Thermomechanical Cutting Model Discretisation. Eulerian or Lagrangian, Mesh or Meshless?

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ABSTRACT. *Despite of the significant technological progress in machining, at present it is very difficult to model cutting processes in order to predict optimal process performance. Usually, Eulerian, Lagrangian and ALE (arbitrary Lagrangian Eulerian) methods have been adopted for modeling cutting process. Lagrangian formulations allow accurate representations of the domain boundary evolution, however it requires a remeshing procedure. Eulerian formulations have been used in order to compute steady-state solution. However, these Eulerian techniques lead to accurate simulations of processes involving large deformations, as presented in this paper. In the context of Lagrangian simulation, the use of a new meshless technique, called the Natural Element Method, is proposed. This technique allows treating large deformations without remeshing requirements.*

KEYWORDS: *Cutting simulation, Eulerian and Lagrangian descriptions, Meshless methods, Natural Element Method.*

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1. Introduction

Despite of the significant technological progress in machining, at present it is very difficult to model cutting processes in order to predict optimal process performance. Material cutting is a very complex forming process involving large strains, severe shear and friction, thermomechanical couplings and thermoelastoviscoplastic behaviour, metallurgical and chemical transformations with material diffusion, material damage and fracture. Moreover, an accurate description of these phenomena requires a multiscale approach as proposed in several works. A good overview of the state of the art on this topic is given, for example, in the proceedings of the second CIRP International Workshop on Modeling of Machining Operations (2000), as well as in the proceedings of the fourth and fifth ESAFORM Conferences (2001 and 2002) where a minisymposium was devoted these problems (see for example Touratier and Lorong, 2000; Ueda et al., 2000; Ceretti et al., 2001; Saanouni et al., 2002).

In the past, Eulerian, Lagrangian and ALE (arbitrary Lagrangian Eulerian) methods have been adopted. Even if the Lagrangian or updated Lagrangian formulations allow accurate representations of the domain boundary evolution (Ali, 2001), the large deformation of the material results in a large distortion of the finite element mesh. Then, a remeshing procedure is frequently carried out. Eulerian formulations have been used in order to compute steady-state analysis (Joyot et al. 1998). However, Eulerian techniques lead to accurate simulations of processes involving large deformation (which occurs in the chip formation zone) using a fixed mesh. The Eulerian techniques, widely used for simulating complex fluid flows, allow to predict accurately the position of free and moving boundaries. In this paper two different Eulerian strategies are considered, and we will focus on surface tracking aspects. The first one concerns a pseudo-behavior or penalty technique, and the second one makes use of a level set function in order to locate the material boundaries.

The main limitation of Eulerian techniques is related to the treatment of convective terms in the motion and constitutive equations. This problem is avoided by the use of a Lagrangian strategy because in this case the mesh moves with the material, and advection terms are simply treated from the difference between the final and the initial configurations. However, in Eulerian simulations the mesh is fixed in space, and the numerical strategies must take into account the hyperbolic character of the advection terms.

The use of a meshless technique is a good choice since it keeps the advantages of moving meshes for the accurate integration of the transport equations along the nodal flow path lines, and for the representation of free and moving boundaries, avoiding the necessity of remeshing. However, the disadvantage of standard meshless techniques (SPH, RKPM or moving least square) is the use of non-nodal

interpolations, which introduces some difficulties in the imposition of essential boundary conditions. To avoid this problem and solve the equations of motion the use of the Natural Element Method (NEM) is proposed. This technique defines a nodal interpolation (that allows us to impose essential boundary conditions) without the necessity of a mesh support.

The main aim of this work is not the establishment of a thermomechanical model for describing the complex phenomena involved in the cutting processes, but the evaluation of the capabilities of different numerical strategies to describe large material transformations (evolution of complex kinematics). For this reason, a very simple constitutive equation (Norton-Hoff) without taking into account other physical phenomena was considered.

