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The statistics of gravitational lenses in a clumpy Universe

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ABSTRACT

We evaluate the effect of small-scale inhomogeneities on large-scale observations within the statistics of gravitationally lensed quasars. To this end, we consider a cosmological model whose large-scale properties (dynamics, matter distribution) are the same as in Friedmann–Lemaître models, but whose matter distribution is locally inhomogeneous. We use the well-known Dyer–Roder distances to allow a simple analytical expression of the optical depth τ, and pay particular attention to the different roles played by the different notions of distance (filled beam angular diameter distance and Dyer–Roder distances) when calculating this quantity, following the 1986 prescription of Ehlers & Schneider for a coherent formalism. We find that the expected number of gravitationally lensed quasars is a decreasing function of the clumpiness parameter α.

Key words: gravitational lensing – cosmological parameters – cosmology: observations.

1 INTRODUCTION

One of the major tasks in modern cosmology is the precise determination of the parameters that characterize the assumed cosmological model. In this *direct approach* (according to the terminology of Ellis 1995), a theoretical description of space–time is postulated, the Friedmann–Lemaître–Robertson–Walker (FLRW) model, and its parameters are determined by fitting the observational data.

The cornerstone of observational support for the FLRW model is the existence and high isotropy of the relic cosmic microwave background radiation (CMWB). Ehlers, Geren & Sachs (1968) showed that, if the background radiation appears to be *exactly* isotropic to a given family of observers, then the space–time is *exactly* FLRW. Therefore, together with the Copernican principle, we can prove the Universe to be FLRW just from our own observations of the CMWB. Stoeger, Maartens & Ellis (1995) extended this result to the case of an almost isotropic background radiation, which implies an almost FLRW space–time. This important result is firmer ground for the assumption of the FLRW model to describe the large-scale structure of the Universe, but it also makes clear that we need to understand the departures from a spatially homogeneous model when interpreting observational data. Indeed, departures from perfect homogeneity change the distance–redshift relation, and this has to be taken into account when fitting the FLRW parameters to observations. However, cosmological observations are usually fitted just using relationships derived for homogeneous models.

In recent years, several authors have addressed this problem in the context of the observations of distant Type Ia supernovae (e.g. Holz 1998; Holz & Wald 1998; Kantowski, Kao & Thomas 2000; Sereno, Piedipalumbo & Sazhin 2002; Wang, Holz & Munshi 2002; Pyne & Birkinshaw 2004). It has been shown that the noise due to weak lensing magnification from small-scale matter inhomogeneities yields large errors on the luminosity measurement of high-z supernovae.

In this paper we investigate the possible systematic errors due to neglect of the effects of local inhomogeneities in the distribution of matter when evaluating the cosmological constant Λ from gravitational lens statistics. The statistics of gravitationally lensed multiply imaged quasars have long been considered a useful tool to constrain the cosmological parameters, in particular the cosmological constant (Turner, Ostriker & Gott 1984; Fukugita et al. 1992; Kochaneck 1996), and the properties of the lensing galaxies (e.g. Maoz & Rix 1993; Kochaneck 1993). Recently, Chae (2003) has shown that the observed gravitational lensing rate in the CLASS radio survey yields strong support to a flat cosmological model dominated by vacuum energy, with $\Omega_{\rm m} \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$. The precision of these results is limited at the moment by the uncertainty in the luminosity function of the lensing galaxy population, the density profile and their evolution since $z \simeq 1$ (e.g. Mao 1991; Chae 2003). For instance, Cheng & Krauss (2000) have shown that constraints on the cosmological parameter are strongly dependent on the choice of galaxy parameters (see also the discussion in Kochaneck et al. 1999).

Another major limitation is the fact that only a few systematic surveys for multiply imaged quasars have been completed up to now (Claeskens & Surdej 2002); today, in fact, the statistical

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uncertainties on Ω_Λ are still dominated by the Poisson errors from the small number of gravitationally lensed quasars. In the near future, the most promising source of new lensed quasars will be wide-field surveys (Kuhlen, Keeton & Madau 2004) and targeted follow-ups of newly discovered quasars (Morgan 2002; Morgan et al. 2004). For instance, the Sloan Digital Sky Survey[1] will almost double the number of known gravitational lenses. The next considerable increase in the number of gravitational lenses is expected by new telescopes like the VLT Survey Telescope,[2] which will allow very wide and deep optical surveys in the Southern hemisphere. See also Kuhlen et al. (2004) for a discussion of other gravitational lensing surveys to become operational in the next future.

While these large observational projects will considerably improve the precision of the results and the importance of the tool, they also make it necessary to consider more realistically the details of light propagation through the observed inhomogeneous Universe. As pointed out by Ellis (1995), small-scale inhomogeneities in the matter distribution have a considerable effect on both observations (Dyer & Oattes 1988) and dynamics (Russ et al. 1997) at a large scale. Moreover, since the lensing effects of small inhomogeneities on the propagation of light change the angular diameter distance–redshift relation (Schneider, Ehlers & Falco 1992), we focus here on this specific problem. Many different approaches have been developed to study gravitational lensing in inhomogeneous cosmological models, but the simplest one from an analytical point of view, and yet efficient, is the one proposed by Dyer & Roeder (1972, 1973), in which the effects of local inhomogeneities along the light bundles are described by the so-called clumpiness parameter α (see definition in next section). In the following, we allow the clumpiness parameter to be a direction-dependent quantity, which is a function of both the line of sight to the source and its redshift (see e.g. Wang 1999).

The statistical properties of a sample of gravitational lenses include the frequency of multiply imaged quasars, the distribution of the lenses and source redshifts, the angular separation distribution and the image multiplicity. In this work we will focus on a discussion of the total lensing probability, leaving a detailed discussion of the other statistical properties for a subsequent paper. Moreover, as we focus our attention on an effect that is independent of our present knowledge of the galaxy luminosity function and the dark matter velocity dispersion, we do not consider these aspects in detail.

The paper is organized as follows. In Section 2 we define the cosmological model and discuss the relevant distances in the study of the propagation of light. In Section 3 we calculate the gravitational lensing rate, and in Section 4 we discuss its behaviour as a function of α, considering different gravitational lens models. Finally, systematic effects on the estimate of the cosmological constant are discussed in Section 5, and in Section 6 we sum up our results.

2 ROLE OF COSMOLOGICAL DISTANCES

While cosmological models that are homogeneous at all scales are very successful in describing the overall dynamics and evolution of the Universe, they do not allow a detailed description of lensing phenomena. In fact, all the gravitational lensing phenomena (the bending of light rays, the deformation of images, and the formation of multiple images) are only possible in a clumpy Universe (see e.g. the discussion by Krasińsky 1997). Therefore, a coherent approach needs an inhomogeneous model.

