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# Collision statistics in a dilute granular gas fluidized by vibrations in low gravity

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**Abstract.** – We report an experimental study of a dilute “gas” of inelastically colliding particles excited by vibrations in low gravity. We show that the mean collision frequency with a container wall does not scale linearly with the particle number  $N$  in the container, and that the probability density function of the impulse distribution has an exponential tail. The anomalous scaling with respect to  $N$  shows that the kinetic energy is not an extensive variable (i.e., the granular temperature is not intensive). This scaling is recovered with numerical simulations and is due to the dissipative nature of the collisions.

*Introduction.* – Since Maxwell and Boltzmann predictions in 1860-70, it is well-known that molecules of a gas move erratically with a Gaussian velocity distribution, as experimentally verified later [1]. This allows thermodynamic and transport properties of usual gases to be described. However, these kinetic theory results do not hold if the particle interactions are dissipative or depend on their velocities (e.g., in relativistic plasmas [2]). A well-known example of dissipative gas is the granular gas (see [3,4] for a recent collection of papers). Since collisions between granular particles are inelastic, a continuous input of energy (by vibrating a piston or the container) is required to reach a nonequilibrium steady state. In this regime, granular matter sometimes seems to behave like an usual gas in which particles follow erratic motions, but several experiments have displayed striking different properties: Instability of the homogeneous density state leading to cluster formation [5–7], non-Gaussian nature of the velocity distribution [8], anomalous scaling of the pressure [7,9]. These effects have been also numerically simulated and some of them have been theoretically understood [10].

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In this paper, we report a 3D experiment of a dilute granular medium fluidized by sinusoidal vibrations in a low gravity environment. The motivation for low gravity is to achieve an experimental situation in which inelastic collisions are the only interaction mechanism, and where only one “input” variable (the inverse vibration frequency) has the dimension of time [6]. This eliminates possible resonances between the time of flight of a particle under gravity and the period of vibration. The aim is to observe new phenomena which result from the inelasticity of the collisions, thus absent in usual gases, and from the absence of scale separation (mean free path comparable to the size of the container). We measure the scaling of the collision number with a container wall, the time lag distribution and the impulse distribution with the vibration velocity,  $V$ , and the particle number,  $N$ . We show in particular that two measurements display significant differences from the behaviors observed in usual gases: the scaling of the collision number with  $N$ , and the scaling of the particle impulse distribution with  $N$  and with  $V$ . For granular gases, the scaling with  $V$  of global quantities (e.g., granular temperature) has been extensively investigated [3], but there exists only one 2D experiment for the scaling with  $N$  [11]. We emphasize that velocity distributions in granular gases have been measured so far only for nearly 2D geometries. In the 3D case, it is much easier to measure the distribution of impact velocities at a boundary as done here. This measurement involves a similar information content and can be easily compared to molecular dynamics simulations.

*Experimental setup.* – A fixed transparent Lexan tube,  $D = 12.7$  mm in inner diameter and  $L = 10$  mm in height, is filled with  $N$  steel spheres,  $d = 2$  mm in diameter. The total number of particles varies from  $N = 12, 24, 36$  to 48 roughly corresponding to  $n = 0.3, 0.6, 0.9$  and 1.2 particle layers at rest (packing fraction from 0.04 to 0.18). A piezoelectric force sensor (PCB 200B02), 12.7 mm in diameter, is fixed at the top of the cell in order to record the particle collisions with the upper wall. A piston made of duralumin, 12 mm in diameter, is driven sinusoidally at the bottom of the cell by an electromagnetic shaker at frequency  $f$ , and maximal displacement amplitude  $A$  in the ranges 40 to 91 Hz and 0.4 to 2 mm, respectively. The vibration parameters during the time line are listed in Table I. Vibration amplitudes are measured by piezoelectric accelerometers (PCB 356A08) screwed in the shaft in a triaxial way. Typical output sensitivities in the vibration direction and in the perpendicular directions are, respectively, 0.1 and 1 V/ $g$ , with  $g = 9.81$  m/s<sup>2</sup> the acceleration of gravity. Typical force sensor characteristics are a 11.4 mV/N output sensitivity, a 70 kHz resonant frequency, and a 10  $\mu$ s rise time. Low gravity environment (about  $\pm 5 \times 10^{-2}g$ ) is repetitively obtained by flying with the specially modified *Airbus A300 Zero-G* aircraft through a series of parabolic trajectories which result in low gravity periods, each of 20 s. An absolute acceleration sensor allows the detection of the low gravity phases and the automatic increment of the vibration parameters after each parabola. During each parabola, the output signals of force, respectively accelerations, are stored on 16 bits on a computer with a 2 MHz, respectively 10 kHz, sampling rate on 12 bits.

