Towards a diagrammatic modeling of the LinBox C++ linear algebra library

Jean-Guillaume Dumas, Dominique Duval

Towards a diagrammatic modeling of the LinBox C++ linear algebra library*

Jean-Guillaume Dumas† and Dominique Duval†

October 20, 2005

Abstract

We propose a new diagrammatic modeling language, DML. The paradigm used is that of the category theory and in particular of the pushout tool. We show that most of the object-oriented structures can be described with this tool and have many examples in C++, ranging from virtual inheritance and polymorphism to template genericity. With this powerful tool, we propose a quite simple description of the C++ LinBox library. This library has been designed for efficiency and genericity and therefore makes heavy usage of complex template and polymorphic mechanism. Be reverse engineering, we are able to describe in a simple manner the complex structure of archetypes in LinBox.

1 Introduction

The LinBox library is a C++ template library for exact, high-performance linear algebra computations with dense, sparse, and structured matrices over the integers and over finite fields. C++ templates are used to provide both high performance and genericity [3]. In particular, LinBox algorithms are generic with respect to the field or ring over which they operate and with respect to the internal organization of the black box matrix. LinBox aims to provide world-class high performance implementations of the most advanced algorithms for exact linear algebra. Combining this high-performance and the genericity resulted in an intricate system of C++ classes.

In this paper, we propose a reverse engineering of this system in order to enlight its underlying mechanism and describe its various functionalities in a unified way. The chosen paradigm is that of *diagrammatic modeling* and *categories*. Our major categorical tool is the notion of *pushout*, which corresponds to several constructions in C++. Pushouts are widely used for describing the combination of two specifications sharing a common part, see for example [6, 11, 2, 10]. However, diagrammatic modeling languages like UML [9] are inadequate for dealing with such pushout constructions. For instance, in UML class diagrams and object diagrams are distinct, while we propose to merge them into a unique kind of diagram, where an instantiation (between a class and an object) is at the same level as an association between two classes or a link between two objects. Moreover, unlike UML, we consider the relation between a generic class and its template parameter as a kind of association, which allows to consider parameter passing as a pushout construction. Therefore, we propose to use a new diagrammatic modeling language (called DML), significantly different from UML. In particular, we identify the object-oriented notions of parameter passing, virtual inheritance, polymorphism template parameter passing, object instantiation,

*supported by the Institut d'Informatique et de Mathématiques Appliquées de Grenoble, InCa project.

†Université de Grenoble, laboratoire de modélisation et calcul, LMC-IMAG BP 53 X, 51 avenue des mathématiques, 38041 Grenoble, France. {Jean-Guillaume.Dumas, Dominique.Duval}@imag.fr .

as pushout constructions in a category. It should be noted that many arrows, for example the inheritance arrows, are directed in the opposite way in UML and in DML.

Some basic features about categories are used in this paper; they can be found in many textbooks, like [8, 1]. They are also given here, in section 2, for the sake of completeness. Then we present in section 3 the new diagrammatic modeling language DML. Since LinBox is a C++ library, this presentation of DML is based on the object-oriented language C++ (and Java for a while), but it could be adapted to another object-oriented language. Finally DML is used for analyzing part of the structure of the LinBox C++ library in section 4.

2 Categories and pushouts

2.1 Categories

A category can be seen as a generalized monoid. For example, let $\mathcal{F}$ denote the functions on the reals, i.e., the functions from $\mathbb{R}$ to $\mathbb{R}$, like $\sin, \cos, \exp : \mathbb{R} \to \mathbb{R}$. Such functions can be composed, like $\exp . \sin$ (or $\exp \circ \sin$), defined by $\exp . \sin(x) = \exp(\sin(x))$. This yields a structure of *monoid* on the set $\mathcal{F}$: this means that the composition is associative, i.e., $(f.g).h = f.(g.h)$, which is therefore denoted $f.g.h$, and that there is a unit for the concatenation, namely the identity id, defined by $id(x) = x$, such that $f.id = f$ and $id.f = f$.

