



HAL
open science

Halo Geometry and Dark Matter Annihilation Signal

E. Athanassoula, F. -S. Ling, E. Nezri

► **To cite this version:**

E. Athanassoula, F. -S. Ling, E. Nezri. Halo Geometry and Dark Matter Annihilation Signal. Physical Review D, 2005, 72, pp.83503. 10.1103/PhysRevD.72.083503 . hal-00014501

HAL Id: hal-00014501

<https://hal.science/hal-00014501>

Submitted on 12 Sep 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Halo Geometry and Dark Matter Annihilation Signal

E. Athanassoula^a, F.-S. Ling^b, E. Nezri^b

^a *Observatoire de Marseille, 2 Place Le Verrier, 13248 Marseille Cédex 04, France*

^b *Université Libre de Bruxelles, Campus de la Plaine CP225, Belgium*

We study the impact of the halo shape and geometry on the expected weakly interacting massive particle (WIMP) dark matter annihilation signal from the galactic center. As the halo profile in the innermost region is still poorly constrained, we focus on geometrical distortions and consider different density behaviors like flat cores, cusps and spikes. We show that asphericity has a strong impact on the annihilation signal when the halo profile near the galactic center is flat, but becomes gradually less significant for cuspy profiles, and negligible in the presence of a central spike. However, the astrophysical factor is strongly dependent on the WIMP mass and annihilation cross-section in the latter case.

PACS numbers: 95.35.+d, 98.35.Gi

I. INTRODUCTION

Flat rotation curves of spiral galaxies [1] can be explained by the presence of a dark matter halo which extends much farther than the luminous disc. While at large distances the gravitational potential is completely dominated by the dark halo, there is still a vivacious debate about whether the dark matter is prevailing in the central parts of bright galaxies, and about whether its radial matter distribution is cuspy, or not [2]. Furthermore, very little is known with certainty about the shape of the halo of disc galaxies and in many cases the halo is simply assumed to be spherical.

However, if the disc is indeed an important component in the central parts, it should, due to its gravity, introduce some flattening of the dark matter distribution. Furthermore, large scale cosmological N -body simulations have shown that, at least at large distances from the center, the natural shape of dark halos is triaxial (see references in [3]) with density axial ratios in the range 0.5 – 0.8 ([4]). It is thus natural to ask whether the flatness and the various complex structures and substructures of the luminous part of the galaxy will affect the dark matter halo, and to what extent the triaxiality of the halo will change a possible dark matter annihilation signal from the central parts of a galaxy.

Usually, two types of asphericity are considered : flattening of the halo and departure from axisymmetry. The halo flattening is quantified by the value of $q = c/a$, where a is the major axis in the galactic plane and c is the axis perpendicular to that plane. Various observational methods have been used to probe the halo flattening in our own Galaxy and in neighboring ones (e.g. [3, 5]). It was found that the measured flattening can vary over a wide range of values, depending on the galaxy and on the method used. A cross-check of the different methods with their systematic biases on the same galaxy would be welcome but is usually not possible. Measurements based on atomic hydrogen favour oblate halos with shortest-to-longest ratios in the very wide range of 0.2 to 0.8 [6]. For our own galaxy and based on the thickness of the Milky Way's gas layer, [7] argue for a rather round halo with

flattening $0.8 \leq q \leq 1$, but their result depends heavily on the values of the galactocentric radius R_0 and of the galactic rotation speed v_0 [7]. Recent studies of the dynamics of the stellar stream coming from the disruption of the Sagittarius dwarf galaxy also give a wide range of values, between 0.5 and 1.7 [8].

