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# $k$ -core decomposition: a tool for the analysis of large scale Internet graphs

Ignacio Alvarez-Hamelin<sup>1</sup>, Luca Dall'Asta<sup>1</sup>, Alain Barrat<sup>1</sup>,  
Alessandro Vespignani<sup>2</sup>

<sup>1</sup> LPT (UMR du CNRS 8627), Université Paris Sud, 91405 Orsay, France

<sup>2</sup> School of Informatics, Indiana University, Bloomington, IN 47408, USA.

**Abstract.** We use the  $k$ -core decomposition, based on a recursive pruning of the least connected vertices, to study large scale Internet graphs at the Autonomous System level. This approach allows the characterization of progressively central cores of networks, conveniently uncovering hierarchical and structural properties. Internet maps show the noticeable property of having all  $k$ -cores consisting of a single connected component with invariant statistical properties (degree distribution, correlation spectrum etc.). This feature suggests that the Internet is organized in a defined hierarchy of connected subgraphs of increasing centrality with self-similar properties. The  $k$ -core decomposition provides also an interesting tool to follow the temporal evolution of Internet maps and test the stability and reliability of different mapping strategies.

## 1 Introduction

In recent times, the possibility of accessing, handling and mining large-scale networks datasets has revamped the interest in their investigation and theoretical characterization along with the definition of new modeling frameworks. In particular, mapping projects of the World Wide Web (WWW) and the physical Internet offered the first chance to study topology and traffic of large-scale networks. Gradually other studies followed describing population networks of practical interest in social science, critical infrastructures and epidemiology [1, 10, 24]. The study of large scale networks, however, faces us with an array of new challenges. The definitions of centrality, hierarchies and structural organizations are hindered by the large size of these networks and the complex interplay of connectivity patterns, traffic flows and geographical, social and economical attributes characterizing their basic elements.

In this paper, we propose the use of the  $k$ -core decomposition to study the hierarchical properties of large scale Internet maps. The  $k$ -core decomposition [4] consists in identifying particular subsets of the network, called  $k$ -cores, each one obtained by recursively removing all the vertices of degree smaller than  $k$ , until the degree of all remaining vertices is larger than or equal to  $k$ . Larger values of  $k$  correspond to vertices with larger degree and more central position in the network's structure. The  $k$ -core decomposition therefore provides a probe to study the properties of the network's regions of increasing centrality. Here we analyze

Internet networks at the Autonomous Systems (AS) level. We first perform an analysis of recent maps of the AS network obtained by various Internet mapping projects. We find that  $k$ -cores are always made by a single connected component, indicating the presence of a hierarchy of well defined regions of which it is possible to investigate the statistical properties. Strikingly, the various distributions and quantities analyzed appear to be invariant in the various  $k$ -cores. The  $k$ -core decomposition therefore exploits the self-similar properties of Internet networks, uncovering the same structural ordering at different hierarchical levels. We also perform a longitudinal analysis of the temporal evolution of Internet maps by comparing the  $k$ -core decompositions of the AS maps at various times between 2001 and 2005. This methodology allows the discussion of the evolution of the Internet and of the reliability and biases of different mapping strategies.

## 2 Related work

The discussion of the topological properties of Internet maps has highlighted a very complex and heterogeneous topology with fluctuations extending over many scale lengths. Starting with the seminal paper by Faloutsos, Faloutsos and Faloutsos [12] an impressive number of papers has dealt with the characterization of the large scale properties and hierarchies of the Internet [18, 19, 24, 26, 28, 29] and the possible sources of errors and biases presented by the experimental data [7, 9, 16, 25]. Here, we introduce the use of the  $k$ -core decomposition as a probe for the hierarchical and self-similar structure of the Internet. This decomposition has been recently used in biologically related contexts, where it was applied to the analysis of protein interaction networks or in the prediction of protein functions [3, 31]. An interesting application in the area of networking has been provided by Gaertler *et al.* [14], where the  $k$ -core decomposition is used for filtering out peripheral Autonomous Systems (ASes) in the case of Internet maps. The  $k$ -core decomposition is also used as a basis for the visualization of large networks, in particular for AS maps [2, 5, 17]. Recently, the  $k$ -core decomposition has been used as a theoretical tool to investigate the structure of random complex networks, with the obtention of a criterion for the emergence of  $k$ -cores and the study of their stability in randomly damaged networks [11]. Finally, recent works using the  $k$ -core analysis have been focused in the analysis of the Internet maps obtained by the DIMES project [20]. In ref. [6], an approach based on the  $k$ -core decomposition and closely related to the one presented here has been used with a different perspective to provide a conceptual and structural model of the Internet; the so-called medusa model for the Internet.