However, in the statistical analysis of gravitational lenses, as well as in the analysis of other astronomical observations, perfect homogeneity is often assumed (see e.g. the discussion in Buchert 2000 on this point). An important reason for this choice is the fact that in homogeneous space–times we have simple relations between the proper distance and cosmological distances, i.e. the luminosity distance and the angular diameter distance (e.g. Kayser, Helbig & Schramm 1997). In inhomogeneous cosmological models, these relations are much more complicated, and Friedmann–Lemaître (FL) distances are not generally a good approximation to be used in the determination of cosmological parameters from a given set of observational data.

The relevant distance in gravitational lensing, the angular diameter distance D, is operationally defined as the square root of the ratio of the area dA of a celestial body to the solid angle dΩ that it subtends at the observer (Schneider et al. 1992):

$$D \equiv \sqrt{\frac{\mathrm{d}A}{\mathrm{d}\Omega}}. \tag{1}$$

In a homogeneous Universe, the angular diameter distance can be derived from the proper distance D_p using the relation

$$D(z) = \frac{D_\mathrm{p}(z)}{1+z}. \tag{2}$$

This relation no longer holds in a clumpy Universe. The basic reason lies in the fact that the proper distance is related to the global geometry of the Universe, while the relation between the angular diameter distance and the redshift is determined mainly by the local matter distribution.

In this paper, we assume a cosmological model that is locally inhomogeneous, but homogeneous at very large scale, according to some density averaging rule (see e.g. Krasińsky 1997). In other words, we assume that the overall dynamics does not differ from the dynamics of a homogeneous cosmological model. This approximation is well justified: By averaging the Friedmann equation, Russ et al. (1997) showed that the influence of small-scale clumpiness on the global expansion factor is negligible (so, for instance, the age of the Universe can be evaluated using the FLRW relation with the Hubble constant, with negligible errors), while the distance–redshift relation is significantly affected (so that measurements of the Hubble constant via measurements of the magnitudes of standard candles do not have negligible systematic errors).

In order to describe the degree of inhomogeneity in the local distribution of matter, we use a generalized notion of the so-called clumpiness parameter (e.g. Dyer & Roeder 1972; Schneider et al. 1992), introduced by Wang (1999). The clumpiness parameter α was first introduced by Dyer & Roeder (1972, 1973, hereafter DR), when writing a differential equation for the angular diameter distance in locally clumpy cosmological models, and it was defined as the fraction of homogeneously distributed matter within a given light cone.

Starting from the equations for scalar optics (e.g. Zel'dovich 1964; Kristian & Sachs 1965), DR derived a second-order differential equation for the diameter of the light ray bundle propagating far away from any clumps (i.e. in regions where $\alpha < 1$), assuming a negligible shear. For a pressure-less Universe with cosmological

[1] http://www.sdss.org/
[2] http://www.na.astro.it/

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constant Λ, the DR equation reads

$$(1+z)\left[\Omega_{\rm m}(1+z)^3+\Omega_{\rm K}(1+z)^2+\Omega_\Lambda\right]\frac{{\rm d}^2D}{{\rm d}z^2} + \left[\tfrac{7}{2}\Omega_{\rm m}(1+z)^3+3\Omega_{\rm K}(1+z)^2+2\Omega_\Lambda\right]\frac{{\rm d}D}{{\rm d}z} + \tfrac{3}{2}\alpha\Omega_{\rm m}(1+z)^2D=0, \quad (3)$$

with initial conditions $D(0)=0$ and $D\prime(0)=c/H_0$. Here $\Omega_{\rm K}\equiv 1-\Omega_{\rm m}-\Omega_\Lambda$, since we neglect the contribution from any relativistic fluid or radiation. For $\alpha=1$ (filled beam case), we recover the angular diameter distance, while for $\alpha=0$ we have the well-known empty beam approximation. For a detailed discussion of the solutions of equation (3) within quintessence cosmological models, we refer the reader to Sereno et al. (2000). For the following, it is useful to introduce the dimensionless distance r,

$$r(z,\Omega_{\rm m},\Omega_\Lambda,\alpha)\equiv\frac{H_0}{c}D(z,H_0,\Omega_{\rm m},\Omega_\Lambda,\alpha), \quad (4)$$

and the symbol r_1 for the dimensionless angular diameter distance in the filled beam case.

Note, however, that the DR equation (3) is well defined for any $\alpha>0$, and in its derivation the mass density is never required to be uniform. This allows us to consider the clumpiness parameter as a local variable, as done in Wang (1999) to describe the weak lensing magnification of distant standard candles. Therefore, in the following, we assume the clumpiness parameter to be a function of both the line of sight and the redshift. Given a source at redshift z in a specific inhomogeneous cosmological model, for any line of sight to the observer, the clumpiness parameter α is calculated via equation (3), where the distance D is given by numerical simulations. As a consequence, a complete description of the light propagation in a clumpy Universe needs knowledge of the probability distribution function (PDF) for values of the clumpiness parameter. We return to this point in Section 5.

Let us now consider in more detail the effect of inhomogeneities in observations meant to measure the cosmological constant. The DR distance r is a strongly decreasing function of α, at fixed redshift (Schneider et al. 1992): therefore, a larger fraction of matter in clumps partially masks the effect of a larger cosmological constant

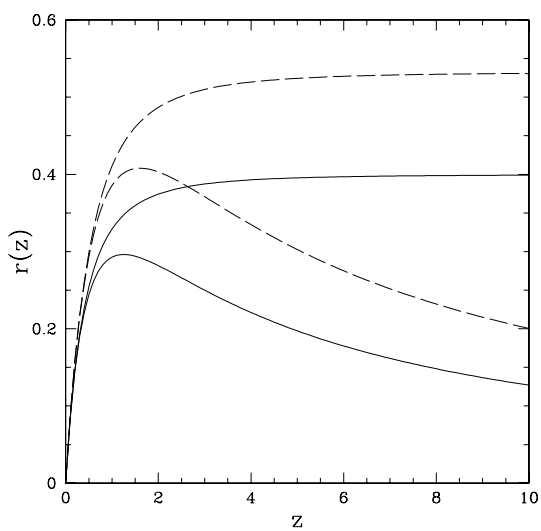

Figure 1. The Dyer–Roeder (DR) distance $r(z)$ for two values of the cosmological constant, $\Omega_\Lambda=0.0$ (full curves) and 0.7 (dashed curves), in the empty beam case (upper curves) and in the filled beam case (lower curves). Space–time is flat.

