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Electromagnetic pulses from an oscillating high finesse cavity

Possible signatures for dynamic Casimir effect experiments

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Vacuum field fluctuations exert radiation pressure on mirrors in quantum vacuum. For a pair of mirrors this effect is well known as the Casimir effect. When a single mirror is moving in vacuum, radiation pressure leads to a dissipative force which opposes itself to the mirrors motion. Accordingly the electromagnetic field does not remain in the vacuum state but photons are emitted by the mirror into vacuum. This paper describes the photon emission of a high-finesse cavity oscillating globally in quantum vacuum. Common features and differences with usual parametric processes in quantum optics are discussed. Interesting signatures of quantum radiation like pulse shaping and frequency up-conversion are predicted, which could be used to experimentally demonstrate motion-induced dissipative effects. The feasibility of an experimental realization is discussed in the end of the paper.

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I. INTRODUCTION

Relativity of motion is one of the basic principles of physics since Galileo. In classical physics this principle applies without further considerations, as vacuum is considered to be completely empty. However, the face of the problem today is changed by quantum theory. Quantum vacuum is no longer empty. It contains irreducible field fluctuations which lead to mechanical effects for any scatterer in vacuum. This paper will focus on the vacuum fluctuations of the electromagnetic field, which are characterized by a mean energy of $\frac{1}{2}\hbar\omega$ per field mode of frequency ω .

Vacuum fluctuations exert mechanical action on scatterers due to their non-vanishing radiation pressure. For macroscopic objects, this leads to the Casimir force, an attractive force arising between two mirrors at rest in vacuum [1]. But vacuum fluctuations produce also mechanical effects related to the motion of scatterers in vacuum. When the mirror is at rest, it already experiences a fluctuating force due to vacuum radiation pressure [2, 3]. But as the radiation pressure is the same on both sides, no mean force appears in this case. However when the mirror is moving, a dissipative force may arise, opposing itself to the mirror's motion [4]. The dependence of the dissipative force is directly connected to the spectral properties of the fluctuating force through the fluctuations-dissipation relations [5].

In order to motivate the origin of the dissipative effect of quantum vacuum, let us consider a mirror in a scalar thermal field. As is well known, in a thermal field, the dissipative force $F_{\text{diss}}(t)$ is proportional to the mirror's velocity $q'(t)$

$$F_{\text{diss}}(t) = -\frac{\hbar\theta^2}{6\pi c^2}q'(t) \quad (1)$$

where $q(t)$ is the mirrors motion and θ the field temperature expressed in frequency units $\theta = 2\pi k_{\text{B}}T_{\text{field}}/\hbar$.

The force may equivalently be written in the frequency domain

$$F_{\text{diss}}[\omega] = \frac{\hbar\theta^2}{6\pi c^2}i\omega q[\omega] \quad (2)$$

where $F_{\text{diss}}[\omega]$ and $q[\omega]$ are the Fourier transform of the force and mirror's displacement. This force is a classical expression which tends towards zero when temperature goes to zero, as it neglects the effect of vacuum fluctuations. When vacuum fluctuations are taken into account, the linear susceptibility is found to scale as the third power of frequency at the limit of zero temperature

$$F_{\text{diss}}[\omega] = \frac{\hbar}{6\pi c^2}i\omega^3 q[\omega] \quad (3)$$

This result, which could be expected from mere dimensional arguments, implies that the force is proportional to the third order time derivative of the mirror's position

$$F_{\text{diss}}(t) = \frac{\hbar}{6\pi c^2}q'''(t) \quad (4)$$

The linear susceptibilities encountered in (2) and (3) are directly connected to the spectral properties of the fluctuating force exerted upon a mirror at rest through the fluctuations-dissipation relations [5]. At arbitrary temperature the dissipative force is just the sum of the two contributions (2) and (3). These expressions can also be generalized to the case of a real mirror with frequency-dependent reflection and transmission amplitudes [6].

The dissipative force arising for a mirror moving in quantum vacuum has interesting consequences with respect to the problem of relativity of motion. In contrast to the dissipative force experienced by a mirror in a thermal field, it vanishes for a motion with uniform velocity. This is a direct consequence of the Lorentz invariance of quantum vacuum. The appearance of vacuum in an accelerated frame is a much debated question [7]. For the present problem of motion of a mirror in vacuum there

is a clear answer at our disposal. No dissipative force arises for a motion with uniform acceleration and this fact may be explained as a consequence of the conformal invariance of electromagnetic vacuum.