## 3 $k$ -core decomposition

Let us consider a graph  $G = (V, E)$  of  $|V| = n$  vertices and  $|E| = e$  edges, the definition from [4] of  $k$ -cores is the following

**Definition 1:** A subgraph  $H = (C, E|C)$  induced by the set  $C \subseteq V$  is a  $k$ -core or a core of order  $k$  iff  $\forall v \in C : \text{degree}_H(v) \geq k$ , and  $H$  is the maximum subgraph with this property.

A  $k$ -core of  $G$  can therefore be obtained by recursively removing all the vertices of degree less than  $k$ , until all vertices in the remaining graph have degree at least  $k$ . It is worth remarking that this process is not equivalent to prune vertices of a certain degree. Indeed, a star-like subgraph with node with a high degree that connect many vertices with degree one and connected only with a single edge to the rest of the graph is going to belong to the first shell no matter how high is the degree of the node. We will also use the following definitions

**Definition 2:** A vertex  $i$  has *shell index*  $k$  if it belongs to the  $k$ -core but not to  $(k + 1)$ -core.

**Definition 3:** A  $k$ -shell  $C_k$  is composed by all the vertices whose shell index is  $k$ . The maximum value  $k$  such that  $C_k$  is not empty is denoted  $k_{\max}$ . The  $k$ -core is thus the union of all shells  $C_c$  with  $c \geq k$ .

The  $k$ -core decomposition therefore identifies progressively internal cores and decomposes the networks layer by layer, revealing the structure of the different  $k$ -shells from the outmost one to the most internal one.

## 4 $k$ -core structure of the AS graphs

In order to obtain Internet connectivity information at the AS level is it possible to inspect routing tables and paths stored in each router (passive measurements) or directly ask the network with a software probe (active measurements). In the following we consider data from three large scale Internet mapping projects using different strategies. The *Oregon route-views* [23] project provides maps of the AS graph based on the knowledge of the routing tables of several Border Gateway Protocol (BGP) peers. An active measurement approach has been implemented by the skitter project at CAIDA [8]. This project deployed several strategically placed probing monitors using a path probing software. All the data are then centrally collected and merged in order to obtain Internet maps that maximizes the estimate of cross-connectivity. The third set we consider is provided by the Distributed Internet Measurements and Simulations (DIMES) project [27]. At the moment the project consists of more than 5,000 measuring agents that perform Internet measurements such as TRACEROUTE and PING. In the following we show how the application of the  $k$ -core decomposition can shed light on important hierarchical properties of Internet graphs focusing on the AS maps obtained by each project in 2005. Finally, in order to show the use of the  $k$ -core decomposition in the model validation process we also apply the same analysis to the graph obtained by the INET3.0 topology generator [30] specifically designed to represent the Internet at the AS level. Table 1 displays a summary of the basic properties of the considered Internet maps, together with the maximal shell index.

source	# of nodes	# of edges	$\langle d \rangle$	$d_{max}$	$k_{max}$
Oregon Route-Views (BGP), 2005/04	18119	32141	3.548	1382	27
CAIDA, 2005/04	8542	25492	5.969	1171	26
DIMES, 2005/05	20455	61760	6.039	2800	39
INET 3.0	10000	19676	3.936	984	8