The second type of asphericity is a departure from axisymmetry in the galactic plane. This is statistically quite common as a large fraction (more than 70%) of present day disc galaxies have bars or ovals [9]. Also, it is now well agreed that our Galaxy is barred in its central parts [10]. Bars form naturally also in N -body simulations, as witnessed already in the early seventies [11]. More recently, it was realized that the presence of a dark halo can play an active role in the formation of the disc bar, if it is non rigid, *i.e.* if it can interact with the disc. Indeed, bars evolve and grow stronger by the redistribution of angular momentum within their galaxy. This is emitted by near-resonant material in the inner disc and absorbed by near-resonant material in the outer disc and in the halo [12]. As a result, the halo also is deformed and acquires a bar structure, which is fatter and shorter than the disc bar, but can concern a considerable amount of mass [13].

On the observational side, departures from axisymmetry can be checked from the orbits of the baryons, in particular the HI gas that has low velocity dispersion. Obtaining a quantitative estimate of such asymmetries is, however, not trivial, since it implies a decoupling of the halo contribution from that of the luminous matter as well as a knowledge of the inclination of the galactic disc [14]. Of course, a direct probe of the halo would enable to see whether the halo deformation follows the barred structure of the disk or not. If the recent EGRET diffuse gamma ray signal above 1 GeV is interpreted as originating from dark matter, it indeed leads to such a structure with an ellipticity value 0.65 ± 0.15 [15].

The purpose of this article is to study the impact of an elliptical deformation of the halo on the expected weakly interacting massive particle (WIMP) dark matter annihilation signal from the galactic center. However, as discussed above, we do not know for certain what the

dark matter radial profile is, so that distributions with or without a cusp, and with or without a spike have to be considered. The density enhancement in a cuspy profile follows the deepening of the central gravitational potential due to the baryon cooling through radiative processes [16, 17, 18, 19]. The presence of a super-massive black hole (SBH) at the galactic center can further create a spike, or an enhancement of the cusp, at very short distances from the galactic center [20, 21] but scatterings on stars and capture of dark matter particles by the SBH could decrease the density in this region [22, 23]. It is clear that the presence of a spike would boost the annihilation signal by several orders of magnitude. In the sequel, we will see how elliptical distortions interplay with density profiles in the dark matter annihilation signal.

II. HALO PARAMETERIZATION AND FLUX CALCULATION

Since the main observable annihilation signal from the galactic center is with γ -rays [24], we will restrict ourselves to this case. The observed gamma-ray flux of energy E , from the annihilation of dark matter particles χ (with mass m_χ and density ρ) and annihilation cross section $\sigma_i v$ (into final state i), can be expressed as (e.g. Ref. [25])

$$\frac{\Phi_\gamma}{d\Omega dE} = \sum_i \frac{1}{2} \frac{dN_\gamma^i}{dE_\gamma} \langle \sigma_i v \rangle \frac{1}{4\pi m_\chi^2} \int_{\text{l.o.s.}} \rho^2 dl, \quad (1)$$

where dN_γ^i/dE_γ is the differential gamma spectrum per annihilation coming from the decay of annihilation products of final state i and the integral is taken along the line of sight. It is customary (see [26]), in order to separate the factors depending on astrophysics from those depending only on particle physics, to introduce the quantity $J(\vec{\Omega})$ for the line of sight $l(\vec{\Omega})$ corresponding to the direction $\vec{\Omega}$,

$$J(\vec{\Omega}) = \frac{1}{8.5 \text{ kpc}} \left(\frac{1}{0.3 \text{ GeV/cm}^3} \right)^2 \int_{l(\vec{\Omega})} \rho^2 dl. \quad (2)$$

We then define the astrophysical factor $\bar{J}(\Delta\Omega)$ as the average of $J(\vec{\Omega})$ over a spherical region of solid angle $\Delta\Omega$, centered on the direction of the galactic center

$$\bar{J}(\Delta\Omega) = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} J(\vec{\Omega}) d\Omega \quad (3)$$

The solid angles $\Delta\Omega = 10^{-3}\text{sr}$ and $\Delta\Omega = 10^{-5}\text{sr}$ correspond to the angular resolutions in the EGRET experiment and the HESS and GLAST ones, respectively. The expected gamma-ray flux in an experiment with threshold energy E_T is finally expressed as

$$\Phi_\gamma(E_T) = 1.87 \times 10^{-13} \text{cm}^{-2} \text{s}^{-1} \bar{J}(\Delta\Omega) \Delta\Omega$$