However, a dissipative force arises for a mirror moving in vacuum with a non-uniform acceleration. For example, an oscillating mirror feels a force proportional to its change of acceleration due to the coupling to vacuum fluctuations. Accordingly, the electromagnetic field does not remain in the vacuum state but photons are emitted by the mirror into vacuum. This motion-induced radiation and the associated radiation reaction force are dissipative effects related to motion in quantum vacuum, although this motion has no further reference than vacuum itself. Since these effects challenge the principle of relativity of motion in vacuum, it would be very important to obtain experimental evidence for them and to study their characteristics in detail. So far, these effects have not yet been observed for macroscopic objects like mirrors. As a matter of fact, the orders of magnitude are exceedingly small for the fluctuating force as well as for the dissipative force. This raises the question as how to increase the order of magnitude of dissipative effects of vacuum fluctuations and eventually render those effects observable.

II. MOTION-INDUCED RADIATION

A first idea in this direction is to observe changes in the field rather than in the mechanical forces. Due to energy conservation, the dissipated energy is transformed into radiation emitted by the mirror. Let us for example consider a mirror oscillating in vacuum at a frequency Ω with an amplitude q_0 as shown in figure 1

$$q(t) = q_0 \sin \Omega t \quad (5)$$

The number of emitted photons N during the measure-

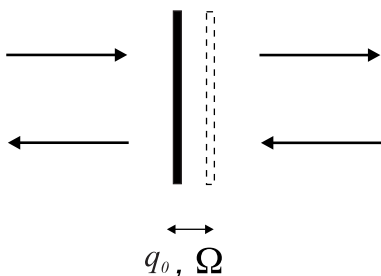


FIG. 1: Single mirror oscillating in vacuum. The arrows represent the vacuum field which, in a onedimensional space, may be considered as two counterpropagating fields.

ment time T is then given by

$$N = \frac{\Omega^3 q_0^2 T}{3\pi c^2} = \frac{\Omega T}{3\pi} \frac{v^2}{c^2} \quad (6)$$

$$(7)$$

The Ω^3 -dependence in the second term comes from the already discussed motional susceptibility (3). $v = q_0 \Omega$ is the mirror's maximal velocity. Since N scales as the square of the ratio of the mirrors mechanical velocity and the speed of light, it remains very small for any possible macroscopic motion.

The emission of photons can also be interpreted using analogies with optical parametric processes. Indeed, photon production by the oscillating mirror can be explained through opto-mechanical coupling between vacuum fluctuations and the mechanical motion. In a linear approximation, the mechanical excitation of frequency Ω is transformed into two photons of frequency ω and ω' emitted into vacuum. Energy conservation imposes the sum of the photon frequencies to be equal to the oscillation frequency $\omega + \omega' = \Omega$. In vacuum only spontaneous

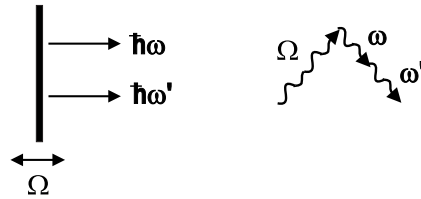


FIG. 2: Emission diagram for a parametric process. Through opto-mechanical coupling between vacuum fluctuations and the mirrors motion, a mechanical excitation Ω is transformed into two photons of frequency ω and ω' which are emitted into vacuum.

parametric processes can take place as it is impossible to extract energy from vacuum.

As motion induced radiation emitted by a single mirror is too small to be observed, a natural idea for improving the orders of magnitude is to study a cavity oscillating in vacuum instead of a single mirror. In this configuration one may profit from the resonant amplification of radiation inside the cavity. The resonant enhancement is determined by the cavity finesse \mathcal{F}

$$\mathcal{F} = \frac{\pi}{1 - r^2} \quad (8)$$

which gives the number of roundtrips of the field before it leaves the cavity, and depends on the mirrors amplitude reflection coefficients r here chosen to be equal for simplicity. Hence the cavity has to be treated as an open system with mirrors having reflection coefficients smaller than unity so that the field can leave the cavity by transmission through the mirrors [8]. This distinguishes the present calculations from the numerous works devoted to photon production between a pair of perfectly reflecting mirrors [4, 9–14] in which case the amount of radiation emitted outside the cavity cannot be evaluated.