**Table 1.** Main properties of the Internet maps considered in the present study.

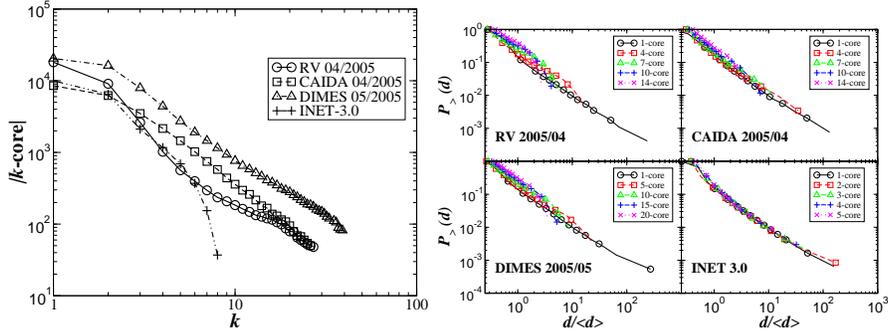
The first observation about the structure of the  $k$ -cores is that they remain connected. This is not a priori an obvious fact since one can easily imagine networks whose  $k$ -core decomposition yields several connected components corresponding, e.g. to various communities. Instead, each decomposition step is just *peeling* the network leaving connected the inner part of the network. Figure 1 displays the size in term of vertices of each  $k$ -core as a function of its number. Power-law like shapes are obtained, meaning that each  $k$ -core is composed of a constant fraction of the  $(k - 1)$ -core, building a hierarchy of more and more central vertices (analogous results have been obtained independently in Ref. [6]). In this figure, the INET generator exhibits a very different behavior with a small number of  $k$ -cores and a size behavior that is exponentially decreasing for increasing core index. It is interesting to note that while the INET network displays size and degree distribution similar to the Internet maps, the  $k$ -core structure appears to be very different, thus discriminating among the model and the actual Internet structure.

Figure 1 also shows the cumulative degree distribution for some  $k$ -cores, for the various AS maps and for the artificial network; namely, the probability  $P_{>}(d)$  that any vertex in the networks has a degree larger than  $d$ . Strikingly, the shape of the distribution, i.e. an approximate power-law, is not affected by the decomposition. In particular the exponent of the power-law is robust although the range of variation of the degree decreases. This feature defines a striking property of statistical self-similarity of the generated  $k$ -cores, which resemble one with each other under the opportune rescaling of the average degree.

In order to better characterize and check this self-similarity, we have computed also the two and three points correlations functions of the various  $k$ -cores. A useful measure to quantify correlations between the degrees of neighboring vertices is the *average degree of nearest neighbors*  $d_{nn}(d)$  of vertices of degree  $d$ :

$$d_{nn}(d) = \frac{1}{n_d} \sum_{j/d_j=d} \frac{1}{d_j} \sum_{i \in V(j)} d_i \quad , \quad (1)$$

where  $V(j)$  is the set of neighbors of vertex  $j$  and  $n_d$  the number of vertices of degree  $d$ . This last quantity is related to the correlations between the degree of connected vertices since on the average it can be expressed as  $d_{nn}(d) = \sum_{d'} d' P(d'|d)$ , where  $P(d'|d)$  is the conditional probability that a vertex with degree  $d$  is connected to a vertex with degree  $d'$ . If degrees of neighboring vertices are uncorrelated,  $P(d'|d)$  is only a function of  $d'$  and thus  $d_{nn}(d)$  is a constant.

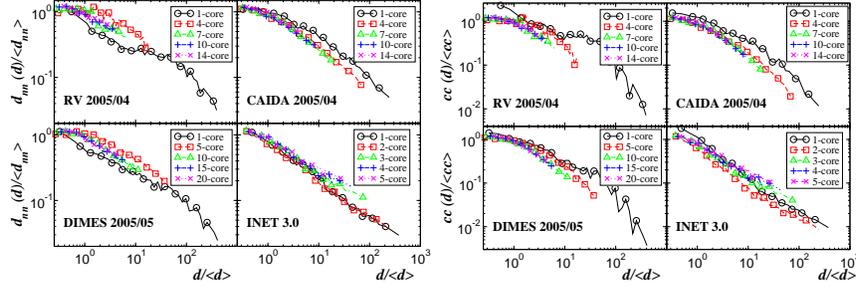


**Fig. 1.** Left: Core size as a function of the core number. Right: Cumulative degree distribution of some  $k$ -cores for the AS maps and models. The degree is normalized by the average degree of each  $k$ -core.