Now, let $\mathcal{F}$ denote the functions from X to Y, where X and Y can be either $\mathbb{R}$ or $\mathbb{C}$, for instance $\mathbb{R} \to \mathbb{R}$ (sine function), $\mathbb{C} \to \mathbb{C}$ (complex conjugate), $\mathbb{C} \to \mathbb{R}$ (modulus) or $\mathbb{R} \to \mathbb{C}$ (inclusion). Such functions can still be composed, but only if they are consecutive: if $f : X \to Y$ and $g : Y \to Z$, then $g.f : X \to Z$. The associativity axiom is still valid, when it makes sense. There are now two identities, $id_R : \mathbb{R} \to \mathbb{R}$ and $id_C : \mathbb{C} \to \mathbb{C}$. The unitarity axioms become: if $f : X \to Y$ then $f.id_X = f$ and $id_Y.f = f$. This is no more a structure of monoid, because of the typing restrictions, but a structure of *category*, as defined below.

Definition 1 A *category* $\mathcal{C}$ is made of *points* $X, Y, \ldots$ and *arrows* $f, g, \ldots$, each arrow has a *source* and a *target* (this is denoted $f : X \to Y$ or $X \xrightarrow{f} Y$), each point X has an *identity* arrow $id_X : X \to X$, each pair of consecutive arrows $X \xrightarrow{f} Y \xrightarrow{g} Z$ has a *composed* arrow $X \xrightarrow{g.f} Z$, and moreover the associativity and unitarity axioms are satisfied: $(h.g).f = h.(g.f)$, $f.id_X = f$ and $id_Y.f = f$, as soon as it makes sense.

2.2 Inheritance

A category can also be seen as a generalized ordering. For example, let us look at the inheritance relation in an object-oriented language. When multiple inheritance is forbidden, as in Java, the inheritance relation defines a partial order on classes: if Z inherits from Y, which inherits from X, then, by transitivity, Z inherits from X. Let us introduce an arrow $X \to Y$ whenever Y inherits from X. **Warning!** *This is the opposite of the notation that can be found in most diagrammatic approaches, e.g. in UML or in [12]; the reasons for this choice will be exposed in section 3.* Now, the transitivity of the inheritance relation corresponds to the composition of arrows: if there are two consecutive arrows $X \to Y \to Z$, then there is a composed arrow $X \to Z$. It is not necessary to give a name to the arrows, since there is at most one arrow with given source and target.

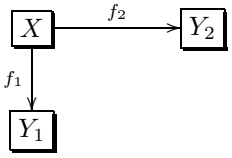
In an object-oriented language that does allow multiple inheritance, two situations may occur, they are called respectively the *ordinary inheritance* and the *virtual inheritance* in C++ [12]. If $X \to Y_1 \to Z$ and $X \to Y_2 \to Z$, in the ordinary inheritance relation Z inherits from X in two different ways, while in the virtual inheritance relation Z inherits from X in only one way. Let us give a name to the inheritance

arrows: $X \xrightarrow{f_1} Y_1 \xrightarrow{g_1} Z$ and $X \xrightarrow{f_2} Y_2 \xrightarrow{g_2} Z$. From a categorical point of view, there are two composed arrows $g_1.f_1 : X \to Z$ and $g_2.f_2 : X \to Z$. If nothing more is said, they are distinct, which corresponds to the ordinary inheritance. But if the equality $g_1.f_1 = g_2.f_2$ is added, this corresponds to the virtual inheritance.

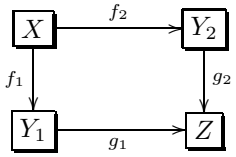
From now on, using C++ terminology, we say that a *derived* class (or subclass) inherits from a *base* class (or superclass). The base class may be *abstract*: it is a class with pure virtual methods in C++, or an `interface` in java; the idea is to provide an interface that the derived classes have to follow (mandatory methods); this kind of inheritance is used in section 3.6. Inheritance can also be an *extension*, where the derived class adds new functionalities or members to the base class, see e.g. [13] for more details on inheritance. In both cases, anyway, the derived class adds something: in the first case, only implementations are added.

