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A SOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM

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According to general relativity, the present analysis shows on geometrical grounds that the cosmological constant problem is an artifact due to the unfounded link of this fundamental constant to vacuum energy density of quantum fluctuations.

Keywords: Cosmology; Vaccum Energy ; Cosmological Constant.

1. Introduction

The status of the cosmological constant Λ has long been discussed[1,2,3,4], whereas it is clearly established in General Relativity (GR) as *universal constant*[5]. Therefore, similarly to Newton constant of gravitation G, its value has to be estimated from observations. However, such an estimate does not agree by hundred orders of magnitude with its expected value as obtained from quantum field theories[6,7,8] by assuming that vacuum energy density of quantum fluctuations is the origin of this constant. The aim of the present investigation is to analyse on geometrical grounds this problem, called *cosmological constant problem* (CCP).

2. Status of the cosmological constant

The cosmological constant was assumed in the field equations for describing the observations in accordance with a static cosmological solution[9] but a general expansion of the universe was observed[10] subsequently. What is usually called "Einstein's biggest blunder" stands probably for the historical reason why Λ was wrongly understood as a free parameter in the field equations (see[7,8] for more details). Such an issue to the cosmological problem has provided us with (authority and/or simplicity) arguments[11] in favor of $\Lambda = 0$ until acceleration of the cosmological expansion

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could not be avoided for the interpretation of recent data (chap. 2.1). On geometrical grounds, the *principle of general relativity* (PGR) applied to gravity provides us with the status of universal constant for Λ, which intervenes in the description of the gravitational field at cosmological scales (chap. 2.2), similarly as for G at smaller scales.

2.1. *Observational status of* Λ

In the past, estimates such as $\Lambda < 2\,10^{-55}\,\mathrm{cm}^{-2}$ from dynamics of galaxies in clusters[12] or $-2\,10^{-56}\,\mathrm{cm}^{-2} \leq \Lambda < 4\,10^{-56}\,\mathrm{cm}^{-2}$ from the minimum age of the universe and the existence of high redshift objects[13], were interpreted with some *a priori* in mind (for arguing) in favor of a vanishing value. Decades later, estimates based on the redshift–distance relation for brightest cluster galaxies[14,15] and for quasars[16,17,14,18] provided us unambiguously with a non zero cosmological constant $\Lambda \sim 3h^2\,10^{-56}\,\mathrm{cm}^{-2}$, where $h = H_\circ/100\,km\,s^{-1}\,Mpc^{-1}$. Nowadays, it is generally believed that $\Lambda \sim 2h^2\,10^{-56}\,\mathrm{cm}^{-2}$ is required for interpreting the CMB temperature fluctuations[19,20,21,22] and for accounting of Hubble diagram of SN[23,24,25,26].

2.2. *Geometrical status of* Λ

The gravitational field and its sources are characterized respectively by the metric tensor $g_{\mu\nu}$ on the space-time manifold V_4 and by a *vanishing divergence* stress-energy tensor $T_{\mu\nu}$. The gravitational field equations satisfy PGR : they must be invariant with respect to the action of diffeomorphism group of V_4[5,27]. In other words, their most general form is an expansion of covariant tensors

$$T_{\mu\nu} = -A_0 F^{(0)}_{\mu\nu} + A_1 F^{(1)}_{\mu\nu} + A_2 F^{(2)}_{\mu\nu} + \dots \tag{1}$$

where $F^{(n)}_{\mu\nu}$ stands for an invariant of degree $2n$, it reads in term of metric tensor $g_{\mu\nu}$ and its derivatives, and A_n is a *coupling constant*. The $n = 0, 1$ order terms are uniquely defined

$$F^{(0)}_{\mu\nu} = g_{\mu\nu}, \quad F^{(1)}_{\mu\nu} = S_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor and R the scalar curvature, whereas $F^{n\geq 2}_{\mu\nu}$ must be derived from additional principles. The values of coupling constants A_n must be estimated from observations.

Schwarzschild solution of Eq. (1) enables us to identify $A_{n=0,1}$ with Newton approximation, what provides us with modified Poisson equation[5]

$$\mathrm{div}\tilde{\mathrm{g}} = -4\pi \mathrm{G}\rho + \Lambda \tag{3}$$

where $\vec{g}$ stands for the gravitational acceleration field due to sources defined by a specific density ρ, and the following identification of constants

$$\mathrm{G} = \frac{1}{8\pi A_1}, \qquad \Lambda = \frac{A_0}{A_1} \tag{4}$$

which shows their common status of *universal constant*. Therefore, one understands that the same treatment has to be applied to both of them for estimating their values from observations but at scales adapted to each of them, as it can be shown from a dimensional analysis of Eq. (1,2).