2. Eulerian formulations

2.1. *Material tracking: a Volume of Fluid technique*

Eulerian formulations usually proceed with a fixed mesh. In this case, the location of moving boundaries remains rather difficult. One possibility, widely used in the area of the fluid mechanics, is based on the definition of a material presence variable which takes a unit value in the region occupied by the material and which vanishes in the empty region. Thus, the equations defining the mechanical model must be imposed only in the fulfilled elements. To illustrate the numerical procedure we focus on a simple mechanical model, the Norton-Hoff viscoplastic model, which, neglectes mass and inertia terms. This model is defined by a set of equations, namely

$$\begin{cases} Div\underline{\underline{\sigma}} = \underline{0} \\ Div\underline{v} = 0 \\ \underline{\underline{\sigma}} = -p\underline{\underline{Id}} + 2\eta\left(\underline{\underline{D}}\right)\underline{\underline{D}} \\ \eta\left(\underline{\underline{D}}\right) = k\left(\sqrt{2\underline{\underline{D}} : \underline{\underline{D}}}\right)^{n-1} \end{cases} \qquad [1]$$

where: $\underline{\underline{\sigma}}$ is the stress tensor, $\underline{\underline{D}}$ is the strain rate tensor (the symmetric part of the gradient of velocity tensor), $\underline{v}$ is the velocity field, p is the Lagrange multiplier associated with the incompressibility constraint, $\underline{\underline{Id}}$ is the unit tensor, and k and n are two material parameters. The symbol ":" denotes the product defined by $\underline{\underline{a}} : \underline{\underline{b}} = \sum_{i,j} a_{ij} b_{ij}$. Thus, $\sqrt{2\underline{\underline{D}} : \underline{\underline{D}}}$ represents the second invariant of the strain rate

tensor. The previous model is defined in the material domain Ω_m, which evolves in time, i.e. $\Omega_m(t)$.

In order to simulate material transformations involving very large displacements, the function of material presence *I* defined in any fixed domain Ω containing the material domain during its transformation was introduced (Pichelin and Coupez, 1998), i.e. $\Omega_m(t) \subset \Omega, \ \forall t$. Thus, the function *I* is defined by

$$I(\underline{x},t) = \begin{cases} 1 & \underline{x} \in \Omega_m(t) \\ 0 & \underline{x} \notin \Omega_m(t) \end{cases} \qquad [2]$$

A possible extended variational formulation defined in the whole domain Ω is given by

$$\int_\Omega \left(f(I)\underline{\underline{\sigma}} : \underline{\underline{D}}^* + \alpha(I)(1-f(I))\underline{v}\underline{v}^* \right) d\Omega = 0 \qquad [3]$$

and

$$\int_\Omega \left(f(I) Div\underline{v}\, p^* + \alpha(I)(1-f(I)) p p^* \right) d\Omega = 0 \qquad [4]$$

where the pseudobehaviour $\underline{v} = \underline{0}$ and $p = 0$ are enforced in the empty region ($f(I=0)=0$ and $f(I=1)=1$). The function $f(I)$ for intermediate values of the material presence function $0 < I < 1$, and the function $\alpha(I)$ must be chosen in order to guarantee the scheme conservation and an accurate moving boundary location (the thickness of the region for which 0<*I*<1 must be minimized). When $f(I) = I$ and $\alpha(I)$ is a constant small enough, the scheme proposed by Pichelin and Coupez, (Pichelin and Coupez, 1998), is obtained. However, Monton (Monton, 2002), proved that taking

$$f(I) = \begin{cases} 1 & I_{th} < I \le 1 \\ 0 & 0 \le I \le I_{th} \end{cases} \text{ and } \alpha(I) = \begin{cases} 1 & I = 0 \\ \varepsilon & I > 0 \end{cases} \qquad [5]$$

where I_{th} is a threshold value close to 1 and ε is a small enough constant, the thickness of the moving boundary is reduced to one or two elements. The variational formulation (Eqs. [3] and [4]) can be solved accurately, in a fixed mesh associated to the whole domain Ω, using a standard mixed finite element formulation. The

functional spaces used in the interpolation of the velocity and pressure variables must verify the "inf-sup" condition. Thus, in our simulations an enriched linear approximation is considered for the velocity interpolation, whereas a linear approximation is used for the pressure interpolation ($P_{1+buble}$-P_1).

On the other hand, the evolution of the function I is given by the following linear advection equation

$$\frac{dI}{dt} = 0 \Rightarrow \frac{\partial I}{\partial t} + \underline{v}\, GradI = 0 \qquad [6]$$

Due to its hyperbolic character, appropriate discretisation techniques are needed. One possible solving is based on the use of the discontinuous finite element formulation (Chinesta, 2002).