For a cavity oscillating in vacuum, it is well known that the cavity field is parametrically amplified when the mechanical cavity length is modulated periodically (breathing motion). If the cavity field is initially in the vacuum state, this excitation leads to a squeezed vacuum state [15] which differs from the pure vacuum state

and in particular contains photons. This corresponds to a usual parametric amplifier or oscillator, which can be easily and efficiently realized in an experiment by using a nonlinear crystal inside the cavity which produces a periodic change of the effective cavity length instead of the mechanical length. A great number of such experiments has been performed in the past [15] and their results are well understood. However, these experiments can give no information about relativity of motion in quantum vacuum.

More strikingly, a resonant enhancement of radiation also exists when the cavity oscillates as a whole, with its mechanical length L kept constant as shown in figure 3. Here both mirrors oscillate with the same amplitude q_0

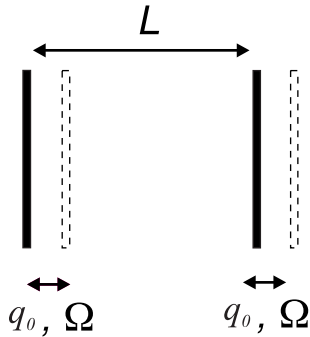


FIG. 3: Cavity oscillating globally in vacuum with an amplitude q_0 at a frequency Ω with constant mechanical length.

and frequency Ω . While the breathing motion of the cavity takes place when the mechanical oscillation frequency equals an even multiple of the fundamental cavity resonance frequency $\pi c/L$, the global motion is realized for frequencies equal to odd multiples of the fundamental resonance frequency :

$$\Omega = K \frac{\pi c}{L}; \quad K = \begin{cases} 2, 4, 6, \dots & \text{breathing} \\ 3, 5, 7, \dots & \text{global} \end{cases} \quad (9)$$

At a first glance, it seems to be paradoxical that both the breathing and the global motion give rise to emission of radiation due to parametric excitation of the cavity field. The basic reason for this is that the optical length (which depends on retardation effects) as seen by the field varies in the same way for both kinds of motion although the mechanical cavity length is modulated in one case and constant in the other.

Motion induced radiation emitted by a cavity oscillating globally is reminiscent of photon emission from a single oscillating mirror. However, the important point is that compared to the situation with a single oscillating mirror in vacuum, radiation from an oscillating cavity is resonantly enhanced by a factor of the order of the cavity finesse [8]. Indeed, at perfectly tuned resonance, the number of photons emitted by the cavity is the product of motion-induced radiation emitted by a single oscillating

mirror (7) by the cavity finesse \mathcal{F}

$$N = \mathcal{F} \frac{\Omega T v^2}{3\pi c^2} \quad (K = 3) \quad (10)$$

Since the cavity finesse can be a very large number, up to $10^9 - 10^{12}$ for instance for microwave cavities [16, 17], this increases considerably the order of magnitude. The effect of detuning from resonance on the number of emitted photons has been studied in [18].

III. SIGNATURES

The previous section has only discussed the linear regime where the field scattering is supposed to be linear in the mirrors motion. For a single reflection, the field scattered by the oscillating mirror undergoes a phase shift of the order of v/c compared to the incoming field. This phase shift is always small for a macroscopic mirror. However, inside the cavity the field is scattered many times by the moving mirrors. For the cavity oscillating globally in vacuum, the single reflection phase shifts accumulate in an optimal way leading to a large total phase shift. A linearization of the scattering in the mirrors motion becomes therefore invalid. We have developed a full nonlinear treatment of this situation which uses the composition of homographic functions of phase exponentials allowing to obtain analytical expressions for the emitted energy density outside and inside the cavity as well as for the frequency spectrum [19]. Our method is a generalization to the leaky cavity of the treatment used by Law [12] and shortly afterwards by Cole and Schieve [13] who calculated the intracavity field for perfectly reflecting mirrors and predicted the formation of wave packets inside the cavity growing with time. In the situation of the leaky cavity that we study, the field can leave the cavity leading to emitted radiation outside the cavity, while the average intracavity energy per oscillation period is stationary. Accordingly, we have found a pulse shaping effect not only in the intracavity field but also for the field radiated outside the cavity, which both grow with increasing cavity finesse. A novel signature related to this pulse shaping effect is a process of frequency up-conversion taking place in the multiple scattering, which can be clearly identified in the radiation spectrum. This signature might become very important in an experimental observation in order to discriminate motion-induced radiation from potential stray effects.

