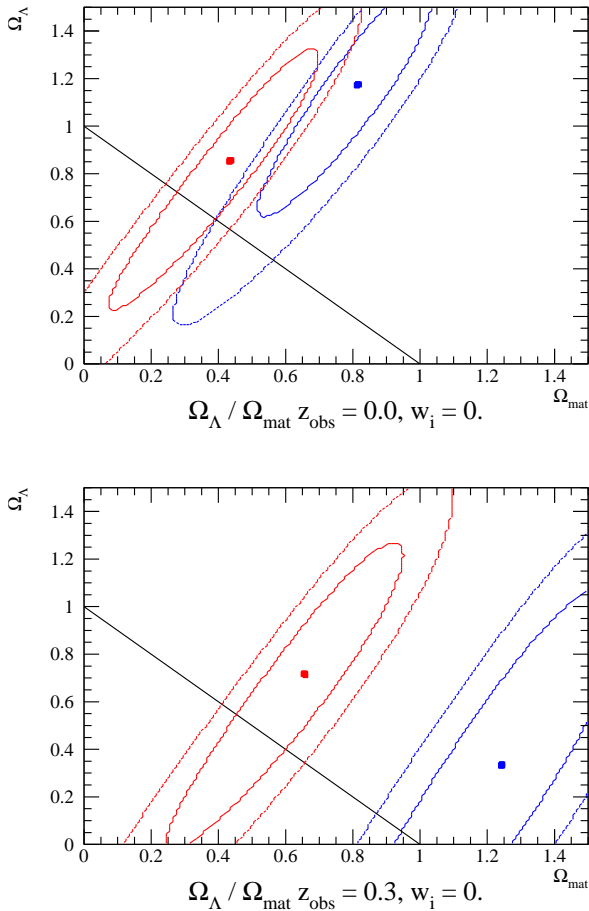


Fig. 1. Comparison of $\Omega_m - \Omega_\Lambda$ fit for the two models discussed in Sec. 3 for an observer living at $z = 0$ (after the transition for the quintessence model) and $z = 0.3$ (during the transition). At $z = 0.3$ the two models are very easily distinguished as one of them corresponds to a pure matter model, whereas one corresponds to a cosmological constant model with $\Omega_\Lambda = 0.5$. On the contrary, at $z = 0$ both models look like a cosmological constant model, and cannot be distinguished at the 2σ level.

4. Analysis

We focus here on constraints that can be set in the transition parameters z_t and Γ , and therefore we set $w_i = -0.2$ and $w_\infty = -1$ as explained above. This reduces the number of free parameters to two and allows to study the complementary aspect of the model, i.e., its transition behavior. Note that a pure cosmological constant behaviour is obtained by considering large z_t and a sufficiently small transition duration (so that it does not last long after z_t).

4.1. Supernovae Hubble diagram

Let us first examine what kind of constraints the Supernovae Hubble diagram allows. The number of well observed SNIa has rapidly increased over the recent years and a significant number of supernovae above redshift one have been detected. In the following we use the Gold sample from Riess et al. (2004), notic-

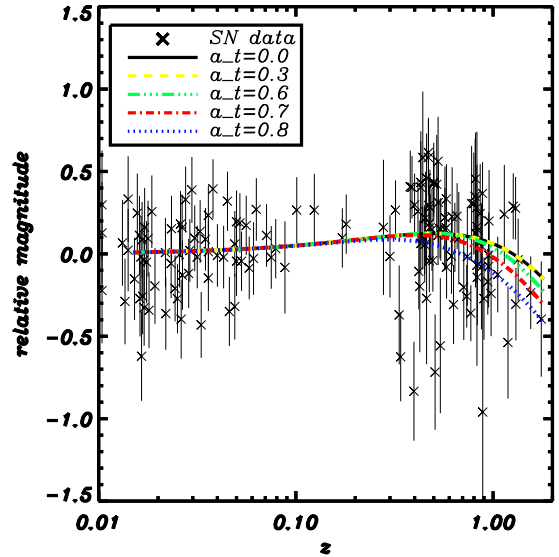


Fig. 2. Residual Hubble diagram with respect to an empty universe for models presenting a transition at epoch $a_t/a_0 = 1/(1+z_t) = 0.0, 0.3, 0.6, 0.7$ & 0.8 compared with the Gold sample from Riess et al. (2004). Note that as explained in the text low values of a_t all give the similar diagram as that of a Λ CDM model.

ing that the inclusion of some Cepheids distance implies that this data set automatically leads to a prior on the Hubble constant, which cannot be entirely disentangled in the following. It should be kept in mind therefore that some of our constraints might be affected by this prior.