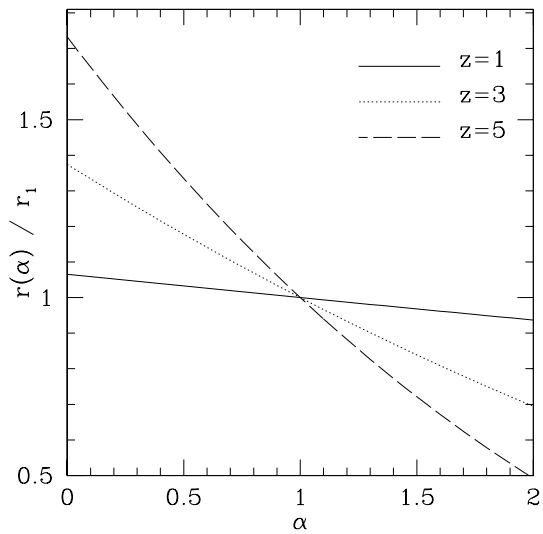

Figure 2. The DR distance versus the clumpiness parameter, relative to the filled beam case at several values of the redshift. Here $\Omega_\Lambda=0.7$ in a flat space–time.

when evaluating angular diameter and luminosity distances (Fig. 1), since there is a smaller contribution from the Ricci convergence. On the other hand, along light beams characterized by $\alpha>1$ (i.e. propagating in overdense regions) angular diameter distances are lower than in the filled beam case. For this reason, a large amount of local clumpiness can result in a lower value for Λ when fitting the observational data. In Fig. 2 we show the ratio of the DR distance for the empty beam case relative to the filled beam for a few values of the source redshift. Up to redshift $z\sim 1$, the differences are not important in the DR distance itself (although they may not be negligible for several observable astronomical quantities). The role of the local clumps becomes more and more important at higher z.

Though the assumed cosmological model is not the most satisfactory to describe inhomogeneities, since it does not have a firm theoretical basis in the framework of general relativity (i.e. it is not a solution of the Einstein field equations), it allows a simple and efficient description of light propagation through a clumpy Universe. This model has been discussed in detail in Schneider et al. (1992) and Seitz, Schneider & Ehlers (1994). Ehlers & Schneider (1986) have introduced a self-consistent formalism to study gravitational lensing in a clumpy Universe. In particular, they stressed the different roles that the different notions of cosmological distance have in this model. Namely, when we consider quantities that are related to the global geometry of the assumed cosmological model (such as volumes), it is necessary to use the FL angular diameter distance. Then, the volume element (i.e. the volume of a shell with proper thickness dl) is

$${\rm d}V=4\pi\left(\frac{c}{H_0}\right)^2 r_1^2\,{\rm d}l. \quad (5)$$

In other words, the volume element dV does not depend on the local degree of inhomogeneity. This is consistent with the fact that, in the locally inhomogeneous model, on large angular scales (i.e. larger than $\theta\sim 10$ arcsec; see Linder 1988), the distance–redshift relation is the one computed in the FLRW models, for any source redshift. This is also in agreement with the results from N-body numerical simulations in cold dark matter (CDM) scenarios (Tomita 1998), where it has been shown that the dispersion in values of α along the different light paths becomes increasingly large as the angle

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becomes as small as a few arcseconds. We also note that equation (5) can be read as the definition of the angular diameter distance in homogeneous models (Schneider et al. 1992).

On the other hand, there is a simple reason why, when considering gravitational lensing phenomena, the DR distance has to be used: light deflection modifies the cross-sectional area of the light bundle, and the Ricci focusing term is a linear function of the amount of matter within the ray bundle (see e.g. Schneider et al. 1992, section 3.4). As a consequence, all the distance-dependent quantities that play a role in the description of the lensing phenomena are functions of the DR distance and, in particular, of the strong lensing cross-section σ.

Let us consider in more detail the cosmological strong lensing probability τ. This is defined as the probability that a light source at redshift z_s is multiply imaged by a deflector at $z < z_s$. In the expression of $\tau(z_s)$ there are two different physical quantities that are functions of combinations of distances: the cross-section for multiple images σ, and the volume element dV.

The cross-section σ is defined as the area in the lens plane for which multiple imaging occurs for sources behind it. This quantity depends on the redshift z of the lens and a set χ of astrophysical parameters that characterize the gravitational lens model. For the most generally used models (point-like mass distributions and isothermal spheres), the cross-section depends on a particular combination of distances:

$$\sigma(z, z_s, \chi) = f\left(\frac{D_{ds}D_d}{D_s}, \chi\right), \tag{6}$$

where D_s, D_d and D_{ds} are the DR angular diameter distances between the observer and the source, the observer and the lens, and the lens and the source, respectively. We remark that the overall effect of the clumpy distribution of matter along the light rays on the lensing probability is not due to any change of the volume element dV (see equation 5), but is only due to the dependence of the strong lensing cross-section σ on the clumpiness parameter.

In the following we will consider the projection of the cross-section on the source plane (located at redshift z_s):

$$\hat{\sigma}(z, z_s, \chi) = \left[\frac{r(z_s, \alpha)}{r(z, \alpha)}\right]^2 \sigma(z, z_s, \chi), \tag{7}$$

where the DR distances are considered. This quantity allows a clearer and more compact definition of the lensing probability, and it is the natural quantity to consider in the assumed cosmological model (Ehlers & Schneider 1986). It is important to note that the quantity $\hat{\sigma}$ is not in general a function of the distance combination $D_{ds}D_d/D_s$: as a consequence, the point-like mass and isothermal sphere cross-sections are functions of different distance combinations (see Section 4). We now calculate the probability of strong lensing phenomena and evaluate its dependence on α, and then determine explicitly the distance functions for these two gravitational lens models.

3 THE STATISTICS OF GRAVITATIONAL LENSES

In this section, we derive the formulae for the statistics of gravitational lensing, following mainly the formalism discussed in Ehlers & Schneider (1986), considering in particular the proper role of the two types of distance. Let us consider a statistical ensemble of cosmological sources at redshift z_s and a set of comoving gravitational lenses with number density $n(z)$. If we neglect gravitational lens evolution and merging, the comoving number density of lenses is conserved: $n(z) = n_0\,(1+z)^3$, where n_0 is the local density. The number of gravitational lenses in a shell with volume dV is then

$$\begin{aligned} N(z) &= n(z)\,dV \\ &= n(z)A(z)\,\frac{dl}{dz}\,dz, \end{aligned} \tag{8}$$

where $A(z)$ is the area of the sphere located at redshift z.