2.3 Pushouts

Let $\mathcal{C}$ be a category. A *span* Sp in $\mathcal{C}$ is made of two arrows with a common source:

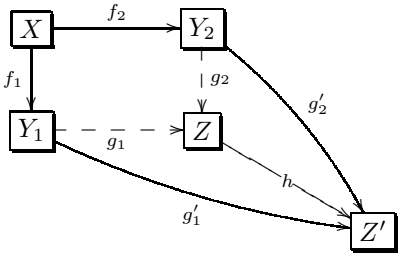


A *cone* Co with *base* Sp in $\mathcal{C}$ is made of the span Sp together with a point Z, called the *vertex* of Co, and two arrows $g_1 : Y_1 \to Z$, $g_2 : Y_2 \to Z$, called the *coprojections* of Co, such that $g_1.f_1 = g_2.f_2$:



The pushout of a span Sp is defined below as a cone with base Sp which satisfies some *initiality* condition (i.e., some kind of "minimality" condition). In this paper, the coprojections of a pushout cone are represented as dashed arrows.

Definition 2 A *pushout* with base Sp is a cone Co with base Sp such that, for each cone Co$'$ with the same base Sp, there is a unique arrow $h : Z \to Z'$ such that $h.g_1 = g'_1$ and $h.g_2 = g'_2$:



A span Sp cannot have more than one pushout (up to isomorphism), which is called *the* pushout with base Sp. The point X will be called the *gluing point* of Sp.

Roughly speaking, this means that Z is obtained by "gluing Y_1 and Y_2 along the image of X".

3 DML: a Diagrammatic Modeling Language

3.1 The Category for DML

In order to model the structure of a C++ piece of software, a category $\mathcal{C}_{dml}$ is described now, in a rather informal way; a more precise definition of the category $\mathcal{C}_{dml}$ would deserve a longer paper.

The points of the category $\mathcal{C}_{dml}$ are called the *specifications*. They are, essentially, the C++ *types*. More precisely, a specification may correspond to a built-in type, a class or a typename, and it may also correspond to a value in a built-in type or an instance of a class. Essentially, a specification A is seen as a collection of *members*, and it determines a set of *models* $\mathcal{M}od(A)$. If the specification is a class A, its models are the instances of the class A. If it is an object a, its unique model is itself. So, one may look at a specification either from a *syntactic* point of view, i.e., as a collection of members, or from a *semantic* point of view, i.e., as a set of models. In this paper, we use the syntactic point of view.

The arrows of the category $\mathcal{C}_{dml}$ are the morphisms between the specifications, they are of various kinds. Since we use the syntactic point of view on specifications, a morphism $\varphi : A \to B$ maps each member of A to a member of B (or to some composition of members of B). For example, a morphism of specifications can be an *inheritance* morphism, between two classes. When B inherits from A, the class B contains all the members of the class A, plus some new ones. From the syntactic point of view, inheritance is an arrow $\varphi : A \to B$. This morphism induces a map $\mathcal{M}od(\varphi) : \mathcal{M}od(B) \to \mathcal{M}od(A)$, by omitting the interpretation of the members of B that are not members of A. For this reason, it is often illustrated as an arrow *from B to A*: this is the case in UML, for instance, but in this paper the syntactic orientation $\varphi : A \to B$ is always chosen. A *template parameterization* is also a morphism of specifications. When a template class T occurs as a template parameter for a class B, the members of T can be used in B, so that there is a morphism $T \to B$. An *instantiation* is another kind of morphism of specifications. When an object a is created as an instance of a class A, then the members of A are instantiated in a, which can be seen as a morphism $A \to a$. An *implementation* of an abstract class A by a class B is also a morphism $A \to B$.

Specifications may be built progressively, by systematic constructions, thanks to pushouts. The aim of the next subsections is to show that some pushouts in the category $\mathcal{C}_{dml}$ correspond to fundamental constructions in C++: virtual inheritance is detailed in section 3.2, and standard parameter passing in section 3.3; now object oriented polymorphism is described in section 3.6, template parameter passing in section 3.4, and object instantiation in section 3.5. Several examples, from the library LinBox, are given in section 4.

3.2 Virtual inheritance

Virtual inheritance gives rise to cones, as explained in section 2.2. Moreover, such a cone is a pushout if and only if the doubly derived class Z is "minimal", in the sense that Z has no additional member, on top of those that are inherited from Y_1 and Y_2. The corresponding piece of C++ code, when the methods are neither constructors nor destructors, is as follows:

```
struct X {void m0(){...} };
struct Y1: public virtual X {void m1(){...} };
struct Y2: public virtual X {void m2(){...} };
struct Z: public virtual Y1, public virtual Y2 { };
```

Then the methods m_1, m_2, and one method m_0, are inherited by Z. When some m_i is a constructor, since it is not inherited in C++, it must appear explicitly in the derived class. We still speak of pushout in the latter, despite this adjunction.