	α	β	γ	a (kpc)	$\bar{J} (10^{-3}\text{sr})$	$\bar{J} (10^{-5}\text{sr})$
Iso	2.0	2.0	0	3.5	2.46×10^1	2.47×10^1
Kra	2.0	3.0	0.4	10.0	1.932×10^1	2.37×10^1
NFW	1.0	3.0	1.0	20	1.21×10^3	1.26×10^4
Moore	1.5	3.0	1.5	28.0	1.60×10^5	1.24×10^7

TABLE I: Parameters of some widely used non spiky density profiles models and corresponding value of $\bar{J}(10^{-3}\text{sr})$ and $\bar{J}(10^{-5}\text{sr})$.

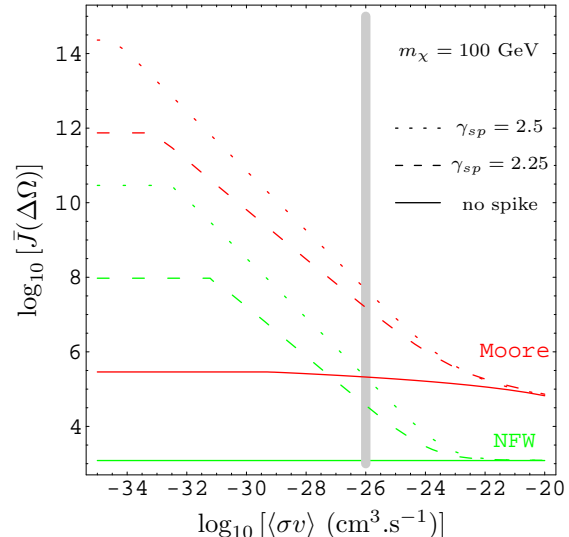


FIG. 1: Variation of $\bar{J}(10^{-3}\text{sr})$ with $\langle \sigma v \rangle$ for NFW and Moore profiles, with and without spike. The WMAP constraint on relic density suggests few orders of magnitude around the vertical grey line.

$$\times \frac{1}{2} \sum_i \int_{E_T}^{m_\chi} dE_\gamma \frac{dN_\gamma^i}{dE_\gamma} \left(\frac{\langle \sigma_i v \rangle}{10^{-29} \text{cm}^3 \text{s}^{-1}} \right) \left(\frac{100 \text{ GeV}}{m_\chi} \right)^2 \quad (4)$$

To compute the quantity \bar{J} , we assume the following effective parameterization for the dark matter halo

$$\rho(r) = \rho_0 \frac{[1 + (R_0/a)^\alpha]^{(\beta-\gamma)/\alpha}}{[1 + (r/a)^\alpha]^{(\beta-\gamma)/\alpha}} \left(\frac{R_0}{r} \right)^\gamma \left(1 + \left(\frac{r_{sp}}{r} \right)^{\gamma_{sp}-\gamma} \right), \quad (5)$$

as suggested by many Λ CDM simulations. R_0 is the Sun's distance to the galactic center, ρ_0 is the solar neighborhood halo density and a is a characteristic length. The exponents α , β and γ can be thought of as power law indices characteristic for $r \simeq a$, $r \gg a$ and $r \ll a$, respectively. Table I gives the parameters for common halo models like the isothermal one which is not cuspy at the center, behaving as $\rho(r) \propto cst$, and the halo models from Λ CDM simulations by Kravtsov *et al* [27], Navarro,