The probability $d\tau(z, z_s, \chi)$ that a quasar at z_s is multiply imaged by gravitational lenses in the redshift range $(z, z + dz)$ is defined as the fraction of the area of the sphere at $z = z_s$ (i.e. the fraction of the sky) covered by the gravitational lens cross-sections $\hat{\sigma}(z, z_s, \chi)$. This definition implicitly assumes that the projected cross-sections do not overlap, which is equivalent to stating that $d\tau\,(z, z_s, \chi) \ll 1$. The area covered by the projected cross-sections of the gravitational lenses in $(z, z + dz)$ is therefore

$$n(z)\hat{\sigma}(z, z_s, \chi)A(z)\,\frac{dl}{dz}\,dz. \tag{9}$$

According to the definition, the differential lensing probability then reads

$$\begin{aligned} d\tau(z, z_s) &= n(z)\hat{\sigma}(z, z_s, \chi)\,\frac{A(z)}{A(z_s)}\,\frac{dl}{dz}\,dz \\ &= n(z)\sigma(z, \chi)\left[\frac{r(z_s, \alpha)}{r(z, \alpha)}\right]^2 \left[\frac{r_1(z)}{r_1(z_s)}\right]^2 \frac{dl}{dz}\,dz. \end{aligned} \tag{10}$$

In the particular case $\alpha = 1$, the distance ratios factorize, so that we obtain the more common expression

$$d\tau(z, z_s) = n(z)\sigma(z, z_s, \chi)\,\frac{dl}{dz}\,dz, \tag{11}$$

i.e. the definition of differential lensing probability in cosmologies that are homogeneous at all scales. Therefore, in this case, the relevant distance combination is exactly the ratio $D_{ds}D_d/D_s$ that appears in equation (6), and does not depend on the particular choice of the gravitational lens model.

Let us now evaluate the explicit expressions for the lensing probabilities in equation (11). The quantity dl can be written in the following way:

$$dl = -c\,dt = c\,\frac{1}{H(z)}\,\frac{dz}{1+z}, \tag{12}$$

where we have considered the past light cone, and $H(z) \equiv \dot{a}/a$, with $a = 1/(1+z)$ being the normalized cosmological scalefactor. In an FLRW cosmological model,

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m-\Omega_\Lambda)(1+z)^2 + \Omega_\Lambda}. \tag{13}$$

Hereafter we focus our attention on flat cosmological models, i.e. with $\Omega_\Lambda + \Omega_m = 1$, as these are preferred by inflationary scenarios and strongly supported by much recent observational evidence (Wang at al. 2000). Then, the differential lensing probability reads

$$\begin{aligned} d\tau(z, z_s) = n_0\frac{c}{H_0}\sigma(z, z_s, \chi)\left[\frac{r(z_s)}{r(z)}\right]^2 \left[\frac{r_1(z)}{r_1(z_s)}\right]^2 \\ \times \frac{(1+z)^2}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}\,dz. \end{aligned} \tag{14}$$

It is evident that the properties of the functions $d\tau$ (and τ) with respect to α are determined only by the strong lensing cross-section $\hat{\sigma}(z, z_s)$. Therefore, it is necessary to consider only those functions of the DR distances that enter their expressions. In other words, the functions τ and $\hat{\sigma}$ have the same qualitative behaviours with respect to the clumpiness parameter. Then, our next point is to investigate the behaviour of the quantity $\hat{\sigma}(\alpha)$.

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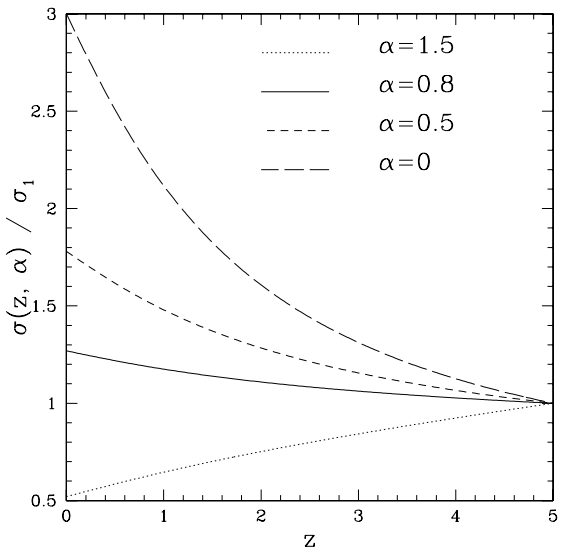

Figure 3. The projected cross-sections of a point-like gravitational lens for $\alpha = 0$, 0.5 and 0.8, normalized to the filled beam case ($\alpha = 1$), versus the lens redshift. The source is at $z_s = 5$, and the cosmological parameters are $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$.

4 PROPERTIES OF DR DISTANCE COMBINATIONS

In this section we analyse in detail the cross-sections of some general models of gravitational lenses and the involved combinations of DR distances, in order to evaluate qualitatively the dependence of the lensing probability τ on the clumpiness parameter. Asada (1998) has investigated the analytical properties of several combinations of DR distances as functions of the clumpiness parameter, and he also deduced some consequences on the observable quantities: the deflection angle, the time delay and the lensing probability. Here we focus on the study of the quantities that directly enter the optical depth. We consider the following models: a point mass distribution, singular isothermal sphere, and isothermal sphere with a non-zero core radius. In this section we only consider the DR distances.

4.1 Point-like gravitational lens

A point-like gravitational lens produces two images, whatever the source position; so, strictly speaking, the given definition of strong lensing cross-section does not apply here. Anyway, it is natural to assume such a cross-section (*on the lens plane*) to be a disc with radius equal to the Einstein radius r_E. So, the cross-section *on the source plane* reads:

$$\hat{\sigma} = \left(\frac{D_s}{D_d}\right)^2 \pi r_E^2 = \frac{4\pi G M}{c^2}\,\frac{D_s D_{ds}}{D_d}, \tag{15}$$

where M is the mass of the lens. This quantity is a decreasing function of the clumpiness parameter α, as shown in Fig. 3. Consequently, at smaller values of the clumpiness parameter, the lensing probability is higher, when considering the lensing cross-section for a point-like distribution of mass.