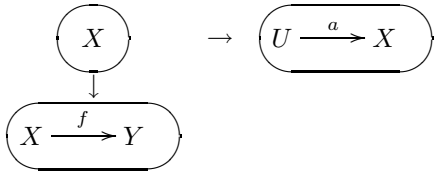
3.3 Parameter passing

The formalization of multiple inheritance by a pushout, as above, is an example of a *symmetric* use of pushouts, where both arrows in the span are of the same nature. In this paper, we are rather interested by several kinds of *dissymmetric* ways to use pushouts [1]. The paradigm for the constructions in the next sections is the parameter passing construction, as described now.

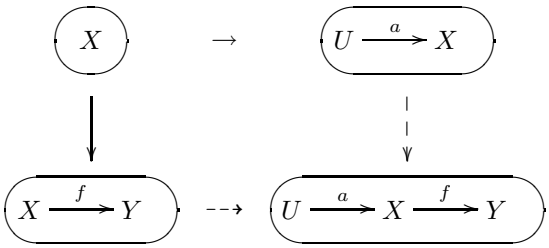
Given some expression $f(x)$ and some value a for x, the parameter passing construction builds the expression $f(a)$. Here $f : X \to Y$ is a function, $x : X$ is a symbol called the *formal parameter*, and $a : X$ is a constant called the *actual parameter*, so that the result $f(a) = f.a : Y$ is also a constant. The parameter and the result are considered as constant functions $a : U \to X$ and $f(a) = f.a : U \to Y$, where U is the *unit* type, which is interpreted as a singleton. So, the parameter passing process is seen as an application of the rule for the composition of arrows:

$$\frac{U \xrightarrow{a} X \qquad X \xrightarrow{f} Y}{U \xrightarrow{f.a} Y}$$

The pushout construction is used for building an instance of the premises of this rule. The category where the pushout takes place is the category $\mathcal{G}$ of (directed multi-)graphs. First the data, made of $f : X \to Y$ and $a : U \to X$, with the same type X, is represented as a span Sp in the category $\mathcal{G}$:

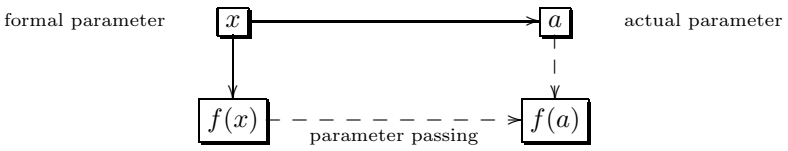


Then the pushout with base Sp is built in the category $\mathcal{G}$:



The vertex of the pushout is an instance of the premises of the rule for composition. Then, the constant $f.a$ is obtained by applying this rule. More about this point of view on deduction rules can be found in [4, 5].

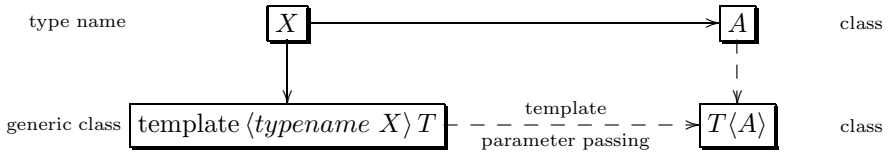
So, schematically, the parameter passing construction corresponds to the following pushout: ((*here, x reappears.*))



3.4 Template parameter passing

In C++, a class can be used as a parameter for building a new class, thanks to the *template* parameters mechanism. This works just like classical parameters, i.e., template parameter passing can be formalized

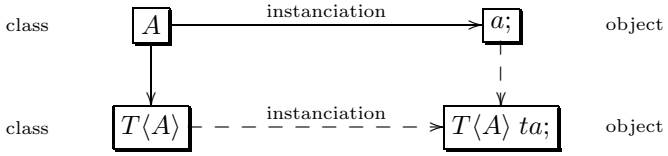
as a pushout of specifications:



Here, X is the name of the formal template parameter, that is used in the definition of the generic class T. The class A is the actual value to be passed, and the vertex of the pushout is the resulting class $T\langle A\rangle$.