Frenk and White (NFW) [28] and Moore *et al* [29], which behave respectively as $\rho(r) \propto r^{-0.4}$, $\rho(r) \propto r^{-1}$ and $\rho(r) \propto r^{-1.5}$ at small r . Adiabatic accretion of dark matter on the SBH could further add a central spike to these profiles [20], or, if more realistic physics as off-centered formation of the SBH is taken into account, the SBH will simply enhance the cusp [21]. Finally, scattering of dark matter particles by stars would substantially decrease the density in the center-most region [22, 23]. In the next section we will consider both the NFW and the Moore profiles with and without spike. The spike's characteristic size and slope are parameterized by r_{sp} and γ_{sp} in Eq. (5). We take (see [20]) $r_{sp} = 0.35$ pc, and the two limiting values for the slope, $\gamma_{sp} = 2.25$ and $\gamma_{sp} = 2.5$, as $\gamma_{sp} = (9 - 2\gamma)/(4 - \gamma)$. It is worth emphasizing that the gamma flux is dominated by the contribution from the *unknown* innermost region, especially for profiles with a steep slope near the center.

A. Annihilation effect on dark matter density

The astrophysical factor \bar{J} for the annihilation flux becomes formally divergent for $\gamma \geq 1.5$ (*a fortiori* with a spike). The presence of a SBH at the center of the galaxy solves the problem in principle since no signal will escape from the region inside the Schwarzschild radius R_S ($R_S \sim 3 \cdot 10^{-10}$ kpc for $M_{BH} = 2.6 \cdot 10^6 M_\odot$). The sphere of influence of the SBH is actually larger, as captured particles in a region of a few R_S are not balanced by particles scattered in from the outer shells. Following Refs.[22],[23], we cut the density below $r_c = 10^{-9}$ kpc:

$$\rho(r < r_c) = 0 \quad (6)$$

For halo profiles with a strong cusp/spike behavior near the center, the density becomes so high that the influence of annihilations on the central density has to be taken into account. A simple bound can be obtained by letting annihilations operate in a static halo (or with an isotropic velocity distribution [30]) initially with an infinite density, and during a time as long as the SBH formation time. By solving the equation

$$\frac{dn}{dt} = -\langle\sigma v\rangle n^2 \quad (7)$$

one obtains the upper bound

$$\rho_{max} = \frac{m_\chi}{\langle\sigma v\rangle t_{BH}}. \quad (8)$$

In a sense, astrophysics and particle physics aspects can no longer be decoupled any more in the flux calculation, especially for profiles with high γ values (> 1.5).

In Fig. 1, we can see that the presence of a spike near the SBH strongly enhances the annihilation signal. The precise value of \bar{J} is very sensitive to r_{sp} , γ_{sp} and $\langle\sigma v\rangle/m_\chi$. For very small values of the annihilation cross-section, the cut due to the capture by the SBH