Assume that the distance ratio $D_d D_{ds}/D_s$ in the expression for the Einstein radius r_E is a decreasing function of the clumpiness parameter (Asada 1998), but is not the correct function of DR distances that enters the expression for the lensing probability in a DR cosmological model, as shown above. In fact, considering this ratio is not consistent with the assumptions on the cosmological model (Ehlers & Schneider 1986), and would wrongly lead to the prediction that the lensing probability decreases for decreasing values of the clumpiness parameter, as in Asada (1998).

4.2 Isothermal spheres

Let us now consider the gravitational lenses to be isothermal spheres, and let us study the dependence of the cross-section on α. Many authors have shown that singular isothermal spheres (SIS) allow a detailed description of the matter distribution in individual gravitational lenses (see e.g. Rusin et al. 2002), and they are therefore used in statistical analysis of the population of lensing galaxies (e.g. Chae 2003). SIS are characterized by the surface mass density

$$\Sigma(\boldsymbol{\xi}) = \frac{\sigma_v^2}{2G}\,\frac{1}{\xi}, \tag{16}$$

where $\boldsymbol{\xi}$ is the position vector on the lens plane, and σ_v is the line-of-sight velocity dispersion [see e.g. Hinshaw & Krauss (1987) and Schneider et al. (1992) for a detailed description of its lensing properties]. The area in the lens plane for multiple lensing is

$$\sigma_0 = 16\pi^3 \frac{\sigma_v^4}{c^4}\left(\frac{D_d D_{ds}}{D_s}\right)^2. \tag{17}$$

Note that the distance combination that enters equation (17) is the same as in the Einstein radius for a point-like gravitational lens. When we consider the projection on the source plane, we get

$$\hat{\sigma}_0 = 16\pi^3\left(\frac{\sigma_v}{c}\right)^4 D_{ds}^2. \tag{18}$$

As the angular diameter distance between two points at redshifts z_1 and $z_2 > z_1$ is a decreasing function of α (Asada 1998), so is the cross-section σ_0 and, consequently, the optical depth τ. In Fig. 4(a) we plot the cross-section $\sigma_0(z;\alpha)$ relative to the filled beam case, evaluated along different lines of sight characterized by different values of the clumpiness parameter.

Let us now consider an isothermal sphere with a non-zero core radius ξ_c. The surface mass density is

$$\Sigma(\boldsymbol{\xi}) = \frac{\sigma_v^2}{2G}\,\frac{1}{\sqrt{\xi^2+\xi_c^2}},$$

which leads to a non-constant deflection angle. The cross-section on the lens plane is (Hinshaw & Krauss 1987)

$$\sigma = \begin{cases} \sigma_0\left[\left(1+5\beta-\frac{1}{2}\beta^2\right)-\frac{1}{2}\sqrt{\beta}(\beta+4)^{3/2}\right], & \beta < \frac{1}{2}, \\ 0, & \beta > \frac{1}{2}, \end{cases} \tag{19}$$

where β is the core radius ξ_c in units of the natural length-scale

$$\xi_0 = 4\pi\left(\frac{\sigma_v}{c}\right)^2 \frac{D_d D_{ds}}{D_s}. \tag{20}$$

The projected cross-section for multiple imaging is, for $\beta < \frac{1}{2}$,

$$\hat{\sigma} = 16\pi^3 C(\beta)\left(\frac{\sigma_v}{c}\right)^4 D_{ds}^2, \tag{21}$$

where we have introduced the quantity $C(\beta) \equiv \sigma/\sigma_0$. The presence of a core radius lowers the lensing probability, but does not change its qualitative properties with respect to the clumpiness parameter (see Fig. 4b). Note that the function $C(\beta)$ is not a constant, since β is not, with respect to the clumpiness parameter. The dimensionless

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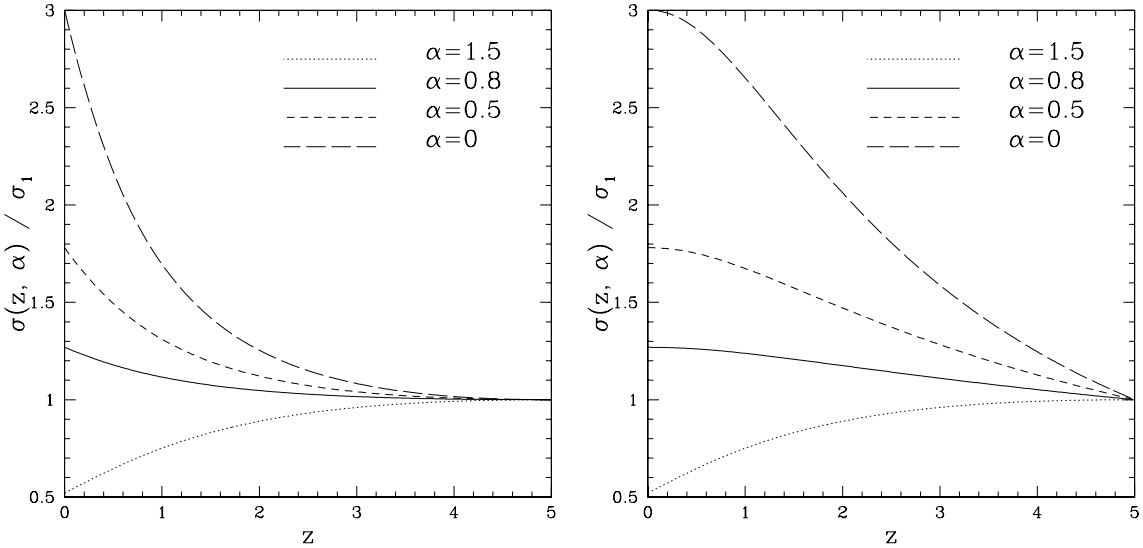

Figure 4. The projected cross-sections relative to the filled beam case for a singular isothermal sphere (left-hand panel) and an isothermal sphere with core radius $\xi_c = 10$ pc (right-hand panel), versus the lens redshift. The source is located at $z_s = 5$ and the cosmological parameters are $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. For lens redshift $z \ll 1$, the two lens models have the same limit. The same qualitative results hold for any values of z_s and Ω_Λ.

core radius β is a function of the redshifts z, z_s, the cosmological parameters and the clumpiness parameter, via the length unit ξ_0:

$$\beta(z, z_s, \Omega_\Lambda, \alpha) = \frac{1}{4\pi}\frac{c^2}{\sigma_v^2}\xi_c\frac{D_s}{D_d D_{ds}}. \tag{22}$$

It is interesting to note that, despite the fact that the cross-sections of the considered gravitational lens models differ in the DR distance ratio, they are both decreasing functions of α. In other words, the monotonic properties of the cosmological optical depth with respect to α are general. Finally, it is evident that neglecting α leads, in the theoretical predictions, to an underestimate of the probability of strong lensing, and, in the statistical analysis of high-redshift quasar catalogues, to an overestimate of the cosmological constant. This is discussed in the next section.