When correlations are present, two main classes of possible correlations have been identified: *assortative* behavior if  $d_{nn}(d)$  increases with  $d$ , which indicates that large degree vertices are preferentially connected with other large degree vertices, and *disassortative* if  $d_{nn}(d)$  decreases with  $d$  [22]. A second, and often studied, relevant quantity is the *clustering coefficient* that measures the local group cohesiveness and is defined for any vertex  $j$  as the fraction of connected neighbors of  $j$ :  $cc_j = 2 \cdot n_{\text{link}} / (d_j(d_j - 1))$ , where  $n_{\text{link}}$  is the number of links between the  $d_j$  neighbors of  $j$ . The study of the clustering spectrum  $cc(d)$  of vertices of degree  $d$  allows e.g. to uncover hierarchies in which low degree vertices belong generally to well interconnected communities (high clustering coefficient), while hubs connect many vertices that are not directly connected (small clustering coefficient).

In Figure 2 we report the  $d_{nn}(d)$  and  $cc(d)$  computed for the various  $k$ -cores. Also in this case the behavior of the two quantities is preserved as the network is recursively pruned of its low-degree vertices. In other words, the overall network topology is invariant for  $k$ -cores of increasing centrality. In summary, the AS networks exhibit a statistical scale invariance with respect to the pruning obtained with the  $k$ -core decomposition. Indeed, while this decomposition identifies subgraphs that progressively correspond to the most central regions of the network, the statistical properties of these subgraphs are preserved at all levels of pruning. This hints to a sort of global self-similarity for regions of increasing centrality of the network, and to a structure in which each region of the Internet as defined in terms of network centrality has the same properties as the whole network. The  $k$ -core decomposition appears therefore as a suitable way to define a pruning procedure equivalent to a scale-change preserving the statistical property of graphs.

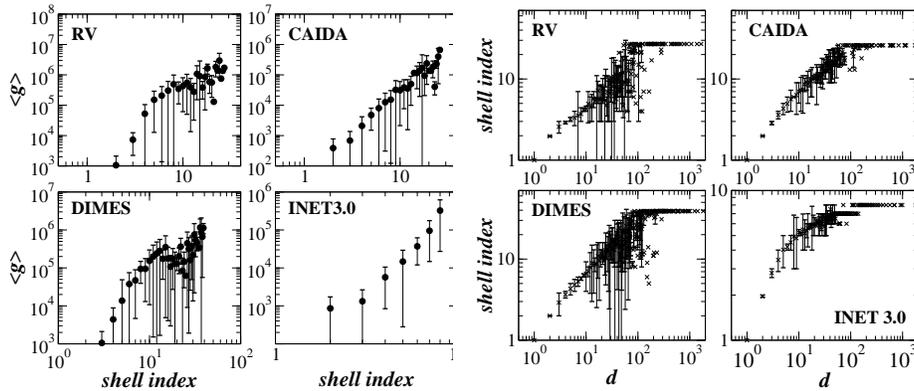
*Shell index and centrality.* The identification of the most central vertices is a major issue in networks characterization [13]. While a first intuitive and imme-



**Fig. 2.** Nearest neighbors degree distribution and clustering coefficient spectrum of some  $k$ -cores, rescaled by the corresponding average values, of the AS maps and model.

diagram measure of the centrality of vertices is given by their degree, more refined investigations are needed in order to characterize the real importance of various vertices: for example, some low-degree vertices may be essential because they provide connections between otherwise separated parts of the network. In order to uncover such important vertices, the concept of betweenness centrality (BC) is now commonly used [13, 15, 21]. The betweenness centrality of a vertex  $v$  is defined as  $g(v) = \sum_{s \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$ , where  $\sigma_{st}$  is the number of shortest paths going from  $s$  to  $t$  and  $\sigma_{st}(v)$  is the number of shortest paths from  $s$  to  $t$  going through  $v$ . This definition means that central vertices are part of more shortest paths within the network than peripheral vertices. Moreover, the betweenness centrality gives in transport networks an estimate of the traffic handled by the vertices, assuming that the number of shortest paths is a zero-th order approximation to the frequency of use of a given vertex, in the case of an all-to-all communication.