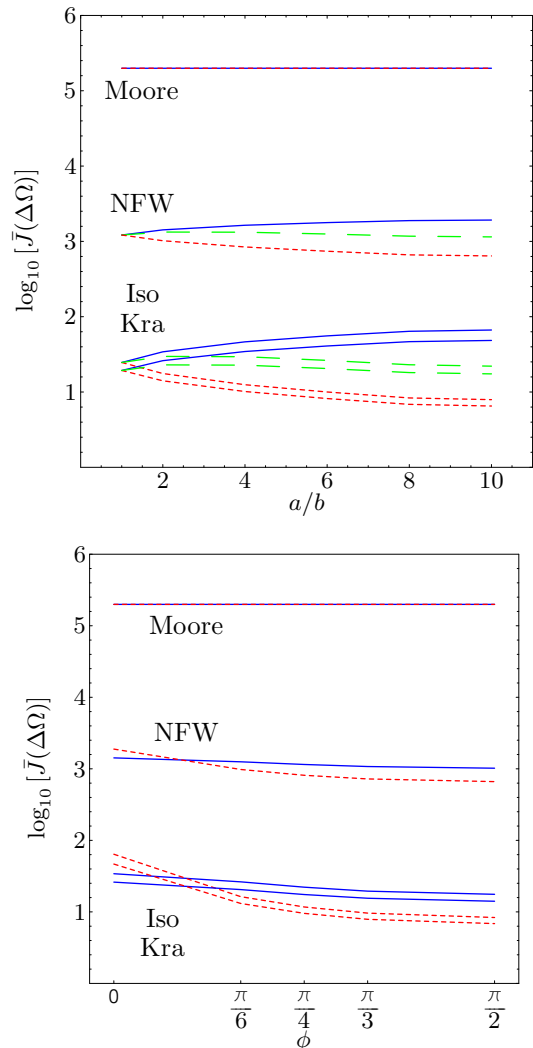


FIG. 2: $\bar{J}(10^{-3} \text{ sr})$ as a function of a) the axis ratio a/b , for $\phi = 0$ (blue solid line), $\phi = 0.35$ ($= 20$ deg, green long-dashed line) and $\phi = \pi/2$ (red short-dashed line); b) the angle ϕ , for $a/b = 2$ (blue solid line) and $a/b = 9$ (red dashed line).

becomes apparent. For large values of the annihilation cross-section, the spike gets washed out and we recover the value for the profile without spike. Additional effects such as scattering of dark matter particles by stars or with baryons have been considered in the literature ([30]), they compete with the annihilations for small values of $\langle\sigma v\rangle$, and need to be included in a precise evaluation of \bar{J} .

Our main purpose, however, is to emphasize that, if spike or strong cusp behaviors are absent, other geometrical effects such as the asphericity of the halo become crucial in the \bar{J} calculation, as will be shown hereafter.

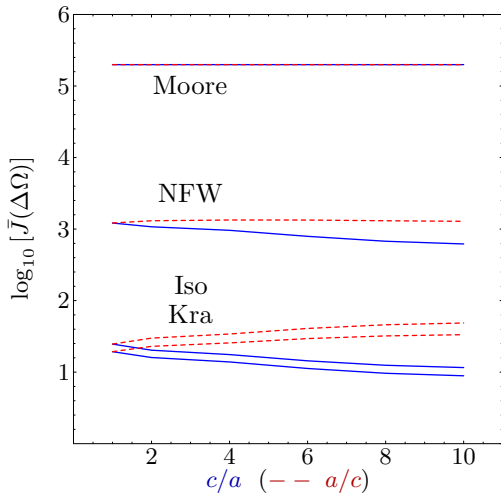


FIG. 3: $\bar{J}(10^{-3} \text{ sr})$ as a function of prolate (blue solid line) and oblate (red dashed line) deformations.