According to GR, the speed of the light $c = 1$ (*i.e.* time can be measured in unit of length[a] 1s = $2.999\,792\,458\,10^{10}$ cm) and then $\mathrm{G} = 7.4243 \times 10^{-29}$ cm g^{-1}. Let us choose units of mass and of length[b], herein denoted respectively by M and L. The correct dimensional analysis of GR sets the covariant metric tensor to have the dimension $[g_{\mu\nu}] = L^2$, and thus $[g^{\mu\nu}] = L^{-2}$, $[R_{\mu\nu}] = 1$ and $[R] = L^{-2}$. Since the specific mass density and the pressure belong to T^{μ}_{ν}, one has $[T_{\mu\nu}] = ML^{-1}$. Hence, according to Eq. (1), the dimensions of A_n are the following

$$[A_0] = ML^{-3}, \quad [A_1] = ML^{-1}, \quad \ldots [A_n] = ML^{2n-3} \tag{5}$$

which shows their relative contributions for describing the gravitational field with respect to scale. Namely, the larger their degree n the smaller their effective scale[c]. Equivalently, the estimation of A_0 demands observational data located at scale larger than the one for A_1, *etc.*... This is the reason why the Λ effect is not discernible at small scale but requires cosmological distances.

2.3. *Modeling gravitational structures*

The space-time geometry is constrained by the presence of gravitational sources as described by means of tensor $T_{\mu\nu}$ in Eq. (1). According to dimensional analysis given in previous subsection, each right hand terms contributes for describing the geometry within its effective scale. The observations show that gravitational structures within scales of order of solar system can be described by limiting the expansion solely to Einstein tensor $S_{\mu\nu}$, when cosmology requires also the first term. The transition scale between A_0 and A_1 is of order of $1/\sqrt{\Lambda} \sim 7h^{-1}$ Gyr. Although GR is preferred for investigating the dynamics of cosmic structures, Newton approximation given in Eq. (3) provides us with an easier schema for realizing the Λ effect. Hence, the acceleration field due to gravity around a point mass m reads

$$\vec{g} = \left(-\mathrm{G}\frac{m}{r^3} + \frac{\Lambda}{3}\right)\vec{r} \tag{6}$$

Since $\Lambda > 0$, the gravity force is attractive at distance $r < r_\circ$ and repulsive at $r > r_\circ$ with a critical distance

$$r_\circ = \sqrt[3]{3m\mathrm{G}/\Lambda} \tag{7}$$

where the gravity vanishes. In accordance with observations, no Λ effect is expected in the sun neighborhood because $r_\circ \sim 10^2 h^{-2/3}$ yr is much larger than the size of

[a]This is the reason why any statement on the variation of c is meaningless in GR.
[b]Only two fundamental units can be chosen, the third one is derived.
[c]In other words, the contribution of A_0 dominates at scale larger than the one of A_1, *etc.*...

solar system and the mean distance between stars. On the other hand, it should be appreciable in the outer parts of the Galaxy since $r_\circ \sim 5\,10^5 h^{-2/3}$ yr is only 5 times larger than the disc diameter[d]. In the case of Local Super Cluster, $r_\circ \sim 4\,10^8 h^{-2/3}$ yr corresponds approximately to its size, what suggests that a Λ effect might intervene in its formation process, and probably for the existence of large scale voids in the distribution of galaxies. The hypothesis that the value of Λ accounts for the smoothing scale $\sim$100 Mpc from which the distribution of cosmological structures becomes homogeneous and isotropic today must be envisaged.

3. The cosmological constant problem

It is assumed that the contribution of quantum fluctuations to the gravitational field is defined by the following stress-energy tensor[e]

$$T^{\rm vac}_{\mu\nu} = \rho_{\rm vac}\, g_{\mu\nu}, \qquad \rho_{\rm vac} = \hbar k_{\rm max} \tag{8}$$

in the field equations Eq.(1), where $k_{\rm max}$ stands for the ultraviolet momentum cutoff up to which the quantum field theory is valid[6]. However, the expected density, *e.g.*

$$\rho^{EW}_{\rm vac} \sim 2\,10^{-4}\ {\rm g\,cm^{-3}}, \quad \rho^{QCD}_{\rm vac} \sim 1.6\,10^{15}\ {\rm g\,cm^{-3}}, \quad \rho^{Pl}_{\rm vac} \sim 2\,10^{89}\ {\rm g\,cm^{-3}} \tag{9}$$

differs from the one measured from astronomical observations at cosmological scale

$$\rho_\Lambda = \frac{\Lambda}{8\pi {\rm G}} \sim h^2\,10^{-29}\ {\rm g\,cm^{-3}} \tag{10}$$

by 25–118 orders of magnitude. Other estimations of this quantum effect from the viewpoint of standard Casimir energy calculation scheme[28] provide us with discrepancies of ~ 37 orders of magnitude[29].

A similar problem happens when

$$\Lambda_{\rm vac} = 8\pi {\rm G} \rho_{\rm vac} \tag{11}$$

is interpreted as a cosmological constant. Indeed, if the quantum field theory which provides us with an estimate of $\rho_{\rm vac}$ is correct then the distance from which the gravity becomes repulsive in the sun neighborhood ranges from $r^{EW}_\circ \sim 2\,10^{-2} h^{-2/3}$ a.u. down to $r^{Pl}_\circ \sim 3\,10^{-11} h^{-2/3}$ Å depending on the quantum field theory, see Eq. (7). Obviously, such results are not consistent with the observations.