In figure 2, we show the effect of a transition on the magnitude difference between a fiducial model and the empty universe. This clearly reveals that a transition from $w_i = -0.2$ to $w_\infty = -1$ occurring at even moderately low redshift is making tiny differences in the observable quantity. This is clearly the reason why a transition even at low redshift cannot be detected easily (i.e., it is not due to degeneracies among the various parameters of our analysis).

It is therefore not surprising that the constraints that can be set from the SNIa Hubble diagram are not very tight. Indeed in Figure 3 the allowed regions for a transition expressed $a_t/a_0 = 1/(1+z_t)$ versus the duration of the transition is presented. Somehow surprisingly, the data prefer a transition at low redshift, a tendency which has been already noticed elsewhere (Bassett et al. 2004; Corasaniti et al. 2004). Note however that the significance level is low and a cosmological constant remains consistent with the data at the 2σ level.

Rapid transitions (corresponding to large Γ) are very weakly constrained: at the two sigma level, transitions are acceptable at any redshift above 0.1. Better constraints are obtained when a strong prior is set on Ω_m : with $\Omega_m = 0.3$ we found that transitions are acceptable at redshift greater than 0.25 with still data preferring models with a transition. In both cases, transitions are preferred, but with a significance level less than two sigmas. This means that the Hubble diagram of distant SNIa seems not very discriminant on the nature of the dark

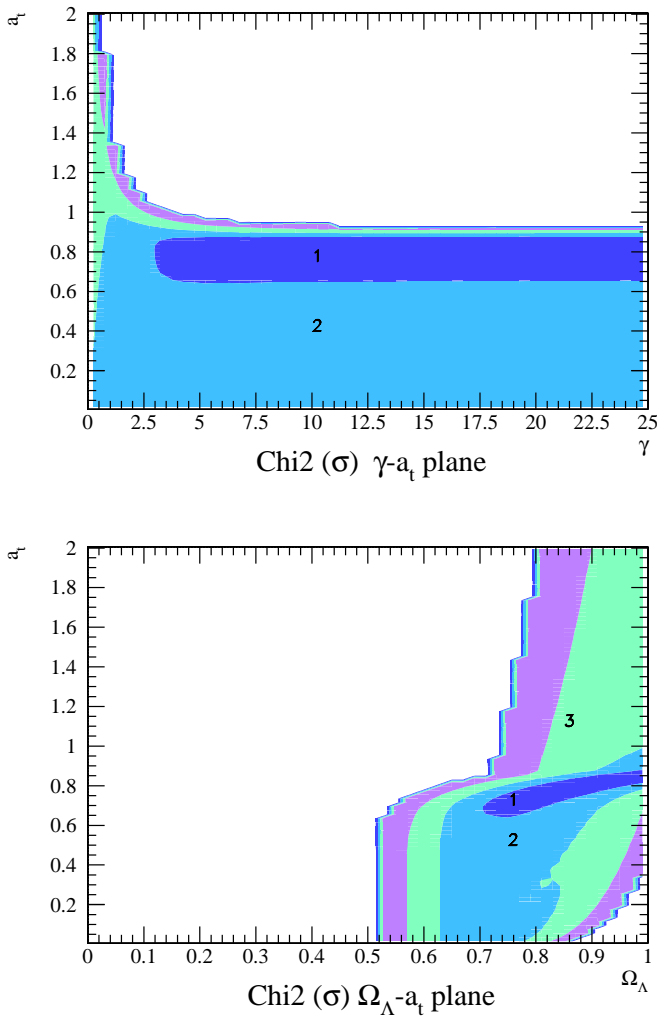


Fig. 3. Contour constraints on the transition at epoch $a_t = 1/(1+z_t)$, and the rate of the transition Γ . Grey area are 1, 2, 3 σ levels on two parameters. Likelihood has been obtained by maximizing on other parameters. The results are independent of Γ as soon as it is sufficiently large since an abrupt transition would necessitate a much thinner redshift resolution in the SN data, which is not available today as the number of supernovae is too small. Bottom figure show the result in the (Ω_Λ, a_t) plane.

energy at high redshift. In order to examine to which level this conclusion is right we have redone the above analysis from a simulated survey having typical precision and statistic expected from space experiments. The input model was a standard concordance Λ CDM model with $w = -1$. The constraints inferred from this sample reveal again that the transition epoch a_t is moderately constrained: transition at redshift as low as 0.16 are acceptable, provided appropriately long duration is taken ($\Gamma \sim 2$, so as to ensure that $w_Q(z)$ can start decreasing long before this late transition epoch), more rapid transitions are better constrained although they are acceptable beyond redshift 0.5, at an epoch where dark energy density is still of the same order of magnitude than the matter density. In this case the addition of a prior on Ω_m does not improve by much the lower acceptable