To discuss these signatures, it is convenient to introduce a single reflection parameter α related to the mirrors velocity v through

$$\text{th} \left(\frac{v}{c} \right) = \alpha \quad (11)$$

and a multiple reflection parameter η , which is the ratio of the mirrors velocity to the velocity of light multiplied by the cavity finesse

$$\eta = \mathcal{F} \frac{v}{c} \quad (12)$$

η corresponds to an effective phase shift acquired by the fields during multiple reflection inside the moving cavity and characterizes the efficiency of the multiple scattering. In contrast to the single scattering parameter which is necessarily very small for macroscopic motions, it can be large thanks to the multiplication by the cavity finesse.

Multiple scattering gives rise to periodic orbits such that the optical length seen by the field bouncing back and forth in the cavity is the same on successive round-trips despite the motion of the mirrors as first described in [12, 13]. As the light rays corresponding to a periodic orbit encounter the mirrors at the same position after an arbitrary number of round-trips, this leads to an interference effect, analogous to that occurring for a motionless cavity. There exist two sets of periodic orbits which respectively attract or repel the neighboring trajectories, giving rise to constructive or destructive interference. Only the attractive orbit is expected to give rise to a large enhancement of the motional radiation. This process leads to the formation of regularly spaced electromagnetic pulses bouncing back and forth the cavity. At each scattering on one of the mirrors, there is a small probability for a photon for escaping the cavity and therefore being detected outside the cavity. This probability is given by the inverse of the cavity finesse.

Based on this qualitative argument one may calculate precisely the energy density emitted into vacuum by the oscillating cavity as a function of time. The result is shown in Figure 4 where is plotted the energy density for

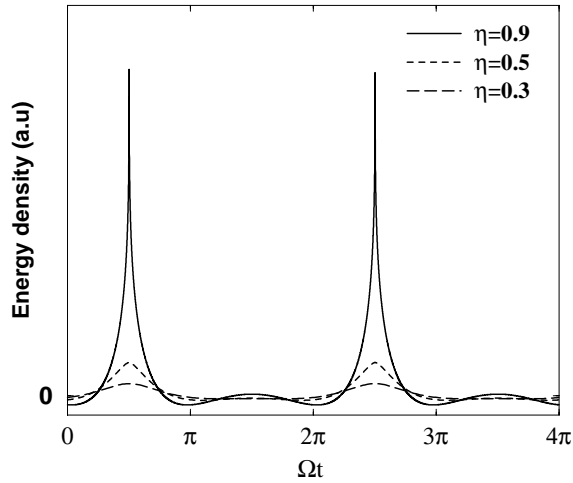


FIG. 4: Energy density emitted by the cavity as a function of time for different effective phase shifts η . With increasing values of η the energy starts to concentrate in pulses emitted periodically by the cavity.

three different values of η . The cavity emits periodically electromagnetic pulses into the vacuum which become higher and sharper with an increasing multiple scattering parameter. The plot is based on an analytical solution of the multiple scattering problem in terms of homographic mappings of phase exponentials [19]. This approach re-

mains valid in the case of interest $\eta \sim 1$ whereas an approach linearizing the fields in the phase shifts would be restricted to $\eta \ll 1$.

The total energy per oscillation period radiated by the cavity can be calculated by averaging the energy density over one oscillation period $2\pi/\Omega$. We will restrict our attention to a realistic case, where the mirrors velocity is small which justifies the use of the homographic relations. This leads then to a total radiated energy per oscillation period

$$\begin{aligned}
 E &= \frac{\hbar\Omega r}{6} \text{sh}^2\alpha + \frac{\hbar\Omega\sqrt{1-r^2}}{24} (\zeta(\alpha) + \zeta(-\alpha) - 2) \\
 &\quad - \frac{\hbar\Omega\sqrt{1-r^2}}{4\pi^2 K^2} (\xi(\alpha)(\zeta(\alpha) - e^{-2\alpha}) \\
 &\quad + \xi(-\alpha)(\zeta(-\alpha) - e^{2\alpha})) \\
 \zeta(\alpha) &= \frac{(1 - e^{-4\rho})e^{2\alpha} + \sqrt{1-r^2}(1 - e^{2\alpha})}{1 - e^{4(\alpha-\rho)}} \\
 \xi(\alpha) &= \sum_{l=1}^{\infty} \frac{e^{2l(\alpha-\rho)}}{l^2 \text{ch}2\alpha l} \tag{13}
 \end{aligned}$$