Another version of the cosmological constant problem points out a fine tuning problem. It consists on arguing on the smallness of $\Lambda = \Lambda_{\rm vac} + \Lambda_\circ$, interpreted as an effective cosmological constant, where $\Lambda_\circ$ stands for a bare cosmological constant in Einsteins field equations.

[d]With this in mind, the dynamics of the extended HI regions of spiral galaxies should be reviewed with respect to the interpretation of rotation curves.

[e]The usual picture which describes the vacuum as an isotropic and homogenous distribution of gravitational sources with energy density $\rho_{\rm vac}$ and pressure $p_{\rm vac} = -\rho_{\rm vac}$ (although this is not an equation of state) is not clear and not necessary for the discussion.

3.1. *Understanding the acceleration of the cosmological expansion*

The observations show that the dynamics of the cosmological expansion after decoupling era agrees with Friedmann-Lemaître-Gamov solution. It describes an uniform distribution of pressureless matter and CMB radiation with a black-body spectra, the field equations are given by Eq. (1) with $n \leq 1$. The present values of related densities are $\rho_{\rm m} = 3h^2\, 10^{-30}\ {\rm g\,cm^{-3}}$ (dark matter included) and $\rho_{\rm r} \sim 5h^2\, 10^{-34}\ {\rm g\,cm^{-3}}$. Their comparison to the expected vacuum energy density $\rho_{\rm vac}$ shows that if quantum fluctuations intervene in the dynamics of the cosmological expansion then their contribution prevails over the other sources (by 26–119 orders of magnitude today). Such an hypothesis provides us with a vacuum dominate cosmological expansion since primordial epochs. Therefore, one might ask whether such disagreements with observations can be removed by taking into account higher order $n \geq 2$ terms in Eq. (1). With this in mind, let us describe the dynamics of structures at scales where gravitational repulsion ($\Lambda > 0$) is observed. Since the values of universal constants G and Λ are provided by observations, it is more convenient to use adapted units of time l_g and of mass m_g defined as follows

$$l_g = 1/\sqrt{\Lambda} \sim h^{-1}\, 10^{28}\ {\rm cm}, \qquad m_g = 1/(8\pi G\sqrt{\Lambda}) \sim 4h^{-1}\, 10^{54}\ {\rm g} \tag{12}$$

herein called *gravitational units*. They are defined such that the field equations read in a normalized form

$$T_{\mu\nu} = -g_{\mu\nu} + S_{\mu\nu} + A_2 F^{(2)}_{\mu\nu} + \ldots \tag{13}$$

i.e. $A_0 = A_1 = 1$, where the stress-energy tensor $T_{\mu\nu}$ accounts for the distribution of gravitational sources. It is important to note that, with gravitational units, Planck constant reads

$$\hbar \sim 10^{-120} \tag{14}$$

Indeed, such a tiny value as *quantum action unit* compared to $\hbar = 1$ when quantum units are used instead, shows clearly that Eq. (13) truncated at order $n \leq 1$ is not adapted for describing quantum physics[30,31]. This is the main reason why it is hopeless to give a quantum status to Λ[32]. As approximation, because of dimensional analysis described above, the contribution of higher order terms being the more significant as the density is large, Eq. (13) can be splited up with respect to scale into two equations systems. The first one corresponds to terms of order $n < 2$ (the usual Einstein equation with Λ) and the second one

$$T^{\rm vac}_{\mu\nu} = A_2 F^{(2)}_{\mu\nu} + \ldots \tag{15}$$

stands for the field equations describing the effect of quantum fluctuations on the gravitational field at an appropriated scale (quantum), interpreted as correction of the RW metric $g_{\mu\nu}$. The identification of constants A_n (*e.g.*, $A_2 = \hbar$) and the derivation of tensors $F^{(n)}_{\mu\nu}$ with $n \geq 2$ requires to model gravitational phenomena at quantum scale, see *e.g.*[33,34]. Unfortunately, the state of the art does not allow yet to provide us with a definite answer for defining the right hand term of Eq. (15), see *e.g.*[35].

4. Conclusion

To rescale the field equations for describing the cosmological expansion prevents us to assume the vacuum acting as a cosmological constant. As a consequence, one understands that such an interpretation turns to be the origin of the cosmological constant problem[f]. Because the understanding of quantum gravity is still an ongoing challenge, the correct field equations describing the contribution to gravity of quantum fluctuations are not yet established. However, the dimensional analysis shows that the related gravitational effects are expected at small (quantum) scales and do not participate to the general expansion of the universe according to observations.

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[f]In other words, CCP is the price to pay for identifying $\Lambda_{\rm vac}$ to the cosmological constant.

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