The  $k$ -core decomposition is offering an alternative definition of centrality as indicated by the shell-index of each node. This intuitively provide a hierarchy of the vertices based on their shell index that is a combination of local and global properties. (e.g.: [31] shows that the shell index is a better criterium for centrality than the degree in protein interaction networks). In this perspective, it becomes very interesting to study the correlation between the degree and betweenness centralities and the shell index of a vertex in order to quantify the statistical level of consistency of the various measures. We show in Fig. 3 the average betweenness centrality (computed on the original graph) of vertices as a function of their shell index, and the shell index as a function of the degree  $d$ . A strong correlation is obtained as expected, however with noticeable fluctuations. These fluctuations are not surprising: while a low-degree node has clearly low shell index, large degree nodes do not always have large shell index. Moreover, the betweenness centrality is a highly non-local quantity which can be large even for small-degree nodes. These quantities are thus pinpointing different kinds of centrality. The shell index appears therefore as a very interesting quantity to uncover central



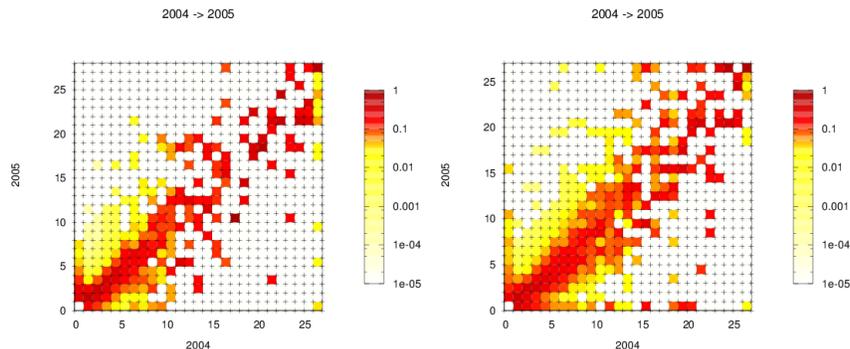
**Fig. 3.** Four figures on the left: average (original) betweenness centrality as a function of shell index. Four figures on the right: average shell index as a function of the degree.

nodes and it has the advantage of a much much faster computation time than those required for the betweenness centrality.

*Time analysis* The availability of data obtained by the various projects (Oregon route-views and CAIDA) makes it possible to study the temporal evolution of the Internet maps. In particular we consider the Oregon route-views (RV) maps of 2001/05, 2002/04, 2003/04, 2004/04 and 2005/04 and the CAIDA maps of 2001/05, 2002/03, 2003/05, 2004/04 and 2005/04. Different maps are thus yearly snapshots from the two projects.

While signatures such as degree distribution, disassortative behavior and clustering spectrum are very stable over the various maps, the  $k$ -core structure analysis however reveals some important differences among the different maps. A first observation concerns the number of nodes and edges and the maximal shell index, which steadily increase in the case of the Oregon RV maps, while these quantities fluctuate in the CAIDA maps. This can be tracked down to the fact that the number of sources used by CAIDA changes (14 for the 2001/05 map, 21 for 2002/03, 24 for 2003/05 and 2004/04, and 22 for 2005/04), and that the locations of some of these sources also change. On the other hand, the Oregon RV uses a more stable set of data sources.

Interesting informations also arise from the study of the change in the composition of the various  $k$ -shells: we show as an example in Fig. 4 the probability for a given AS to change from a shell of index  $x$  in a map obtained at a given time to a shell of index  $y$  in the successive map. While most nodes do not change their shell index, as shown by the dark area around the diagonal, some suffer an important change of status, from a highly central shell to a peripheral one or vice-versa. As Fig. 4 shows, such fluctuations are much stronger in the evolution of the CAIDA maps than for the Oregon RV maps. A further fingerprint of



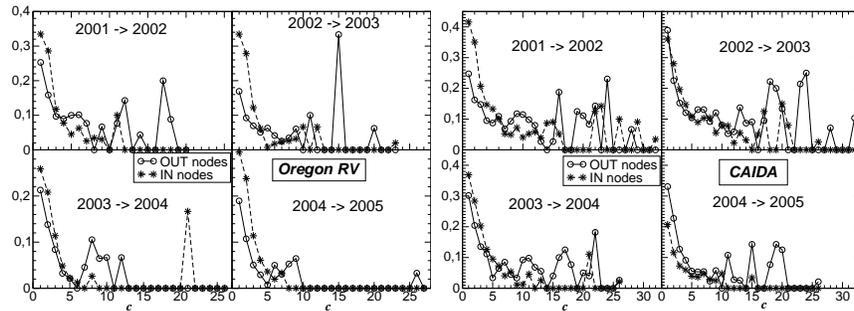
**Fig. 4.** The color code gives the probability of a change in shell index, from 2004/04 Oregon RV ( $x$  axis) to 2005/04 Oregon RV ( $y$  axis) in the left figure. The points in line 0 correspond to ASes that are present in 2005 but not in 2004, and the column 0 corresponds to the reverse situation. The right figure shows similar data for CAIDA maps.