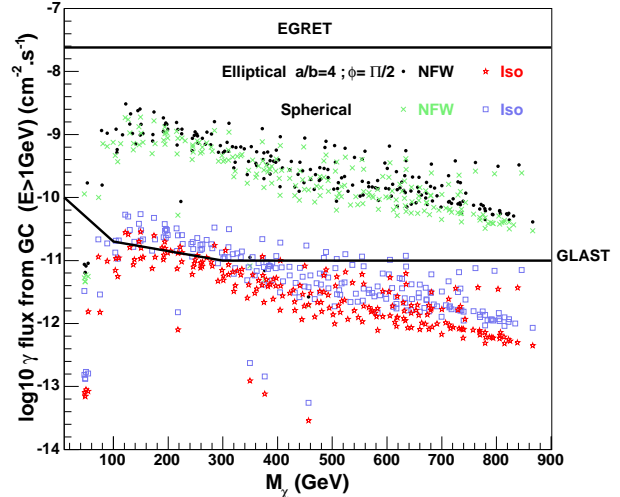
B. Halo asphericity

A general triaxial halo is modeled by taking

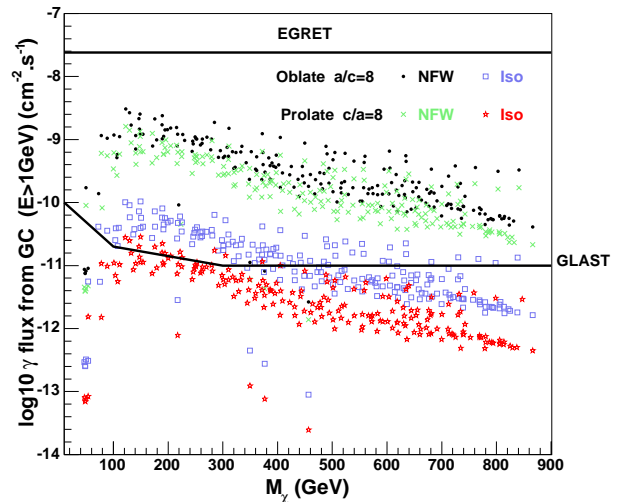
$$r = [(x/a)^2 + (y/b)^2 + (z/c)^2]^{1/2}, \quad (9)$$

with the semi-major axis in the galactic plane aligned with the x axis, $a > b$, and $abc = 1$ so that the overall halo mass is fixed. To study the influence of the non axisymmetry of the halo in the galactic plane, we set $c = 1$ and derive the variation of \bar{J} as a function of a/b and/or the angle ϕ between the x axis and the direction of the Sun. Note that an elliptical deformation of the halo can impact substantially the dark matter annihilation flux without jeopardizing the rotation curve fit. As shown in numerical simulations [12], the elliptical deformation is powered by the angular momentum exchange between the galactic bar and the dark halo. When this mechanism is acting solely, the deformation is strongest near the galactic center and decreases outwards down to a spherical symmetry. In principle, one could use any radius-dependent axis ratio function $(a/b)(r)$ and recover axisymmetry at large distances (*i.e.* away from the bar). For simplicity, however, we will consider a constant ellipticity factor. This will not introduce a strong deviation from a realistic case because the annihilation signal is dominated by the contribution near the galactic center. Axial ratios factors of up to 5 have been obtained for the bar in the baryonic disk component both in observations and in numerical simulations, but the oval deformation of the dark halo is expected to be milder [13]. Given the uncertainties, however, and the unknowns from the history of bar formation that could well vary from one galaxy to another, we examine here a wider range of possible values.

In Fig. 2, the variations of $\bar{J}(\Delta\Omega = 10^{-3} \text{ sr})$ as a func-



a)



b)

FIG. 4: Effects on gamma fluxes of NFW and Isothermal halo elliptical deformations : a) in the disc plane b) prolate and oblate cases .

tion of a/b for $\phi = 0, 0.35$ ($= 20 \text{ deg}$; see review [31] and references therein) and $\pi/2$, and as a function of ϕ for $a/b = 2$ and 9 are given for the isothermal, Kravtsov, NFW and Moore halo profiles. For $\phi = 0$, \bar{J} increases with a/b , which is expected since the higher density region is stretched along the line of sight in this case. For larger values of ϕ , the variation becomes negative, as the stretching in the x axis is misaligned with the line of sight. Therefore, \bar{J} is a decreasing function of ϕ for a fixed value of a/b .

It is important to notice that the impact of the ellip-

ticity is stronger for less cuspy halo profiles (which are favored by observations). Indeed, the relative contribution to \bar{J} coming from inner regions inside a small radius r increases with γ . Therefore, for higher values of γ , a larger fraction of the volume integral $\int \rho^2 dV$ is not affected by a change in ellipticity, as the observation solid angle is taken constant.

To study the influence of the prolate-oblate shape of the halo, we set $a = b$ in Eq. (9) and let c/a vary. The result is given in Fig. 3, for the isothermal, Kravtsov, NFW and Moore halo profiles. The variation of \bar{J} with the prolate-oblate shape of the halo is again stronger for smaller values of γ . As we can see on Fig. 3, an oblate deformation induces an enhancement of dark matter density along the line of sight increasing the signal, whereas the prolate shape decreases it and thus could be understood with arguments similar to those above.