In spite of the reduced numerical diffusion and mass conservation accuracy resulting from the application of the numerical schema described, the exact position of the moving boundary remains uncertain. Thus, the imposition of surface tractions remains a delicate matter. In order to improve the domain boundary localization the level set method can be applied as noticed in the next section.

2.2. *Material tracking: the Level Set Method*

In order to improve the interface localization, we can define a smooth function (level set function) ϕ (see references of Sussman et al.) whose zero value determines the position of the flow front. The equation governing the evolution of this function is given by

$$\frac{\partial \phi}{\partial t} + \underline{u}\, Grad\phi = 0 \qquad [7]$$

where $\underline{u}$ is an arbitrary velocity field which takes the material velocity $\underline{v}$ on the moving boundary $\Gamma(t)$ $(\Gamma(t) = \{\underline{x} / \phi(\underline{x}, t) = 0\})$. This equation can be solved using a numerical technique, taking into account its hyperbolic character. The main difficulty in the application of such techniques is that the quality of the function $\phi(\underline{x}, t)$ is degraded during the process simulation. Thus, the slope of the level function can reach very high or very small values. In both cases a significant error can be introduced in the interface updating (determination of the curve where ϕ vanishes). In order to reduce these numerical errors some researchers proposed to modify the level function at each iteration in order to obtain a unit slope in the flow

direction on the moving boundary. For this purpose, and knowing the level function at time *t*, we need to solve the following non-linear advection equation

$$\frac{\partial \bar{\phi}}{\partial t} + sgn(\phi)\frac{Grad\bar{\phi}}{\left\|Grad\bar{\phi}\right\|}Grad\bar{\phi} = sgn(\phi) \qquad [8]$$

where the function $\bar{\phi}$ is equal to ϕ along the moving boundary. Today, the numerical treatment of non-linear advection equations is a difficult matter (see the works of Sussman et al.). Moreover, an accurate velocity description on $\Gamma(t)$ is also required, which is not simple if one proceeds using a penalization technique as described in the previous section.

2.3. *Taking into account the material history*

Using an Eulerian description, the material history can be defined from some variables whose evolution is governed by an advection equation. One of these variables is the gradient of deformation $\underline{\underline{F}}$, whose evolution verifies the equation

$$\frac{d\underline{\underline{F}}}{dt} = \frac{\partial \underline{\underline{F}}}{\partial t} + \underline{v}\,Grad\underline{\underline{F}} = Grad\underline{v}\;\underline{\underline{F}}; \qquad \underline{\underline{F}}(t=0) = \underline{\underline{Id}} \qquad [9]$$

Equation [9] can be solved applying the same numerical scheme used to integrate Eq. [6]. The main difficulties found in this explicit integration are related to the field updating through the moving boundary (Chinesta, 2002).

3. The natural element method: an efficient meshless Lagrangian strategy

Concerning the treatment of convective terms, one of the most accurate integration techniques is the method of characteristics. This technique proceeds integrating along the material trajectories, which correspond to the nodal pathlines in a Lagrangian description. However, it is well known that the use of a Lagrangian formulation in finite elements requires frequent remeshing. This is because the mesh becomes quickly highly distorted. In this case a field projection between the old and new meshes is required. Moreover, in 3D simulation this remeshing procedure is not an easy matter. It is in this context where the so-called meshless methods have been introduced. The most popular are the SPH (Smooth Particle Hydrodynamics) and the Moving Least Square techniques (MLS), for which the main limitation is the imposition of the essential boundary conditions. One possibility to avoid this problem and solve the equations of motion is to consider the Natural Element Method (NEM). This technique defines a nodal interpolation (that facilitate the

imposition of essential boundary conditions). However, the main advantage of using this technique is that the interpolation accuracy does not depend on the regularity of the nodal distribution. Thus, we can simulate large material deformations without remeshing.