the structural changes is provided by the analysis of the shell index of nodes that appear in or disappear from the maps between one snapshot and the other, as shown in Fig.5: while for Oregon RV maps such nodes are mostly from low index shells, and can therefore be thought of as mostly peripheral, central nodes with large shell index disappear from the CAIDA maps even in the most recent maps, between 2004 and 2005. The fluctuations observed in the shell index of ASes may be related to two factors. A first one is the natural evolution of the Internet structure. A second factor is the uncertainty and bias in the data collection. In this respect, CAIDA maps exhibit the highest level of instability, indicative of a mapping process less stable in time. In this context, the  $k$ -core analysis appears as an interesting tool to highlight the temporal changes of the Internet structure as well as the measurement reliability in each particular experimental set-up. It will certainly be of interest in the future to study similar data for evolving DIMES maps.

## 5 Conclusions

We have presented the application of the  $k$ -core decomposition to the analysis of large scale Internet maps. The  $k$ -core decomposition allows the progressive pruning of the networks and the identification of subgraphs of increasing centrality. The study of the obtained subgraphs uncovers the main hierarchical layers of the network and allows for their statistical characterization. Strikingly, we observe for the Internet at the Autonomous System a statistical self-similarity of the topological properties for cores of increasing centrality.

The  $k$ -core decomposition proves useful to uncover not only the hierarchical decomposition of real maps, but also for model validations. For example, many



**Fig. 5.** Probabilities that the out- or in- nodes have shell index  $c$  for Oregon RV and CAIDA.

models, although having e.g. degree distribution and clustering properties similar to those of real maps, do not present shell index values as large as the real data. It would be interesting to further compare various recently proposed models or topology generators and real maps of the Internet. It is also worth mentioning that the router level  $k$ -core structure of the Internet appears to have different properties than those appearing at the AS level [20]. This calls for repeating the present analysis for different router level maps available at the moment in order to better emphasize the structural difference exhibited by the two different mapping granularities.

Finally, the  $k$ -core decomposition may be used also to define a computational feasible centrality measure. It combines the degree ranking with more global structural properties providing a centrality measure that is highly correlated with the various standard definitions such as degree and betweenness centrality. Indeed, the  $k$ -core centrality finds a correspondence in the actual data of the Internet maps, where ASes in highest core are typically Tier-1 ASes; i.e. rank high in the AS classification by <http://www.netconfigs.com/>.

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## References

1. R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. *Rev. Mod. Phys.*, 74:47–, 2000.
2. J.I. Alvarez-Hamelin, L. Dall’Asta, A. Barrat, and A. Vespignani.  $k$ -core decomposition: a tool for the visualization of large scale networks. *Preprint cs.NI/0504107*, 2005.
3. G. D. Bader and C. W. V. Hogue. An automated method for finding molecular complexes in large protein interaction networks. *BMC Bioinformatics*, 4(2), 2003.
4. V. Batagelj and M. Zaversnik. Generalized Cores. *CoRR*, cs.DS/0202039, 2002.
5. M. Baur, U. Brandes, M. Gaertler, and D. Wagner. Drawing the AS Graph in 2.5 Dimensions. In *12th International Symposium on Graph Drawing, Springer-Verlag editor*, pages 43–48, 2004.