ρ is related to the reflection coefficients $r^2 = e^{-2\rho}$ and corresponds to the inverse of the cavity finesse for a high finesse cavity $\rho \ll 1$ via eqn. (8). The last equation in (13) suggests the interpretation of α as a gain for the motion-induced radiation while ρ signifies the cavity losses. The total radiated energy would diverge for $\alpha = \rho$. However this limit is not reached as the energy density already diverges when $2\alpha/\rho$ approaches unity. The reason for this is simply that the widths of the various contributions to the energy density decrease when the number of roundtrips inside the cavity increases, so that the contribution to the integrated energy increases less rapidly than the peak value of the energy density. As a consequence, the divergence of the peak density occurs before the divergence of the integrated energy. Our calculation does not remain valid above the oscillation threshold for the energy density. Still it may be expected that a large amount of radiation is emitted above threshold.

For experimental reasons one might also be interested in the amount of energy stored inside the cavity. The intracavity energy, integrated over the cavity length $L = K\pi/\Omega$, is with the same notations as above found to be

$$\begin{aligned}
 \mathcal{E} &= \frac{\hbar\Omega K}{48} (\zeta(\alpha) + \zeta(-\alpha) - 2) \\
 &\quad - \frac{\hbar\Omega}{8\pi^2 K} (\xi(\alpha)\zeta(\alpha) + \xi(-\alpha)\zeta(-\alpha) - 2\xi(0)) \tag{14}
 \end{aligned}$$

The energy is here directly expressed with respect to the static Casimir energy which is recovered when the cavity is motionless [4]. This result is due to the fact that the vacuum outside and inside the cavity is not the same but differs exactly by this amount of energy.

In order to obtain an appreciable value for the radiated energy per period if the cavity is moving at a small velocity, it is necessary to consider a high finesse cavity

$\rho \ll 1$. In this case equation (13,,14) may be approximated as follows by expanding separately the common denominator and numerators

$$\begin{aligned} E &\approx \frac{\hbar\Omega}{6}\alpha^2 + \frac{\hbar\Omega}{6}\left(1 - \frac{1}{K^2}\right)\frac{\rho\alpha^2}{\rho^2 - \alpha^2} \\ \mathcal{E} &\approx \frac{\hbar\Omega}{24}\left(K - \frac{1}{K}\right)\frac{\alpha^2}{\rho^2 - \alpha^2} \\ \alpha &\leq \frac{\rho}{2} \ll 1 \end{aligned} \quad (15)$$

The first term in the radiated energy is due to the field which is directly reflected by the two mirrors without entering the cavity. This term corresponds to the expression for motion-induced radiation from a single perfectly reflecting mirror. The second term has its origin in the field which has traversed the cavity and thus accumulated a much more important dephasing than the singly reflected field. Neglecting α^2 in the denominator and deducing the total number of emitted photons during the detection time T leads to the linear result (10). The linear approximation is found to be rigorously valid if the gain is much smaller than the cavity losses ($\alpha \ll \rho$).

A particular case is the excitation at the fundamental cavity mode $K = 1$. Clearly equation (15) shows that for a high finesse cavity no enhancement of photon production inside the cavity can be obtained in accordance with results in ref. [11, 12]. In this case the energy radiated by the cavity corresponds to the one emitted by a single mirror and the motional intracavity energy vanishes. However, in the general case of arbitrary cavity finesse motion-induced photons are also found for the $K = 1$ mode (cf. equation (13)). Although this motion gives rise to periodic orbits as pointed out in [13], the key point is that the cavity plays the part of a filtering function and suppresses photons at zero frequency. Due to the coupling to radiation pressure photons are not emitted singly but in pairs. Thus motion-induced radiation is enhanced by the cavity if all photons are emitted into a cavity mode, the sum of their frequencies being equal to the mechanical oscillation frequency. In order to fulfill this condition when the cavity oscillates with the frequency of the lowest cavity mode photons have to be emitted at zero frequency. The cavity suppresses those photons the more efficiently the higher is the cavity finesse. Thus motion-induced photons for the $K = 1$ mode can be found in the bad cavity limit but not in the high finesse limit.