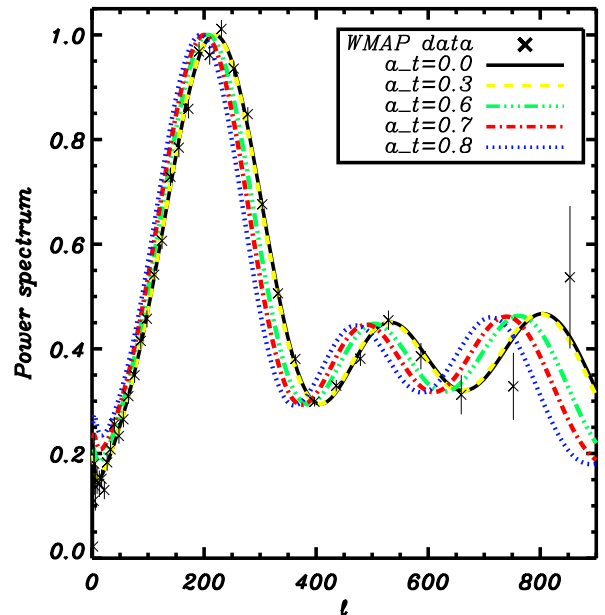


Fig. 4. Angular power spectrum of CMB fluctuations for models presenting a transition at epoch $a_t = 1/(1+z_t) = 0, 0.3, 0.6, 0.7$ & 0.8 compared with WMAP TT spectrum. Apart from the large scale (low l) power, the only difference between the spectra is a shift in the spectrum as a result of the modification of the angular distance to the last scattering surface.

redshift for a transition which remains of the order of 0.5. The situation is therefore rather paradoxical: although space survey precision improves the constraints by pushing the acceptable redshift from 0.16 to 0.5, this last result appears intrinsically rather modest as a significant fraction of the high precision data provided by the experiment is above this redshift.

4.2. Constraints from the CMB

CMB is known to provide interesting constraint on quintessence scenario although the best constraints are obtained through combinations of various data (Spergel et al. 2003; Douspis et al. 2003; Ödman et al. 2004). It is natural therefore to examine CMB constraint on the type of models we have introduced above. For this purpose we use a version of the CAMB cosmological code (Lewis et al. 2000) that we have modified. Modification of the code are quite straightforward since the public version of it includes models with constant w_Q in which what has to be put by hand is the value of the energy density $\rho_Q(z)$ and the pressure $P_Q(z)$ as a function of the redshift. Since with our ansatz for the equation of state parameter $w_Q(z)$ allows to integrate the conservation equation to obtain an analytical form for $\rho_Q(z)$ it almost only suffices to include the solution of Eq. 3 in the code¹.

¹ One has also in addition to make the distinction between the equation of state parameter $w_Q = P_Q/\rho_Q$ and the “sound speed” squared $c_{sQ}^2 \equiv \dot{P}_Q/\dot{\rho}_Q$, which are identical when w is constant.