*Detection of collisions.* – A typical time recording of the force sensor shows a succession of peaks corresponding to particle collisions with this “wall”, as displayed in Fig. 1 for 10 periods of vibration. Bursts of peaks roughly occur in phase with the vibration but the number of peaks in each burst and their amplitude are random (see Fig. 1). A peak corresponds to the collision of a single sphere, which leads to an almost constant impact duration from 5 to 6  $\mu$ s for our range of particle velocities  $v_{imp}$  (assumed of the order of  $V$ ). Indeed, the Hertz’s law of contact between a sphere of radius  $R$  and a plane made of same material, leads to a duration of collision  $\tau = ER/v_{imp}^{1/5}$ , where  $E = 6.9 \times 10^{-3}$  (s/m)<sup>4/5</sup> for steel [12]. The signal recorded

$N$	$A$ (mm)	$f$ (Hz)	$V$ (m/s)	$\Gamma$	$N_c$	Symbols in Fig. 3
12	0.92	40	0.23	5.9	2591	$\times$
12	0.65	59.7	0.24	9.3	2605	$\circ$
12	0.88	80	0.44	22.7	5756	$\bullet$
12	0.64	90.9	0.37	21.4	4617	$+$
24	0.96	40	0.24	6.2	5097	$*$
24	0.67	59.7	0.25	9.6	4078	$\diamond$
24	0.89	80	0.44	22.8	8362	$\nabla$
36	0.44	40	0.11	2.8	2538	$\triangle$
36	0.67	59.7	0.25	9.7	6496	<i>penta.</i>
36	0.89	80	0.44	22.8	9744	$\circ$
36	0.69	90.9	0.39	22.9	9741	$\times$
48	0.42	40	0.11	2.7	2728	$\times$
48	0.69	59.7	0.26	9.9	8650	<i>hexa.</i>
48	0.89	80	0.45	22.9	10906	$\circ$
48	0.73	90.9	0.41	24.2	12512	$\square$

TABLE I – *Vibration parameters during each parabola of 20 s of low gravity.  $V = 2\pi Af$  and  $\Gamma = 4\pi^2 Af^2/g$  are respectively the maximal piston velocity and the dimensionless acceleration of vibration. The collision number  $N_c$  on the sensor is detected during 16 s of low gravity to avoid transient states.*

by the sensor corresponds to an impulse response,  $I(t)$ . Each peak due to a collision is thus followed by an oscillatory tail at the sensor resonance frequency (roughly 100 kHz) damped over  $500 \mu\text{s}$  (see inset of Fig. 1). A thresholding technique is applied to the signal in order to detect the collisions. We have to discard a time interval of  $100 \mu\text{s}$  around each detected peak in order to avoid counting the first maxima of each oscillatory tail as additional collisions. Thus an additional weak collision occurring in the oscillatory tail due to the previous one may be missed by our detection process. However, the discarded time interval is small compared to the mean time lag (a few ms) between successive collisions if their statistics is assumed Poissonian (see below). Consequently, the probability of possibly discarded collisions is small.

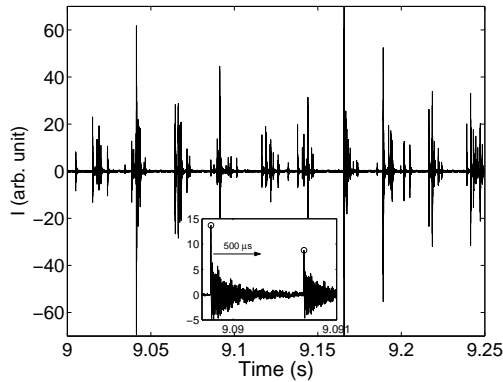


Fig. 1 – Typical time recording of the force sensor [impulse response  $I(t)$ ] during 10 periods of vibration showing 106 collisions. Inset: zoom of this signal during 1.3 ms showing two collision peaks detected ( $\circ$ ) and the typical damping time of the oscillatory response of the sensor. The parameters of vibration are:  $N = 12$ ,  $f = 40$  Hz,  $A = 1.96$  mm (not listed in Table I).