5 ON THE EXPECTED NUMBER OF GRAVITATIONAL LENSES

In this section we evaluate numerically the optical depth in a flat cosmology as a function of the cosmological parameter Ω_Λ and of the clumpiness parameter α. In particular, we analyse quantitatively the effect of neglecting local inhomogeneity in calculating the lensing probability in a clumpy Universe, along lines of sight with $\alpha < 1$. For this purpose, we consider gravitational lenses to be singular isothermal spheres, because they allow a very simple and analytical treatment of the problem. Also, they have been used in a variety of studies of the statistical properties of gravitational lenses (see Chae 2003, and references therein), thus allowing a direct comparison of the results. Moreover, this particular choice does not affect our qualitative results.

As we are mainly interested in the effects of varying α, we compare the optical depth $\tau\ (z_s, \Omega_\Lambda, \alpha)$ with τ_0, the numerical value in the case of vanishing cosmological constant and light propagation through filled beams. Thus, our discussion is independent of the numerical parameter

$$F \equiv 16\pi^3\left(\frac{\sigma_v}{c}\right)^4\left(\frac{c}{H_0}\right)^3,$$

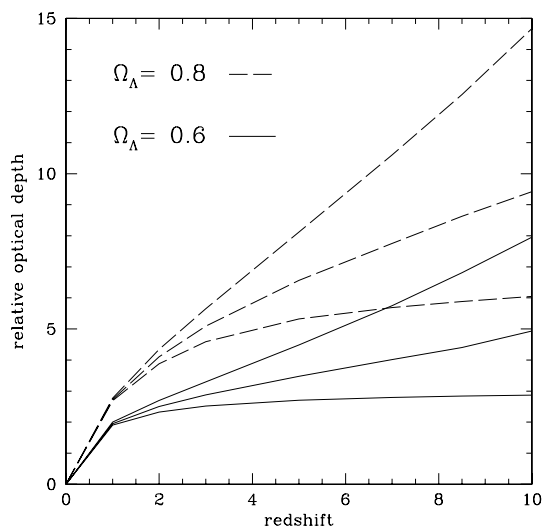

Figure 5. The optical depth as a function of the redshift for different values of α and Ω_Λ relative to τ_0, the optical depth, in the case $\Omega_\Lambda = 0$ and $\alpha = 1$. The full and dashed curves are for $\Omega_\Lambda = 0.6$ and 0.8, respectively. The clumpiness parameter is $\alpha = 0$, 0.5 and 1.0, from the top curve to the bottom one for each value of Ω_Λ. The gravitational lenses are SIS, and space–time is flat.

which controls the lensing probability. In Fig. 5 we plot the relative optical depth τ/τ_0 as a function of the source redshift, for six different cosmological models, in which $\Omega_\Lambda = 0.6$ and 0.8, considering lines of sight with $\alpha = 0$, 0.5 and 1 for each case. As is well known, the strong lensing probability is very sensitive to the value of the vacuum energy (e.g. Fukugita et al. 1992). For any value of α, the probability that a source at given redshift z_s is multiply imaged grows by a factor of $\sim$2 when Ω_Λ goes from $\sim$0.6 to $\sim$0.8.

In order to disentangle the effect of the clumpy distribution of matter from that due to the cosmological constant, we consider $\Delta\tau/\tau_0$, i.e. the relative variation of the lensing probability with respect to the case $\alpha = 1$, as a function of Ω_Λ along lines of sight

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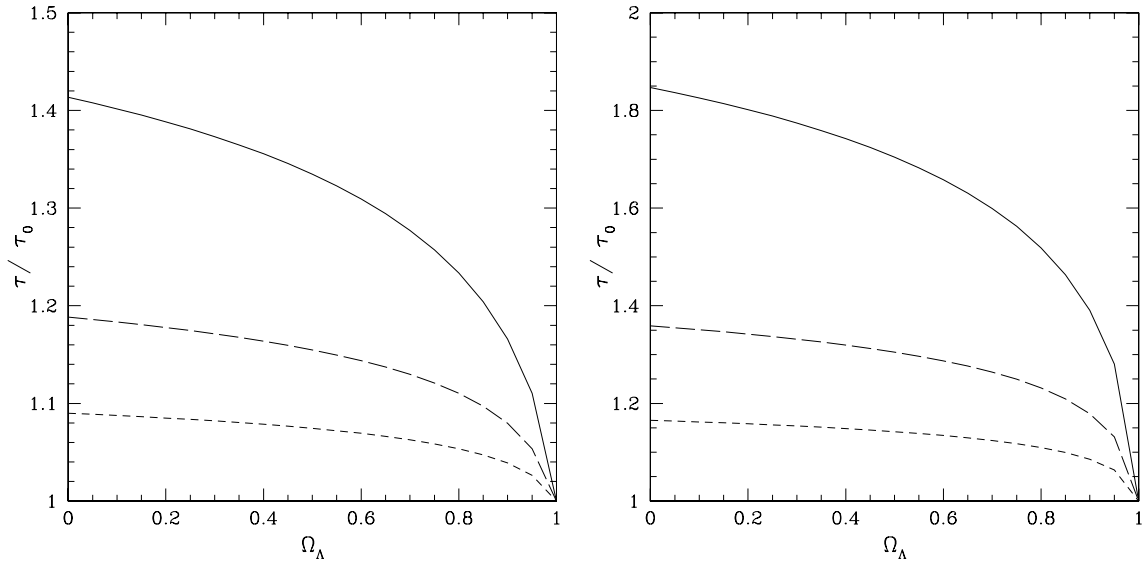

Figure 6. Relative variation of the optical depth with respect to the filled beam case. The clumpiness parameter is $\alpha = \{0, 0.5, 0.75\}$. Sources are at redshifts $z_s = 3$ (left-hand panel) and 5 (right-hand panel). Here $\Omega_\Lambda = 0.7$ in a flat space–time.

characterized by $\alpha < 1$. We plot this quantity, evaluated at different source redshifts, in Fig. 6. As shown above, for any z_s and Ω_Λ, the lensing probability is a decreasing function of the clumpiness parameter. We also notice that the effect of the clumpiness parameter increases with the redshift of the sources, if a given direction with $\alpha < 1$ is considered.