3.5 Object instantiation

Object instantiation can be obtained from a pushout of specifications, in many different ways. For instance, the morphism above, from the class A to the class $T\langle A\rangle$, can be used for building an instance of $T\langle A\rangle$ from an instance of A, as follows:



This pushout yields an instance ta of $T\langle A\rangle$ using the empty constructor of T, which might call constructors of A.

3.6 Object oriented polymorphism

In this section we propose to see the polymorphism mechanism of an object oriented language, involving inheritance, as a pushout. According to [12, §12.2.6], object oriented polymorphism behaves as follows in C++: the member functions called must be *virtual* and objects must be manipulated through pointers or references. So, polymorphism is obtained when a derived class is used via a pointer to its base class, and when this base class contains virtual member functions, for example it can be an abstract class. The idea is to write algorithms on the base class and pass afterwards derived values. Note that the effect is close to that of the `template` mechanism of C++. Here is a C++ example :

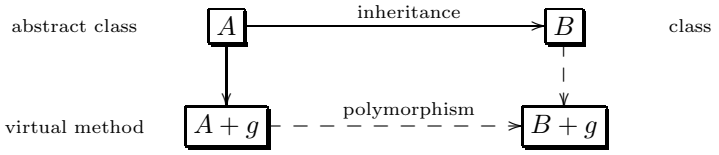
```
#include <iostream>
struct A { virtual void f() = 0; }; // abstract class
void g(A * a) { a->f(); }           // global function

// derived class adds an implementation of method f.
struct B : public A { void f() { std::cout << "f of B"; } };

int main() {
    B b;
    g( &b );  // a pushout is used
    return 0;
}
```

In this example, the class A is an abstract class, with a virtual method f; on the one hand, the class B inherits from A, and adds an implementation of f; on the other hand, the function g is implemented knowing only the interface of f, as given in A. Later on, a pointer on an object b of type B can be passed

as an argument to this function g. The corresponding pushout is then between A and g on one side, and the derived class B on the other side, with the abstract class A as gluing point:



3.7 A C++ example

The next C++ piece of code provides an example of template parameter passing followed by object instantiation.

```
#include <iostream>
template <typename X> struct T { // Template class
    // All X class are supposed to have a "g" method
    // The code in T, defines the required interface
    void f(X x) { x.g(); }
};

// An actual implantation matching the "X" interface
struct A {
    void g() { std::cout << "g of A"; }
};

int main() {
    T<A> ta;   // The class A is given as a parameter of T
    ta.f();
    return 0;
}
```

4 Application to a linear algebra C++ library: LinBox

LinBox is a C++ template library of routines for solving linear algebra problems. It has been designed for dealing with matrices over a variety of domains, in particular over finite fields or rings. Genericity and high performance are the twin goals of LinBox. The genericity is achieved by use of a small set of interfaces. Algorithms are implemented with C++ template parameters which may be instantiated with any class adhering to the specified interface. High performance is achieved by judicious specializations of the generic algorithms. It is entirely within the spirit of the project to introduce new implementations. Thus a user of the library may invoke a LinBox algorithm, say for determinant or rank of a matrix, but providing a black box class of her own design and perhaps even providing the underlying field (or commutative ring) representation. Conversely, the LinBox field and ring interfaces and the many specific representations can be used for purposes other than linear algebra computation or with algorithms not provided by LinBox.

In order to solve simultaneously the genericity and performance issues, the LinBox library has designed a complex structure, involving five distinct classes, that each LinBox domain must respect. This system is extremely efficient [14, Table 5.1], but it is quite complex to describe and to use. The aim of this paper is to give an abstract view on this architecture, thanks to the Diagrammatic Modeling Language, in order to get a simple user interface description. We now focus on the case where the underlying domain is a field, but a similar structure holds for commutative rings, for example.

4.1 A class "Field" for the algorithms

LinBox algorithms have been conceived to function with a variety of fields, in particular over finite fields or rings. To carry out its computations, any algorithm may need additional parameters, such as the modulus `p` (a prime number) for computing in the field of integers modulo `p`. For this purpose, LinBox offers a special object for the field which defines its methods (e.g., the arithmetic operators). A `Field` class thus contains the actual computational code. An instance `F` of the class `Field` corresponds to an actual field with its parameters. Moreover, an internal class `Field::Element` is used to deal with the storage of the elements of this field: for example, the call `F.add(x,y,z)` adds the elements `y` and `z` in the field `F` and stores the result in the element `x`. This `Field::Element` type can be a `longint` of C++, for the field of integers modulo `p` when `p` is a word size prime, or it can be a more complex data structure.