*Collision frequency scaling.* – The collision number  $N_c$  with the top wall (i.e., the sensor) is obtained by the previous thresholding technique, for each parameter listed in Table I, during  $\theta = 16$  s of low gravity to avoid possible transient states. For a fixed number of particles,  $N$ , Fig. 2 shows that  $N_c$  is proportional to the maximal piston velocity,  $V$ , for  $0.1 \leq V \leq 0.5$  m/s. As also shown in Fig. 2, the collision frequency,  $\nu_c = N_c/\theta$ , is found to scale like  $\nu_c \propto VN^\alpha$ , with  $\alpha = 0.6 \pm 0.1$  for our range of  $N$ . This result strongly differs from the kinetic theory of usual gases for which  $N_c$  varies linearly with  $N$ . It cannot be explained either in the very dilute limit (Knudsen regime). Indeed, assuming that each particle mostly collides with the boundaries of the container and does not interact with others, leads to  $\nu_c \propto VN/[2(L-d)]$ . Therefore particles do interact significantly with each other through inelastic collisions. We will show below that this anomalous scaling is recovered in numerical simulations and is a consequence of the dissipative nature of collisions.

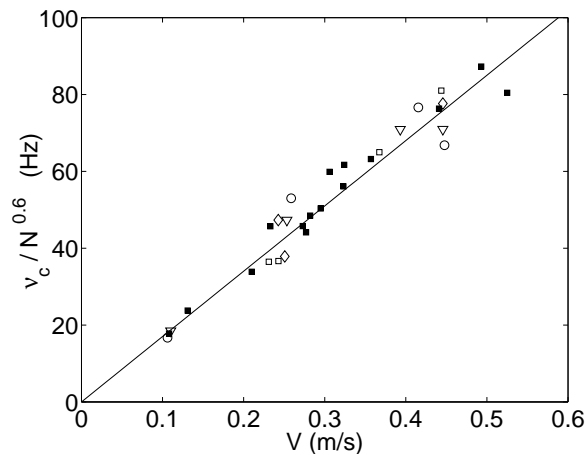


Fig. 2 – Frequency of collision rescaled by the number of particles,  $\nu_c/N^{0.6}$ , as a function of  $V$  for  $N = 12$  ( $\square$ ) and ( $\blacksquare$ );  $24$  ( $\diamond$ );  $36$  ( $\nabla$ );  $48$  ( $\circ$ ).  $\blacksquare$ -marks are from a previous set of experiments at fixed  $N = 12$  for 15 different velocities which are not listed in Table I. Solid line corresponds to the fit  $\nu_c/N^{0.6} = V/l_0$  where  $l_0 \simeq 5.9$  mm (see text for details).

*Time lag distribution.* – The probability density functions (PDF) of the time lag  $\Delta t$  between two successive collisions with the top wall is displayed in Fig. 3 for 4 different values of  $N$  and various parameters of vibration. These PDFs are found to be exponential and to scale like  $V$  for our range of  $V$ . This exponential distribution for the time lag statistics is the expected one for Poissonian statistics. As already shown for the data of Fig. 2, these PDFs can be collapsed by the  $N^{0.6}$  rescaling. Since  $V\Delta t$  has the dimension of length, this PDF can be seen as the distribution of the particle mean free path  $l_S$  between 2 collisions *with the sensor*. Thus, for each value of  $N$ , we can extract  $l_S$  from the inverse of the slope of the PDF in Fig. 3. This leads to  $l_S = V/\nu_c = l_0/N^{0.6}$ , where  $l_0 \simeq 5.9$  mm. Thus,  $l_S$  is roughly an order of magnitude smaller than the box size ( $L = 10$  mm). We also observe in Fig. 3 that even the largest values of  $\Delta tV$  are smaller than  $L$ . In our range of  $N$ ,  $l_S$  is a few times smaller than the mean free path  $l = \Omega/(N\pi d^2)$  as usually defined in kinetic theory. Thus, the Knudsen number,  $K = l/L$ , is in the range  $0.1 - 1$  in our experiments. We are thus in a transition regime from a Knudsen regime to a kinetic regime. It corresponds to a crossover between the very dilute regime for which each particle mostly collides with the boundaries (mean free path