The most important feature to note is that the variation rapidly becomes less important at larger values of the cosmological constant. The reason for this effect is two-fold. First, for larger values of the cosmological constant, the influence of all other astrophysical and cosmological parameters is expected to be less important, since at high Ω_Λ the optical depth is very sensitive to any small change in the value of the cosmological constant. Secondly, α enters equation (3) as a coefficient of the matter density parameter, determined (in flat cosmological models) by the relation $\Omega_m = 1 - \Omega_\Lambda$; so it is relatively less important for small values of the density parameter. In the most commonly accepted range for the cosmological constant ($0.6 \leqslant \Omega_\Lambda \leqslant 0.85$), the lensing probability increases (relative to a completely homogeneous matter distribution) by a factor of about 7 per cent, 17 per cent and 30 per cent if we consider the clumpiness parameter $\alpha = 0.75$, 0.5 and 0, respectively.

As stated above, in order to have a coherent description of the small-scale clumpiness, we need to consider the PDF for α, since this is a direction-dependent quantity. The PDF depends on the background FLRW cosmological model, and can be derived from the PDF of the lensing magnification $\mu\,(z)$, calculated in numerical simulations (e.g. Wambsganss et al. 1997; Holz & Wald 1998; Tomita 1998; Mortsell 2002), via the relation

$$\mu(z) = \left[\frac{r_1(z)}{r(z,\alpha)}\right]^2, \tag{23}$$

and the DR equation. As shown by Wang (1999), in general the α PDF is peaked at $\alpha < 1$ for any z, but it tends to be more symmetric and shows smaller scatter around the peak value as the redshift increases (see e.g. fig. 3 in Wang 1999). In Fig. 7 we plot, for a flat space–time with $\Omega_m = 0.4$, both the most likely value and the average value, as calculated via the approximate analytic expressions of the PDF (α) given in Wang (1999).

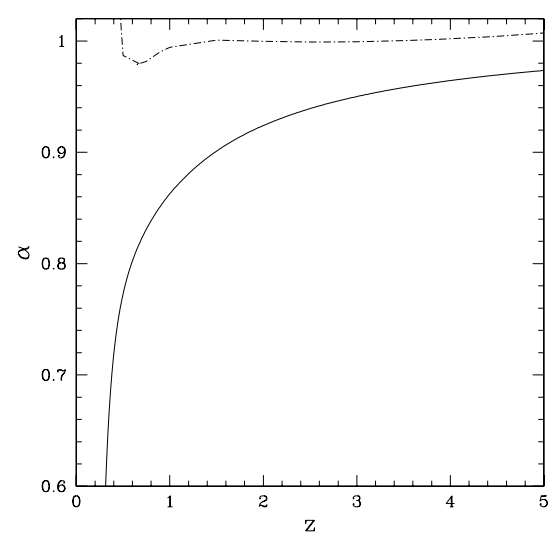

Figure 7. The peak value (full curve) and the mean value (broken curve) of the clumpiness parameter in a flat cosmological model with $\Omega_m = 0.4$, calculated via the analytic approximation given in Wang (1999).

The mean value of α is 1 at any redshift in all cosmological models (if they are FLRW on average, as assumed here). This is indeed the same property that is known to hold for the PDF of the lensing magnification μz, whose basic motivation is flux conservation (Weinberg 1976). Therefore we expect that the effect of inhomogeneities on the lensing cross-sections is reduced in a large ensemble of gravitational lenses, or for very distant sources.

For a gravitational lens at $z \sim 1$, the most likely value is $\alpha \sim 0.85$; this translates into an underestimate of the lensing cross-section for an SIS (using the filled beam distances and considering a source at $z_s \gtrsim 4$) of a factor of $\sim$1.15 (see Fig. 4). These systematic errors decrease for more distant gravitational lenses or sources. However, since the most likely value is always less than 1, and because, in a

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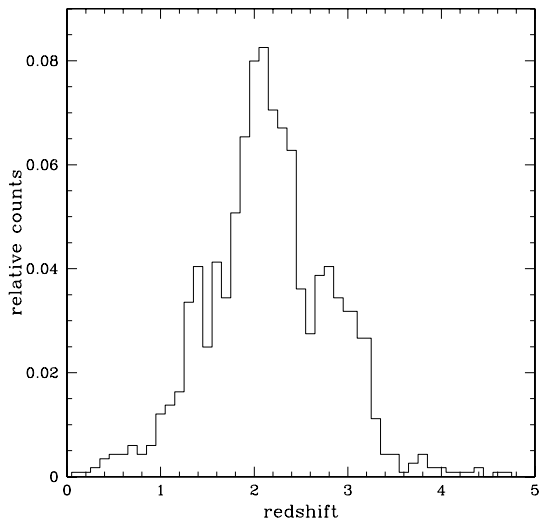

Figure 8. Redshift distribution of the 1163 luminous quasars in the catalogue used to calculate the effect of a clumpiness parameter $\alpha \neq 1$. See text for details.

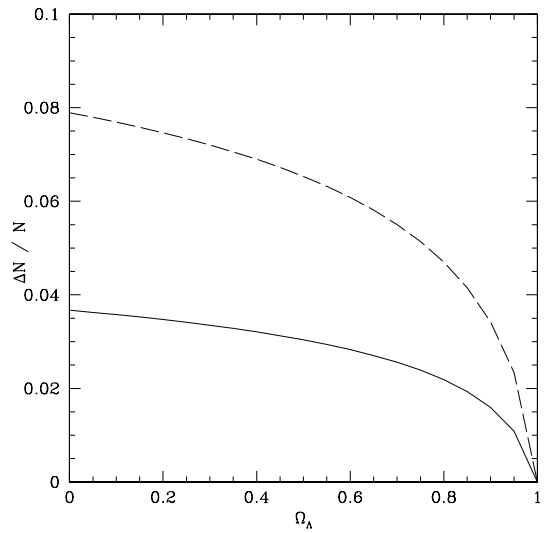

Figure 9. Relative variation of the expected number of strong lensed quasars in the given sample of high-luminosity quasars with respect to the filled beam case, as a function of the cosmological constant. The clumpiness parameter is $\alpha = 0.75$ (dashed curve) and $\alpha = 0.95$ (full curve). Lenses are modelled as SIS.

relatively small sample of gravitational lenses, the mean of a non-symmetric probability distribution is not likely to be the best estimator, such errors may become non-negligible when evaluating the cosmological parameters.