4.2 An abstract class "FieldAbstract" for genericity

Since all the algorithms are generic with respect to the field, they must follow a common interface. This is carried out in LinBox by a common inheritance to an abstract class, `FieldAbstract`.

4.3 An archetype "FieldArchetype" class to control code bloat

The number of generic levels in LinBox induces the need to control code bloat. The solution developed by LinBox is to define a non generic additional class, `FieldArchetype` [3]. This class encloses a pointer towards a `FieldAbstract` object. The generic algorithms can thus be instantiated on this single class. If the explosion of executable code makes it necessary, they are thus linearly and separately compiled. Thus, it is not directly the abstract class which plays the interface part. This prototype [14, 7] fulfills three roles in the library: it describes the common object interface, it provides a compiled instance of executable code, and it controls code bloat. Separating this archetype from the abstract class is mandatory in the field case for efficiency. Indeed, while polymorphism could be directly used, it induces the overhead dereferencing the pointers. This dereferencing can be too much a price to pay for every arithmetic operation. Thus, in LinBox, one can choose between best efficiency without archetypes, or better control of code bloat to the price of an overhead in computational timings.

4.4 A generic envelope class "FieldEnvelope" for external fields

Two problems result from this organization. First of all, every field, even if comes from an external library, must inherit from the abstract class. Second, the constructors, and in particular the copy constructor, cannot be inherited in C++. It is thus necessary to add a virtual method `clone` fulfilling this job in the abstract class. This induces a different interface between the abstract class and the archetype class. These two problems of inheritance and interface are solved by the creation of an additional wrapping class `FieldEnvelope`, which we describe precisely now. Then, for every field, its envelope inherits from the abstract class `FieldAbstract`.

An envelope is a generic adaptor [15], matching the interface of its wrapped object. In C++, we distinguish between several variants of this design. All of them are templated structures, depending on a template parameter B.

1. *Envelope without inheritance:* the envelope class is related to B via the type of a member, either directly or via a pointer. Since there is no inheritance, object oriented polymorphism is impossible.

   (a) *Copy envelope:* the template parameter B is the type of a member of the envelope class.

   ```
   template <typename B> struct Env {
   ```

```
private:
   B _b;
};
```

(b) *Pointer envelope:* this is a variant of the latter, without copy: it is $B*$, instead of B, which is the type of a member of the envelope class.

```
template <typename B> struct Env {
private:
   B* _b;
};
```

2. *Inheritance envelope:* the object inherits from its template. Every template characteristic is preserved except for the constructors and destructors. Object-oriented polymorphism is also preserved.

```
template <typename B> struct Env : public B;
```

The envelope fulfills two functionalities. On the one hand, an envelope gives an internal inheritance to immutable external classes: with multiple inheritance, an immutable class can nonetheless use polymorphism.

```
// Every Env<B> class inherits from a class A.
template <typename B> struct Env : public B, public A;
```

On the other hand, an envelope allows generic method abstraction. Indeed, it is conceptually impossible to define an abstract class with templated methods: the virtual table mechanism cannot resolve an associated abstract method call, since the actual method is not known when the object is created. The envelope mechanism can simulate an abstract class by forcing the template parameter interface.

```
template <typename B> struct Env : public B {
   template <typename V> void mygenericmethod(V a) {
        // This method is required for every B even if it is generic
        B::mygenericmethod(a);
   }
};
```

4.5 Envelopes in Java

This envelope formalism is not restricted to C++. In Java, for instance, one can build similar classes. First, an external and independent class:

```
public class External {
        int _a;
        External(int a) {
                _a = a;
        }
        public void amethod() {
        System.out.println("a: " + _a);
        }
}
```

Then an abstract class:

```
public interface Abstract{
        public void Themethod();
}
```

And finally an envelope `EnvelopeInherit` that implements `Abstract` and extends `External`:

```
public class EnvelopeInherit extends External implements Abstract {
        EnvelopeInherit(int a) {
                super(a);
        }
        public void Themethod() {
                super.amethode();
        }
}
```

The `EnvelopeInherit` class is therefore an `External` class implementing the internal `Abstract` interface. The copy envelope is defined similarly. Now it is less impressive in Java, since the envelope cannot be templated. In C++ a single envelope can be used by many external classes.