of order  $L$  independent of  $N$ ), to the kinetic regime (mean free path  $l$  inversely proportional to  $N$ ).

Finally, if the amplitude of vibration,  $A$ , is not negligible with respect to  $L$  (i.e.,  $A/L \geq 0.17$ ), the time lag distributions are no longer exponential (not shown here). This transition to a non exponential behavior occurs when the tail of distribution of  $V\Delta t$  reaches the box size  $L$  as  $A$  is increased. We also note that this transition value of  $A/L$  is only slightly smaller than the one corresponding to the first resonance leading to the formation of a shock wave [13].

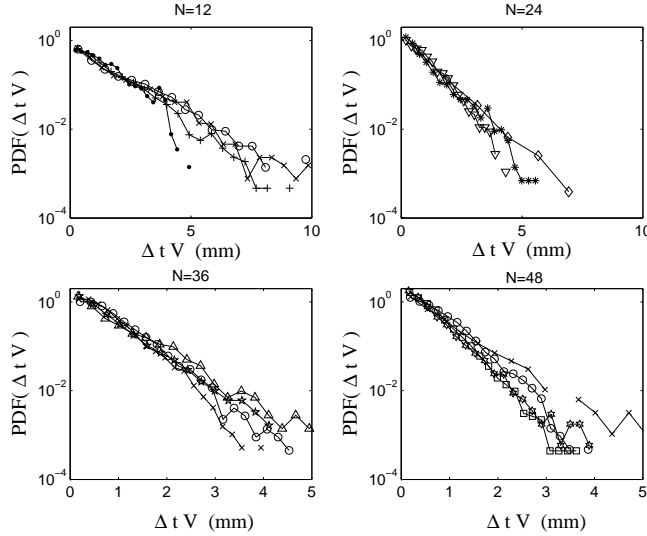


Fig. 3 – Probability density functions of the time lag  $\Delta t$  between two successive collisions rescaled by  $V$  ( $V = 2\pi Af$ ), for  $N = 12, 24, 36$  and  $48$  particles, and for different vibration parameters (see symbols in Table I).

*Impulse distribution.* – The PDFs of the maxima  $I$  of the impacts recorded in Fig. 1 are displayed in Fig. 4 for 4 different values of  $N$  and various parameters of vibration. Note that the low impact events are not result because of noise. Indeed, we expect the PDFs to vanish to zero for  $I = 0$ . We first observe that they scale like  $V$ , for our range of  $V$  (see Fig. 4). Second, they display exponential tails with a slope increasing with the particle number  $N$ . This again shows that there is a significant effect of the interparticle collisions and can be understood because the probability of collisions with large velocities parallel to the one of the piston is decreased for an increasing number of particles. The PDF for different values of  $N$  can be roughly collapsed when  $I$  is scaled like  $V/N^\beta$  with  $\beta \approx 0.8 \pm 0.2$ . We observe that these distributions also differ from kinetic theory of usual gas first because of the dependence on the particle number  $N$  and second due to their exponential tail instead of the Gaussian one. It may be partly linked to a crossover between the Knudsen and the kinetic regimes although this deserves further studies.