The NEM is based on the natural neighbor interpolation scheme (Sibson, 1981; Watson 1981), which relies on the concepts of Voronoi diagrams (Voronoy, 1908) and Delaunay triangulations (Delaunay, 1934) (see figure 1), to build Galerkin trial and test functions. These are defined as the Natural Neighbor coordinates (also known as Sibson's coordinates) of the point under consideration, that is, with respect to figure 2, the value of the shape function associated with the node 1 at point $\boldsymbol{x}$ is defined as

$$\phi_1(\boldsymbol{x}) = \frac{A_{abf\,e}}{A_{abcd}} \qquad [10]$$

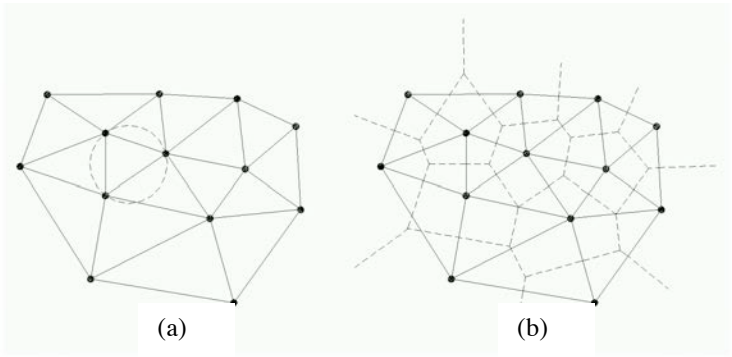

Figure 1. Delaunay triangulation (a) and Voronoy diagram (b)

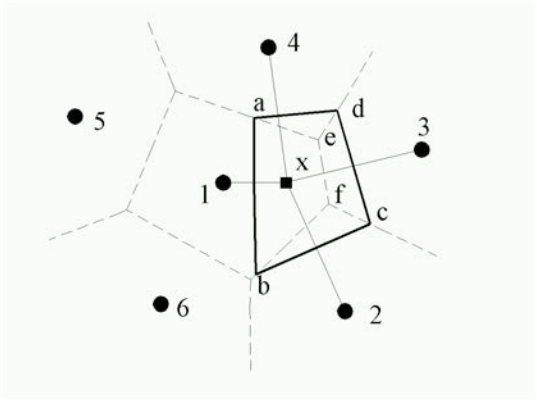

Figure 2. Definition of the Natural Element shape functions

These functions are used to build the discrete system of equations arising from the application of the Galerkin method in the usual way. It has been proved (see Sukumar et al., 1998) that angles of the Delaunay triangulation are not influencing the accuracy of the results, in contrast to the standard FEM.

In addition, the NEM has interesting properties such as linear consistency and smoothness of the shape functions (natural neighbor coordinates are C^{∞} everywhere except at the nodes, where they are C^0). These functions are dependent on the position and density of nodes, leading to standard FE constant strain triangle shape functions, bilinear shape functions or rational quartic functions in different situations.

But, perhaps, the most interesting property of the Natural Element Method is the Kronecker delta property, i.e. $\phi_i(x_j)=\delta_{ij}$. In contrast to the vast majority of meshless methods, the NEM shape functions are strictly interpolant. This property allows an exact reproduction of linear (even bilinear in some 3D cases) fields on the boundary of convex domains, since the influence of interior points vanishes along convex boundaries (this is not true in the non-convex ones). Proofs of this behaviour can be found in (Sukumar, 1998), and references therein.

A Galerkin technique is considered for the discretisation of the previous mixed variational formulation (Eqs. [3] and [4]). In this work we have chosen a C^0-C^{-1} interpolation for velocity and pressure fields, respectively. A discontinuous field has been considered for the pressure approximation, given by

$$p^h(\underline{x}) = \sum_{I=1}^{n} \frac{1}{n} p_I \qquad [11]$$

where n is the number of natural neighbors of point $\underline{x}$ and p_I denotes the nodal pressures.

No spurious pressure modes and no locking have been detected in the implementation of the method. Although it is known that this approximation does not verify the "inf-sup" condition, its behavior has been proved to be similar to the bilinear quadrilateral (BLQ) finite element with discontinuous constant pressure interpolation for incompressible elasticity in a regular nodal distribution.

In (Cueto et al., 2000) the issue of imposing essential boundary conditions is analyzed. Two main approaches are possible: in the first one the boundary of the material domain is not explicitly defined (in a CAD sense) and in the second one it is defined. The first approach leads to the alpha-shape based Natural Elements Method, which was applied in the examples presented in this paper. Thus, the material domain boundary can be automatically extracted from the cloud of nodes by using the alpha-shape concepts, and an exact reproduction of linear fields on the boundary of convex or non-convex domains can be obtained.

4. Numerical results

In order to illustrate the ideas introduced previously we consider a simple geometry (see Fig. 3) of a material piece (its behaviour is modeled by the Norton-Hoff constitutive equation), which moves from the left to the right with an imposed velocity.