4.6 A DML description of the LinBox architecture for domains

In order to visualize the relations between the various classes that are used by LinBox for representing fields, we use the Diagrammatic Modeling Language of section 3. The specifications that are used by LinBox for dealing with one field (here the field with two elements), except for the class `Field::Element`, are represented by the diagram in figure 1.

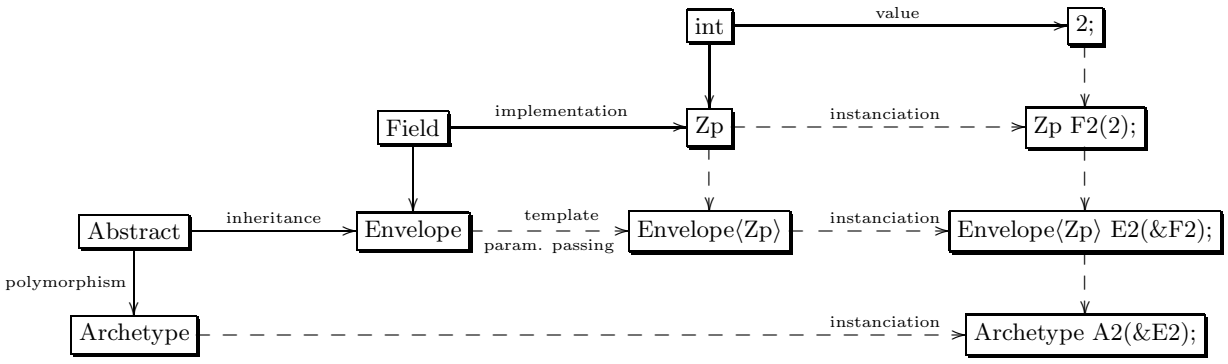


Figure 1: A DML diagram for the LinBox architecture for fields, with a copy envelope

This diagram can be read from the left to the right, i.e., from the abstraction to the instances. With the exception of the first line, the specifications (i.e., the boxes) in the rightmost column are objects, the other specifications are classes (`Zp` denotes a class of finite prime fields), and the rightmost horizontal arrows are instantiations. The first line is slightly different: it has a built-in type `int` instead of a class, and a value `2` of type `int` instead of an instance. The three objects `F2`, `E2` and `A2` represent the field with two elements, from three different points of view. The three pushouts on the right, with vertices `F2`, `E2` and `A2`, build object instantiations, as in section 3.5. The pushout in the middle, with vertex `Envelope`$\langle$`Zp`$\rangle$, corresponds to a template parameter passing, as in section 3.4. The horizontal coprojection of the bottom pushout, from `Archetype` to `Archetype A2(`, is composed of three morphisms of different nature: first an inheritance, then a template parameter passing, and finally an instantiation.

In this diagram, the envelope is a copy one: the construction of the envelope `E2` requires a copy of the field `F2`. This can be compared with the next diagram, in figure 2, with an inheritance envelope.

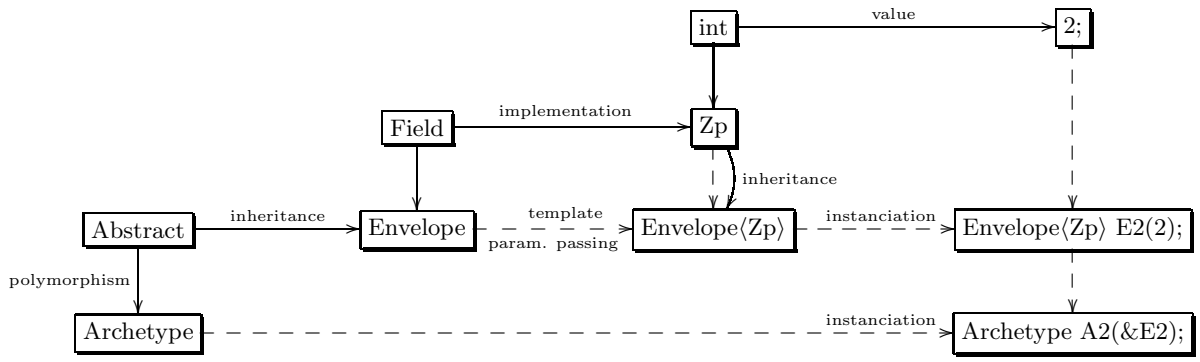


Figure 2: A DML diagram for the LinBox architecture for fields, with an inheritance envelope