6. S. Carmi, S. Havlin, S. Kirkpatrick, Y. Shavitt, and E. Shir, 2005. [http://www.cs.huji.ac.il/~kirk/Jellyfish\\_Dimes.ppt](http://www.cs.huji.ac.il/~kirk/Jellyfish_Dimes.ppt).
7. A. Clauset and C. Moore. Accuracy and scaling phenomena in the internet. *Phys. Rev. Lett.*, 94:018701, 2005.
8. Router-Level Topology Measurements” ”Cooperative Association for Internet Data Analysis. [http://www.caida.org/tools/measurement/skitter/router\\_topology/](http://www.caida.org/tools/measurement/skitter/router_topology/).
9. L. Dall’Asta, I. Alvarez Hamelin, A. Barrat, A. Vazquez, and A. Vespignani. Traceroute-like exploration of unknown networks: A statistical analysis. *LNCS*, 3405:140–153, 2005.
10. S. N. Dorogovtsev and J. F. F. Mendes. *Evolution of networks: From biological nets to the Internet and WWW*. Oxford University Press, 2003.
11. S.N. Dorogovtsev, A.V. Goltsev, and J.F.F. Mendes. k-core percolation and k-core organization of complex networks. *cond-mat/0509102*, 2005.
12. M. Faloutsos, P. Faloutsos, and C. Faloutsos. On power-law relationship of the Internet topology. *Comput. Commun. Rev.*, 29:251–263, 1999.
13. L. C. Freeman. A Set of Measures of Centrality Based on Betweenness. *Sociometry*, 40:35–41, 1977.
14. M. Gaertler and M. Patrignani. Dynamic Analysis of the Autonomous System Graph. In *IPS 2004, International Workshop on Inter-domain Performance and Simulation, Budapest, Hungary*, pages 13–24, 2004.
15. K.-I. Goh, B. Kahng, and D. Kim. Universal behavior of load distribution in scale-free networks. *Phys. Rev. Lett.*, 87:278701, 2001.
16. A. Lakhina, J. W. Byers, M. Crovella, and P. Xie. Sampling biases in IP topology measurements, 2002. Technical report BUCS-TR-2002-021, Department of Computer Sciences, Boston University.
17. LARge NETwork VISualization tool. <http://xavier.informatics.indiana.edu/lanet-vi/>.
18. L. Li, D. Alderson, W. Willinger, and J. C. Doyle. A first-principles approach to understanding the internet’s router-level topology, 2005. Proceedings of ACM Sigcomm, Portland, OR, August 2004.
19. P. Mahadevan, D. Krioukov, M. Fomenkov, B. Huffaker, X. Dimitripoulos, Kc. Claffy, and A. Vahdat. Lessons from three views of the internet topology. *Preprint cs.NI/0508033*, 2005.
20. ”Distributed Internet MEasurements and Simulations”. <http://www.netdimes.org>.
21. M. E. J. Newman. Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality. *Phys. Rev. E*, 64:016132, 2001.
22. M. E. J. Newman. Assortative Mixing in Networks. *Phys. Rev. Lett.*, 89:208701–, 2002.
23. University of Oregon Route Views Project. <http://www.routeviews.org/>.
24. R. Pastor-Satorras and A. Vespignani. *Evolution and structure of the Internet: A statistical physics approach*. Cambridge University Press, 2004.
25. T. Petermann and P. De Los Rios. Exploration of scale free networks. *Eur. Phys.J. B*, 38:201, 2004.
26. C. Qian, H. Chang, R. Govindan, S. Jamin, S.J. Shenker, and W. Willinger. The origin of power laws in Internet topologies revisited. In *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings IEEE , Volume 2*, pages 608–617. IEEE Computer Society Press, 2002.

27. Y. Shavitt and E. Shir. Dimes: Let the internet measure itself. *Preprint cs.NI/0506099*, 2005.
28. L. Subramanian, S. Agarwal, J. Rexford, and R.H. Katz. Characterizing the internet hierarchy from multiple vantage points. In *IEEE INFOCOM 2002*, page 618, June 2002.
29. H. Tangmunarunkit, R Govindan, S. Jamin, S. Shenker, and W. Willinger. Network topologies, power laws, and hierarchy. *Comput. Commun. Rev.*, 32:76–76, 2002.
30. J. Winick and S. Jamin. Inet-3.0: Internet topology generator. Technical Report UM-CSE-TR-456-02, Department of EECS, University of Michigan, 2002.
31. S. Wuchty and E. Almaas. Peeling the Yeast protein network. *Proteomics. 2005 Feb;5(2):444-9.*, 5(2):444–449, 2005.