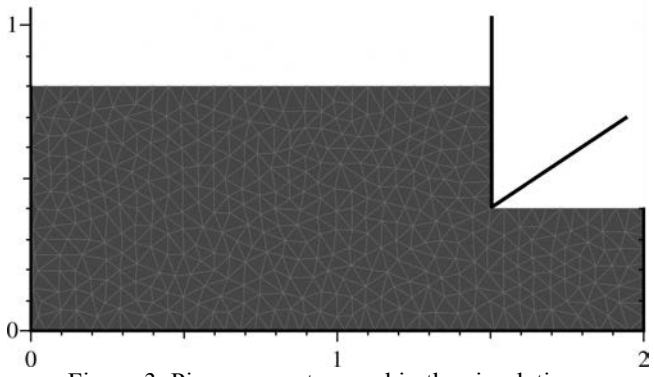

Figure 3. Piece geometry used in the simulations

The chip formation has been widely simulated using Lagrangian techniques for simple material behaviour (Norton-Hoff) and for more realistic elastoviscoplastic behaviors (Ali, 2001). Remeshing operations require a non negligible CPU time, and as previously indicated, the efficient treatment of 3D models remains today an open problem.

A first possibility lies in using an Eulerian description. In this case, as previously described, we consider a domain Ω containing the piece and a fixed mesh of the whole domain (see Fig. 4)

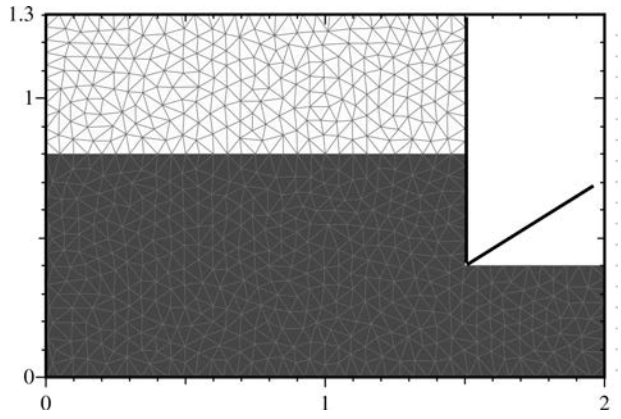

Figure 4. Mesh of the pseudo-domain containing the material piece

Now, we can solve the extended variational formulation (Eqs. [3] and [4]) using a mixed finite element technique and a Norton-Hoff behavior law. The Newton's technique is used to treat the non-linearity. Figures 5 and 6 depict the first velocity field obtained for values of the power index n=1 (Newtonian fluid) and n=0.5 respectively.

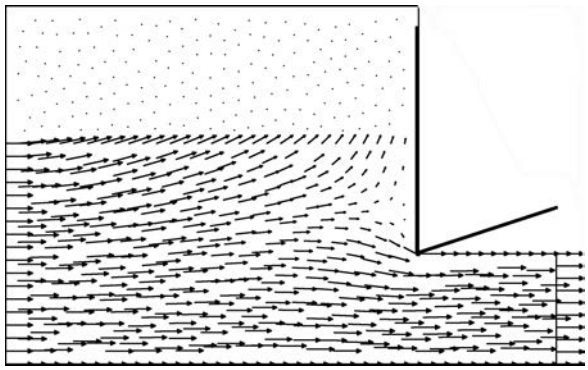

Figure 5. Velocity field at the first time step obtained for a power index value n=1

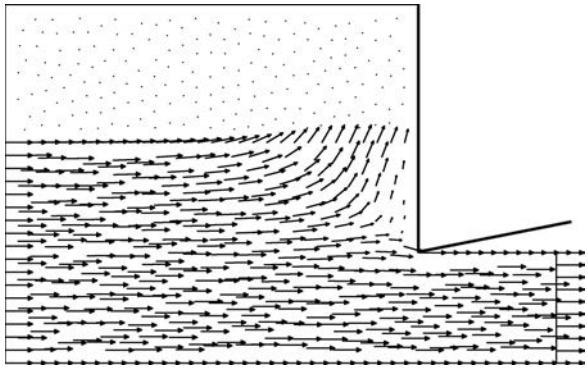

Figure 6. Velocity field at the first time step obtained for a power index value n=0.5

It can be noticed froms Figs. 5 and 6 that the deformation is much more localized for small values of power index n. The same effect can be obtained using the Bingham model (which introduces a yield stress) or its regularized version (Papanastasiou, 1987). It can also be observed that the introduction of the pseudobehaviour in the empty region does not affect the mechanical state of the piece. From the computed velocity field we can update the function I defining the

material domain, to solve again the extended variational formulation to obtain the kinematics and the stresses in the updated material domain. Thus, the chip formation can be simulated with proper accuracy in a fixed mesh. Figure 7 illustrates the beginning of the chip formation when the power index is equal to n=0.5.