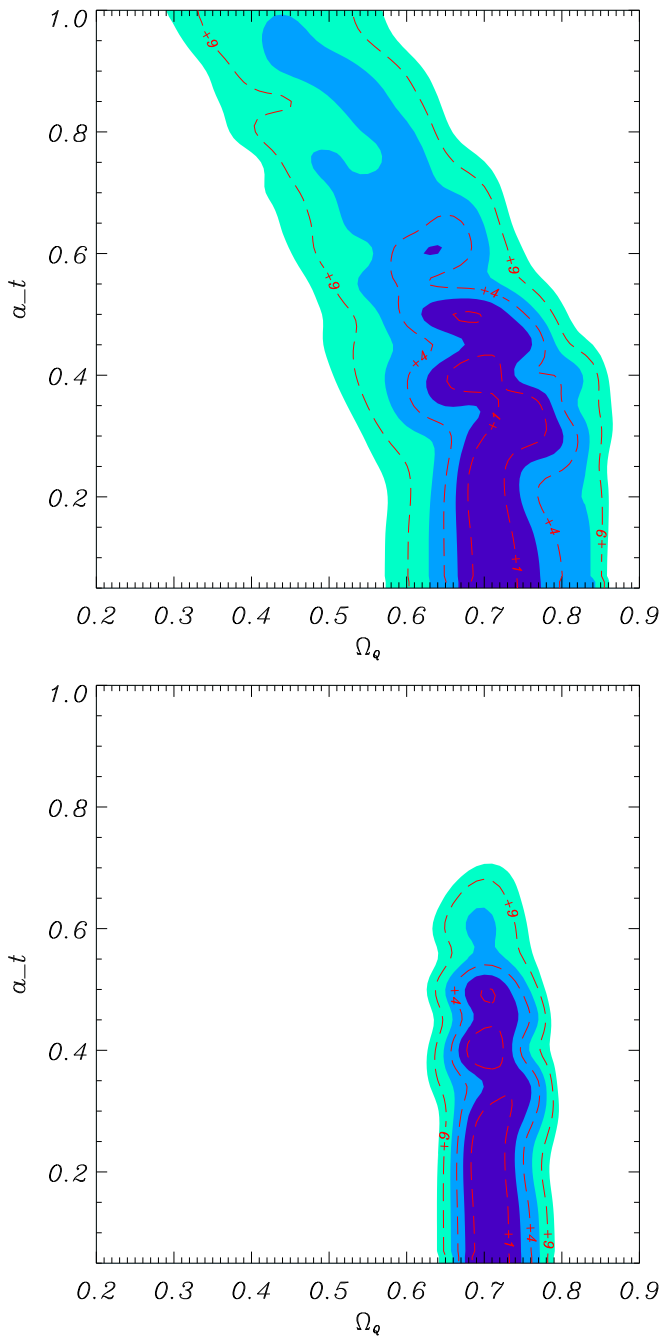


Fig. 5. Contour constraints on the transition epoch $a_t/a_0 = 1/(1+z_t)$ versus the value of the dark energy density parameter at present time, Ω_Q from the CMB alone (upper) using WMAP, CBI, VSA, boomerang). Constraints on the same quantities when combined with supernovae (lower). Areas are 1, 2, 3 σ levels contours on two parameters. Lines are 1, 2, 3 σ levels contours on one parameter. Likelihood has been obtained by maximizing on other parameters.

Angular power spectrum of CMB fluctuations in the presence of dark energy is modified mainly through the modification of the angular distance (Blanchard 1984). The angular distance to the CMB being known very accurately from WMAP results, tight constraints are expected. In addition, ISW

will contribute to a lower and lower level as the transition is assumed to be at lower and lower redshift, and this effect also contribute to modify the angular power spectrum of the CMB fluctuations. We have investigated the CMB constraints on models with the above type of rapid transition. For this purpose Γ was set to 10. We have checked that varying Γ above 5 produces no appreciable differences. The others parameters which were let free were the Hubble constant H_0 , the dark energy density at present day Ω_Q , the index of the primordial spectrum n , the amplitude of fluctuations and the transition epoch $a_t = 1/(1+z_t)$. From the contours obtained on figure 5, one can see that the constraints that can be set on the transition redshift z_t from the CMB are rather stringent, $z_t > 1.8$ (2σ on parameter) when Ω_Q represents less than 10% of the total density. These constraints being slightly dependent on H_0 we have also examined whether a combination of CMB and supernova allows to improve the transition epoch constraints, but although the SNIa data restricted the dark energy density much more around $\Omega_Q \sim 0.7$ the final constraints does not represent a significant improvement: as SNIa slightly prefer a transition the final constraint is $z_t > 1.5$ (2σ on one parameter). One can wonder whether additional constraints currently used in combination on dark energy model may help to improve the above limit. For instance, the amplitude of matter fluctuations is known to be a sensitive function of the nature of dark energy (Caldwell et al. 1998; Douspis et al. 2003), and one can wonder whether such additional constraint could be useful. However, as transition is allowed only when dark energy represents less than 10% of the total density, observables are likely to have values very close to those obtained with the standard cosmological constant. We have checked explicitly that the amplitude of matter fluctuations is correlated with the transition epoch, but that for models within the two σ contours (corresponding to models with $z_t > 1.8$), the amplitude does not change by more than 10%, a precision which is not obtainable robustly by present days given the systematics uncertainties on σ_8 (Blanchard & Douspis 2005).

5. Conclusion

We have investigated a class of models which undergo a rapid transition in the equation of state of their dark energy component. In order to establish the constraints that can be obtained on the characteristics of the transition we have concentrated our study on a class of models which dark energy transit rapidly between $w \sim 0$ and $w \sim -1$. Quite logically we found that the duration of the transition cannot be constrained as soon as it is shorter than the Hubble time. More surprisingly we found that SNIa Hubble diagram does not constraint very much this type of scenario, as even with the data expected from space experiments as transition can still be allowed at epoch when the dark energy density represents 40% of the density of the Universe. This suggests that the SNIa diagram would hardly catch dynamic of dark energy at redshift above 0.5. On the contrary, we found that by adding existing CMB data we already provide tight constraints on this type of scenario, rejecting possible transition at redshift beyond 1.5 when dark energy represents less than 10% of the total density of the Universe.

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