Finally, let's consider the most popular dark matter candidate, *i.e.* the neutralino (χ), which comes from the neutral gauge and Higgs boson superpartners in the *Minimal Supersymmetric Standard Model* (MSSM) framework. We show in Fig. 4 the neutralino dark matter resulting fluxes for a wide sample of supersymmetric models, *i.e.* we take parameters of the MSSM to get bino as well as mixed bino-wino and bino-higgsino neutralino which have higher couplings and cross sections (see *e.g.* [32]). All the points shown satisfy the WMAP requirement on relic density and accelerator constraints. As the GLAST experiment sensitivity will probe a wide range

of halo profiles, we clearly see that in addition to the (essentially inner) power law behavior of the halo, the geometry also alters the estimation of the fluxes and has to be included in flux calculations.

III. CONCLUSIONS

The dark matter annihilation signal from the galactic center has been calculated for different halo characteristics. In particular, we have shown some possible effects of the halo asphericity. The induced corrections are more relevant for flat than for cuspy cores.

Although a plausible elliptical deformation of the dark matter halo does not change the expected annihilation signal by orders of magnitude, a consistent prediction of the flux from the halo shape or conversely of the halo shape from the signal should take those effects into account.

Acknowledgments

The authors thank Albert Bosma and Jean Orloff for interesting and motivating discussions and O. Gnedin and H. Zhao for useful email exchanges. F.-S. L. and E.N. work is supported by the I.I.S.N. and the Belgian Federal Science Policy (return grant and IAP 5/27).

-
- [1] A. Bosma (1978) PhD thesis, Groningen Univ; A. Bosma *Astron. J.* **86**, 1825 (1981)
 - [2] A. Bosma, *in* S. D. Ryder *et al.*, International Astronomical Union Symposium **220**, 39-48 (2004);
 - [3] P. Natarajan, (ed.) 2002, *The shapes of galaxies and their dark halos*, Proceedings of the 1st Yale Cosmology Workshop (Singapore: World Scientific)
 - [4] C. S. Frenk, S. D. M. White, M. Davis, G. Efstathiou, *Astrophys. J.* **327**, 507 (1988); J. Dubinski, *Astrophys. J.* **431**, 617 (1994); P.A. Thomas *et al.*, *Mon. Not. Roy. Astron. Soc.* **296**, 1061 (1998); M. S. Warren, P. J. Quinn, J. K. Salmon, W. H. Zurek, *Astrophys. J.* **399**, 405 (1992); Y. P. Jing, Y. Suto, *Astrophys. J.* **574**, 538 (2002); J. S. Bullock, *The shapes of galaxies and their dark halos*, P. Natarajan, (ed.), Proceedings of the 1st Yale Cosmology Workshop (Singapore: World Scientific 2002)
 - [5] A. Bosma, *Celestial Mechanics & Dynamical Astronomy*, **72**, 69-90 (1998).
 - [6] M.R. Merrifield, "The Shape of Galaxies and their Halos", Yale May 2001, ed. World Scientific (2002).
 - [7] R.P. Olling and M.R. Merrifield, *Mon. Not. Roy. Astron. Soc.* **311**, 361-369 (2000)
 - [8] R. Ibata, G. F. Lewis, M. Irwin, E. Totten, T. Quinn, *Astrophys. J.* **551**, 294 (2001); S. R. Majewski, M. F. S krutskie, M.D. Weinberg, J. C. Ostheimer *et al* 2003, *Astrophys. J.* 599, 1082; D. Martinez-Delgado, M. A. Gomez-Flechosa, A. Aparicio, R. Carrera, *Astrophys. J.* **601**, 242 (2004) ; A. Helmi, *Astrophys. J.* **610** (2004a) L97-L100; A. Helmi, *Mon. Not. Roy. Astron. Soc.* **351**, 642 (2004b); Johnston K. V. Law D. R. Majewski S. R. *Astrophys. J.* **619** 800 (2005)
 - [9] P.B. Eskridge *et al.*, *Astron. J.*, **119** 536-544 (2000); Grosbol, P., Pompei, E., Patsis, P. A., *in* E. Athanassoula *et al.*, *Disks of Galaxies : Kinematics, dynamics and perturbations*, pp 305-310, *Astron. Soc. Pac. Conf. Series* 275, 2002.
 - [10] L. Blitz, D. Spergel, *Astrophys. J.* **379** 631 (1991); J. L. Weiland *et al* *Astrophys. J.* **425** L81 (1994); E. Dwek *et al* *Astrophys. J.* **445**, 716 (1995) ; J. Binney, O. Gerhard, D. Spergel *Mon. Not. Roy. Astron. Soc.* **288**, 365 (1997).
 - [11] R.H. Miller *et al.*, *Astrophys. J.* **161**, 903-916 (1970); F. Hohl, *Astrophys. J.* **168**, 343-359 (1971).
 - [12] E. Athanassoula, *Astrophysical Journal letters*, 569 (2002), L83-L86; E. Athanassoula, *Mon. Not. Roy. Astron. Soc.* **341**, 1179-1198 (2003).
 - [13] E. Athanassoula, *Celestial Mechanics and Dynamical Astronomy* **91**, 9-31 (2005); E. Athanassoula, *Nonlinear Dynamics in Astronomy and Physics (In Memory of Henry E. Kandrup)* Stephen T. Gottesman, J.-R. Buchler, and M.E. Mahon, Editors. *Annals of the New York Academy of Sciences*, vol.1045, 168-192, (2005); Colin P., Valenzuela O., Klypin A., *astro-ph/0506627*
 - [14] M.R. Merrifield, "The Shape of Galaxies and their Halos", Yale May 2001, ed. World Scientific (2002); H.-W. Rix and D. Zaritsky, *Astrophys. J.* **447**, 82 (1995).
 - [15] W. de Boer *et al.*, *Contributed Paper to ICHEP04, Bei-*