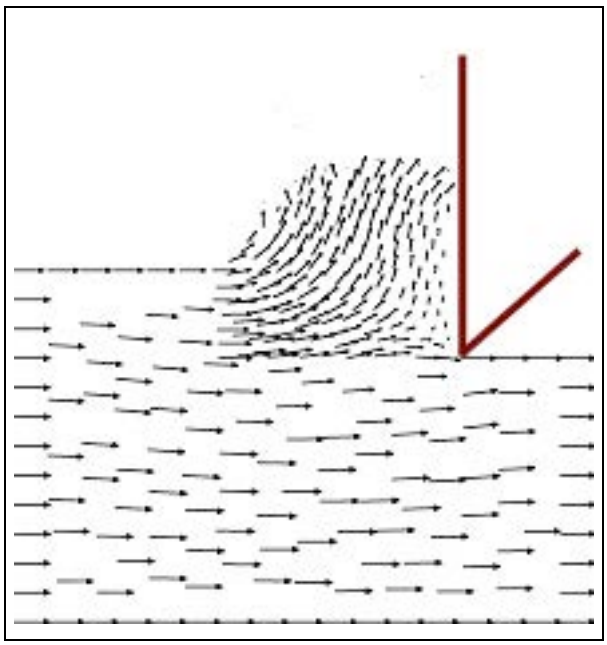

Figure 7. Chip formation based on a fixed mesh description

Figure 8 shows the solution found by using the NEM for a power index value of n=0.3. In spite of the very large deformations that can be appreciated in the sequence shown in figure 8, no nodal redistribution is required in order to assure the computation accuracy, even if the Delaunay's triangulation (which is the dual structure of the Voronoy tessellation from which the interpolation is defined) contains after few iterations a lot of triangles extremely distorted. Thus, the nodal position is updated directly from the velocity field.