While a detailed statistical analysis including an accurate determination for the PDF for any cosmological model is beyond the scope of the present work, it is none the less important to estimate an upper limit to the possible errors on the predicted number of gravitational lenses. We have therefore considered the list of 1163 luminous quasars[3] that have been observed in the following optical surveys: CFHT (Crampton, McClure & Fletcher 1992), CFHT (Yee, Filippenko & Tang 1993), *HST* (Maoz et al. 1993), and NOT (Jaunsen et al. 1995). This catalogue contains seven confirmed gravitational lenses, and its redshift distribution is plotted in Fig. 8: the peak is at redshift $z \simeq 2$, and only a small fraction of sources are beyond $z = 3$. We have calculated the relative variation in the expected number of multiply imaged quasars considering different values for the clumpiness parameter, corresponding to peak values of its PDF for $z \simeq 0.5$ and 3.0. The lensing galaxies are being modelled as SIS (note that including a small core makes the clumpiness effect slightly larger). In Fig. 9, we plot the relative variation of the expected number of lenses as a function of Ω_Λ for the peak values $\alpha = 0.75$ and 0.95. This figure shows the upper limit for the systematic errors that can be found for the predicted number of gravitational lenses when adopting the simple filled beam hypothesis; given the property $\bar{\alpha} = 1$, this could be further reduced in upcoming larger surveys. Note that, since we are considering a flat space–time and the effect of local inhomogeneities increases with Ω_m, the variation of the expected number N of multiply imaged quasars is a decreasing function of the cosmological constant. However, for very large values of the cosmological constant ($\Omega_\Lambda \gtrsim 0.8$) this variation rapidly becomes small. This makes clear the point that, if in the evaluation of τ we use angular diameter distances for a perfectly homogeneous cosmological model, we can underestimate the lensing probability and, consequently, overestimate Ω_Λ.

[3] This catalogue is available at the web address http://vela.astro.ulg.ac.be/themes/extragal/gravlens/bibdat

6 CONCLUSIONS

In this work we have investigated whether local departures from a completely homogeneous cosmological model can have observable effects in the statistical study of high-z gravitational lenses. Following the work by Ehlers & Schneider (1986), we derived expressions for the cosmological optical depth in the framework of a cosmological model that is FLRW on very large scales (i.e. whose overall dynamics is very well described by FLRW models) and whose matter distribution is locally inhomogeneous. The direction-dependent clumpiness parameter α quantifies the fraction of matter in compact objects along a given line of sight, and its peak and mean values (as a function of the source redshift) are calculated via the analytical approximation of the PDF given in Wang (1999). We have paid particular attention to disentangling the various roles played by the different notions of distance in the definition of the optical depth $\tau(\alpha)$: small-scale inhomogeneities along the line of sight do not change the volume element $\mathrm{d}V$, but do affect the strong lensing cross-section (see equation 5 and Fig. 4).

Up to redshift $z \sim 3$, the most probable value of the clumpiness parameter is very different from the average value, which is constrained to be 1 in a model homogeneous on a large scale (see Fig. 7), and the effect may be important in statistical analysis of relatively small sets of gravitationally lensed sources.

Asada (1998) presented a similar calculation to that in Section 4. He discussed the influence of the clumpiness parameter on several observable gravitational lensing quantities, showing that in a clumpy Universe deflection angles are smaller and time delays are longer than in a homogeneous Universe, given the same lens–source configuration. However, in contradiction to Asada's result, we have found that the gravitational lensing rate is a decreasing function of the clumpiness parameter. We have shown that in the empty and filled beam cases different angular distance ratios enter the

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optical depth expression (Section 3), and this leads to a higher number of expected gravitational lenses when light beams with $\alpha < 1$ are considered.

While a detailed statistical analysis including the effect of small-scale inhomogeneities on the determination of the cosmological constant is beyond the scope of the present work, we can already draw some important conclusions and compare our findings with previous works. Since the mean value of α tends to 1 when a sufficiently large number of different lines of sight are considered, the effect described here tends to be less important in large surveys for gravitationally lensed sources. Moreover, since the peak value also tends to 1 for $z \gtrsim 5$, statistical study of very high-redshift sources is less affected by local clumpiness along different lines of sight. On a small set of lensed sources, since the peak value of the clumpiness parameter is always less than 1, using the filled beam angular diameter distances leads to an overestimate of the number of expected gravitational lenses (Fig. 9), and, consequently, to an overestimate of the cosmological constant.

It is interesting to note that the same qualitative result (on the determination of the cosmological constant) has been found in previous works in which the authors have taken into account the effect on the distance–redshift relation played by inhomogeneities in the matter distribution. Kantowski (1998) described the effects of such inhomogeneities on the determination of the cosmological constant and $\Omega_{\rm m}$ using a 'Swiss cheese' model to derive analytic expressions for the distance–redshift relation. He showed that, when analysing high-z standard candles, assuming a completely homogeneous matter distribution leads to overestimating the cosmological constant if the filled beam hypothesis is always used (see e.g. fig. 8 in Kantowski 1998).

In a recent work, Barris et al. (2004) analyse a set of 194 Type Ia supernovae (SNe) at $z > 0.01$ (including the 23 discovered in the IfA Deep Survey in the range $z = 0.34$ –1.03) using both the filled beam and the empty beam assumptions for *all* the lines of sight to calculate luminosity distances.[4] Although the presence of small-scale inhomogeneities is far from eliminating the need for a vacuum energy contribution to explain the new data, they do change the final estimate of the cosmological parameters $\Omega_{\rm m}$ and Ω_Λ. Indeed, when using the angular diameter distances with $\alpha = 0$, the confidence contours for the cosmological parameters are shifted to lower values of Ω_Λ and higher values of $\Omega_{\rm m}$

The same effect has been found by authors who investigated the effect of large-scale inhomogeneity in the matter distribution. Tomita (2001) considered a local void on scales of 200–300 Mpc around our Galaxy to interpret the high-z SN data, effectively describing it with $\alpha < 1$ in the DR distances. He finds that the data (available at the time) could be well fitted with $\Omega_\Lambda \sim 0.4$ (the value estimated in the overdense outer region). It is certainly interesting to test these findings with the more recent and larger set of data available at the moment. In conclusion, a precise determination of the cosmological parameters using both the Hubble diagram of the high-z Type Ia SN and the statistical properties of gravitational lenses requires an accurate determination of the effect of local clumpiness in the matter distribution on the light propagation.

[4] Note, however, that using the empty beam approximation for all the lines of sight is not a coherent description, since in an inhomogeneous Universe not all the light beams can be devoid of matter, and a more realistic distribution for α has to be chosen to describe the clumpiness effects using the DR distances.

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This work is dedicated to the beloved memory of Professor Ruggiero de Ritis.

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