It appears clearly now that the difference lies in the instantiation of the field. Indeed, the field construction is not required anymore, it is automatically made during the construction of the envelope. In figure 2, we see that one can still instantiate `Zp(2)` if needed elsewhere, but this is actually now automatically done within the constructor of the envelope.

5 Conclusion

We have designed a new diagrammatic modeling language, DML. The paradigm used is that of the category theory and in particular of the pushout tool. We have shown that most of the object-oriented structures can be described with this tool and have many examples in C++, ranging from virtual inheritance and polymorphism to template genericity.

With this powerful tool, we propose a quite simple description of the LinBox library. This library has been designed for efficiency and genericity and therefore makes heavy usage of complex template and polymorphic mechanism. Be reverse engineering, we were able to describe the fundamental structure of archetypes in LinBox. This structure contains several classes generic or not, polymorphic or not and our description requires four pushouts. We believe that our description with pushouts is quite clear and enables better understanding of the behavior of the archetypes.

Next work will be to have tools to manipulate the diagrams and to generate object oriented skeletons. Prototypes of such softwares ("Dessiner les Calculs" for diagram manipulations and "SketchUML" for generating UML for diagrams) are available on the web page of the InCa project[1].

References

[1] Michael Barr and Charles Wells. *Category Theory for Computer Science.* International Series in Computer Science. Prentice Hall, 1990.

[2] R.M. Burstall and J.A. Goguen. Putting theories together to make specifications. In *Proc. 5th Internat. Joint Conf. on Artificial Intelligence*, pages 1045–1058, 1997.

[1] `http://www-lmc.imag.fr/MOSAIC/InCa`

[3] Jean-Guillaume Dumas, Thierry Gautier, Mark Giesbrecht, Pascal Giorgi, Bradford Hovinen, Erich Kaltofen, B. David Saunders, Will J. Turner, and Gilles Villard. LinBox: A generic library for exact linear algebra. In Arjeh M. Cohen, Xiao-Shan Gao, and Nobuki Takayama, editors, *Proceedings of the 2002 International Congress of Mathematical Software, Beijing, China*, pages 40–50. World Scientific Pub, August 2002.

[4] D. Duval and C. Lair. Diagrammatic specifications. Rapport de recherche 1043 m, IMAG-LMC, January 2002.

[5] Dominique Duval. Diagrammatic specifications. *Mathematical Structures in Computer Science*, 13(6):857–890, 2003.

[6] J.A. Goguen. Categorical foundations for general systems theory. In *Advances in Cybernetics and System Research*, pages 121–130. Transcripta Books, 1973.

[7] Erich Kaltofen, Dmitriy Morozov, and George Yuhasz. Generic matrix multiplication and memory management in linbox. In Manuel Kauers, editor, *Proceedings of the 2005 International Symposium on Symbolic and Algebraic Computation, Beijing, China*. ACM Press, New York, July 2005.

[8] Saunders Mac Lane. *Categories for the Working Mathematician*, volume 5 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2nd edition, 1997. (1st ed., 1971).

[9] Pierre-Alain Muller and Nathalie Gaertner. *Modélisation objet avec UML*. Eyrolles, 2000.

[10] Catherine Oriat. Detecting equivalence of modular specifications with categorical diagrams. *TCS*, 247(1–2):141–190, 2000.

[11] Y.V. Srinivas and R. Jüllig. Specware language manual, 1995.

[12] Bjarne Stroustrup. *The C++ Programming Language: Third Edition*. Addison-Wesley Publishing Co., Reading, Mass., 1997.

[13] Antero Taivalsaari. On the notion of inheritance. *ACM Computing Surveys*, 28(3):438–479, September 1996.

[14] Will J. Turner. *Blackbox linear algebra with the LinBox library*. PhD thesis, North Carolina State University, May 2002.

[15] G. Bowden Wise. An overview of the standard template library. *SIGPLAN Not.*, 31(4):4–10, 1996.