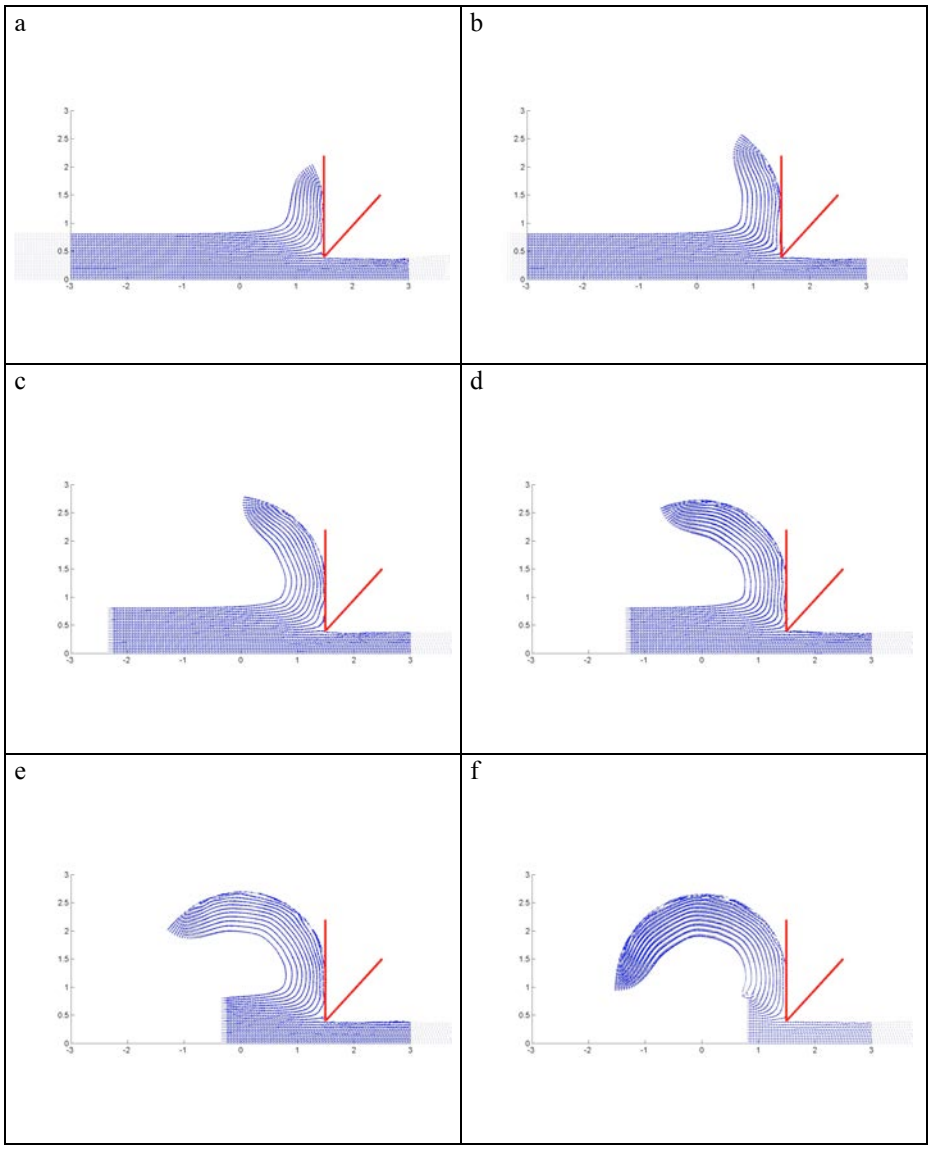

Figure 8. Chip formation simulation using the Natural Element Method

Figure 9 depicts a function of the equivalent strain rate. It can be noticed that the localization of the strain rate is well predicted in spite of the coarse and uniform nodal distribution. However, as it could be expected, the introduction of new nodes or their elimination is a very easy task, because in both cases any nodal redistribution to ensure an interpolation accuracy is required. Thus, the introduction

of a new node in a region of the domain to improve the solution accuracy at that zone (for example to approximate high gradients) only requires the interpolation of the different fields at that node from their values at the neighbor nodes using the standard natural elements interpolation.

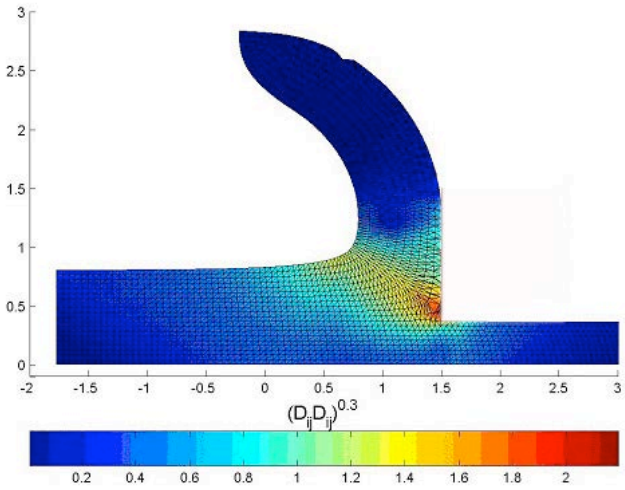

Figure 9. Strain rate localization in the chip formation zone

6. Conclusions

Eulerian simulations of cutting processes using a fixed mesh can be carried out accurately, however some difficulties persist related to the treatment of the advection equations governing the material domain evolution and consequently the position of the domain boundaries. Moreover, the constitutive equations, taking into account the material deformation history, also introduce advection terms.

The use of a meshless technique is a good choice since it keeps the advantages of moving meshes for the accurate integration of the transport equations along the nodal flow path lines, and for the representation of free and moving boundaries, avoiding the necessity of remeshing. However, the disadvantage of standard meshless techniques (SPH, RKPM or moving least square) is the use of non-nodal interpolations, which introduces some difficulties in the imposition of essential boundary conditions. To avoid this problem and solve the equations of motion the use of the Natural Element Method (NEM) is proposed. This technique defines a nodal interpolation (that allows us to impose essential boundary conditions) without the necessity of a mesh support. Preliminar results involving a simple constitutive model (Norton-Hoff) in the simulation of a chip formation have been presented,

which prove the capabilities of that technique to the discretisation of more complex thermomechanical cutting models.

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