- jing, astro-ph/0408272.
- [16] G. R. Blumenthal, S. M. Faber, R. Flores and J. R. Primack, *Astrophys. J.* **301**, 27 (1986).
- [17] J. Edsjo, M. Schelke and P. Ullio, arXiv:astro-ph/0405414.
- [18] F. Prada, A. Klypin, J. Flix, M. Martinez and E. Simonneau, *Phys. Rev. Lett.* **93**, 241301 (2004).
- [19] O. Y. Gnedin, A. V. Kravtsov, A. A. Klypin and D. Nagai, *Astrophys. J.* **616**, 16 (2004)
- [20] P. Gondolo and J. Silk, *Phys. Rev. Lett.* **83**, 1719 (1999)
- [21] P. Ullio, H. Zhao, M. Kamionkowski, *Phys. Rev. D* **64**, 043504 (2001)
- [22] O.Y. Gnedin and J.R. Primack, *Phys.Rev.Lett.* **93**, 061302 (2004).
- [23] D. Merritt, *Phys. Rev. Lett.* **92**, 201304 (2004)
- [24] G. Bertone, E. Nezri, J. Orloff and J. Silk, *Phys. Rev. D* **70**, 063503 (2004)
- [25] G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rept.* **267**, 195 (1996).
- [26] L. Bergstrom, P. Ullio, J. Buckley, *Astropart.Phys.* **9**, 137-162 (1998).
- [27] A.V. Kravtsov *et al.*, *Astrophys. J.* **502**, 48 (1998).
- [28] J.F. Navarro, C.S. Frenk, S.D.M. White, *Astrophys. J.* **490**, 493 (1997).
- [29] B. Moore *et al.*, *Mon. Not. Roy. Astron. Soc.* **310**, 1147-1152 (1999).
- [30] G. Bertone and D. Merritt, astro-ph/0501555.
- [31] W. Dehnen in E. Athanassoula, A. Bosma and R. Mujica, *Disks of Galaxies : Kinematics, Dynamics and Perturbations*, *Publ. Astron. Soc. Pacific* 275, p. 105 (2002)
- [32] V. Bertin, E. Nezri and J. Orloff, *JHEP* **0302**, 046 (2003); Y. Mambriani and C. Munoz, arXiv:hep-ph/0407158.