*Numerical simulations.* – We have performed numerical simulations in order to understand the scaling with respect to the particle number. An event driven method has been used [14].  $N$  disks of unit mass and diameter  $d$  are enclosed in a square box of size  $L = 100d$  with one boundary vibrating with amplitude  $A = 1.5d$  and unit frequency [14]. The frequency of collision on the opposite boundary is displayed in Fig. 5 as a function of the number of

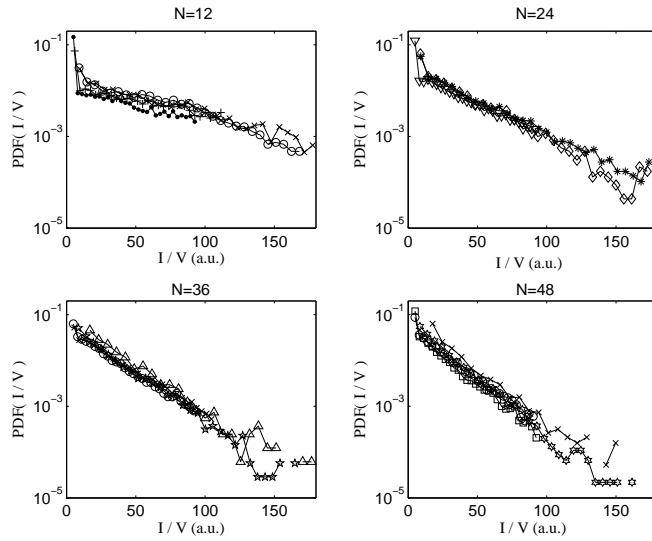


Fig. 4 – Probability density functions of the impulse  $I$  of the impacts on the sensor rescaled by  $V$ , for different vibration parameters. Symbols are the same as Fig. 3.

particle layers  $n = Nd/L$  and the restitution coefficient  $r$ . We first observe that for realistic values of the restitution coefficient for stainless steel ( $r \approx 0.95$  for velocities up to 0.5 m/s), the collision frequency increases like  $N^{0.55}$  from  $N = 2$  up to  $N = 500$  particles (i.e.,  $n = 5$  particle layers). Collision frequencies with different restitution coefficients can be collapsed on a single curve if plotted versus the parameter  $n/(1-r)$ . This is true up to a critical number of particle layers (dependent on  $r$ ) for which a cluster is generated. The collision frequency then abruptly increases (see arrows in Fig. 5).

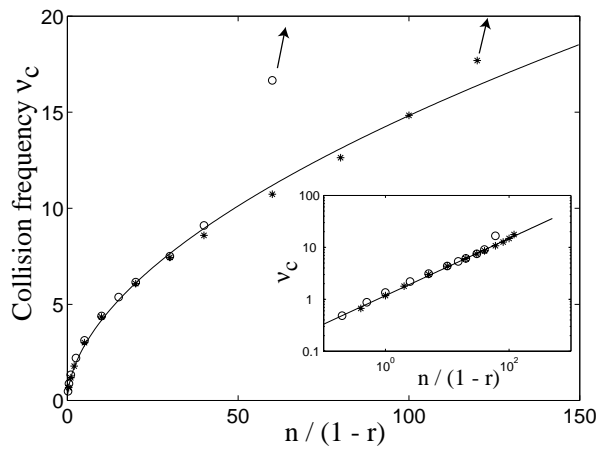


Fig. 5 – Frequency of collision  $\nu_c$  as a function of the number of particle layers  $n$  and the restitution coefficient  $r$ :  $r = 0.9$  ( $\circ$ ) and  $r = 0.95$  ( $\star$ ). Using the variable  $n/(1-r)$  collapses the data up to the formation of a cluster in the granular gas.

Although the simulation is 2D, the behavior of the collision frequency with  $N$  is in very good agreement with the experiment. This behavior results from the dependence of the mean kinetic energy  $\langle E \rangle$  and dissipated power as  $N$  increases [15].

*Conclusion.* – We have reported a 3D experiment of a dilute granular medium fluidized by sinusoidal vibrations in a low gravity environment. We have measured the collision frequency, the time lag distribution and the impulse distribution with the vibration velocity and the particle number. We have shown that two measurements differ significantly from both the kinetic theory of usual gases and the limit of the Knudsen regime  $K > 1$ :

- The scaling of the collision frequency  $\nu_c \sim N^{0.6}V$  with respect to the particle number  $N$ , which is an important consequence of the dissipative nature of collisions, as recovered by numerical simulations.
- The probability density functions of the particle impulse  $\sim \exp(-N^\beta I/V)$  ( $\beta = 0.8 \pm 0.2$ ) also differs from the case with elastic collisions.

\* \* \*

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