A second interesting feature is the frequency spectrum of the emitted radiation which can be evaluated analytically

through Fourier series developments [19]

$$\begin{aligned} n_\nu &= \frac{\sin^2(\pi\nu)}{\pi^2} \sum_{m>\nu} \nu(m-\nu) \\ &\times \left\{ \left| r(1-r^2) \sum_{n\geq 0} r^{2n} e^{-2i\pi K\nu(n+1)} G_m(\nu, \beta_{2n+1}) \right. \right. \\ &- \left. \left. r G_m(\nu, \beta_{-1}) \right|^2 \right. \\ &\left. + (1-r^2)^2 \left| \sum_{n\geq 0} r^{2n} e^{-2i\pi K\nu n} G_m(\nu, \beta_{2n}) \right|^2 \right\} \end{aligned} \quad (16)$$

$$\beta_p = (-1)^K \text{th}(p\alpha)$$

$$\nu = \frac{\omega}{\Omega}$$

where $G_m(\nu, x)$ is the hypergeometric series

$$G_m(\nu, \beta) = \beta^m \sum_{l\geq 0} \frac{\Gamma(\nu+l)\Gamma(m-\nu+l)}{\Gamma(m+1+l)} \frac{\beta^{2l}}{l!} \quad (17)$$

Figure 5 shows the upper radiation spectrum for an effective phase shift $\eta = 0.9$ and a mechanical oscillation frequency $\Omega = 5\pi c/L$. Radiation is emitted at the res-

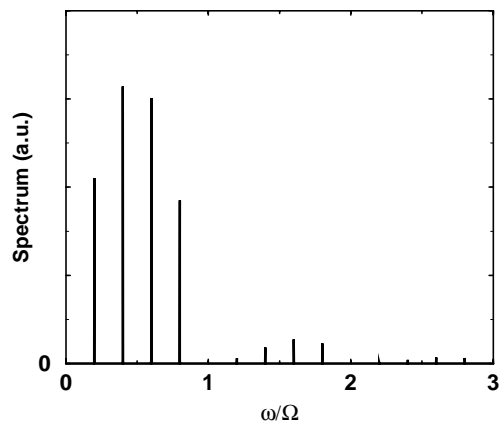


FIG. 5: Spectrum of the radiation emitted by the cavity for $\eta = 0.9$. The peaks correspond to cavity resonance frequencies. The spectrum is plotted for a cavity oscillating globally at a mechanical frequency $\Omega = 5\pi c/L$.

onance frequencies of the cavity corresponding to the peaks in the spectrum. The spectrum shown here is plotted for a cavity oscillating at a frequency $\Omega = 5\pi c/L$. Photons are emitted at multiple integers of the fundamental cavity frequency, that is at specific rational multiples of the mechanical excitation frequency Ω

$$\omega = \frac{\Omega}{5}, \frac{2\Omega}{5}, \frac{3\Omega}{5}, \frac{4\Omega}{5}, \frac{6\Omega}{5}, \dots \quad (18)$$

A striking feature is that no radiation is emitted at multiple integers of Ω . There the spectrum vanishes rigorously.

This constitutes an interesting property in so far as it is very different from the expected behavior of any pick-up or spurious field which would be strong at multiples integers of the mechanical oscillation frequency. In addition, photons are emitted not only for frequencies lower but also for frequencies higher than the oscillation frequency Ω which means that a process of frequency up-conversion takes place in the system.

The simple interpretation given in section II of emission of motion induced radiation in terms of a parametric process is therefore not valid anymore in the nonlinear regime, where the field inside the cavity acquires a large phase shift due to numerous reflections on the two oscillating mirrors. The process of frequency up-conversion can be explained in the following way. For a single reflection only spontaneous parametric processes are possible. However, inside the cavity the field undergoes a great number of roundtrips while at each reflection spontaneous parametric photons are emitted. Before these photons leave the cavity through one of the mirrors, they can be re-absorbed in a higher order parametric process as shown in figure 6 which produces a photon of a frequency larger than the mechanical oscillation frequency.

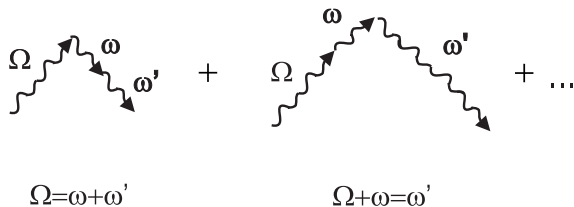


FIG. 6: Emission diagram in the nonlinear regime. Spontaneous parametric photons are emitted from a single reflection on the moving mirrors. Before these photons leave the cavity, they can be re-absorbed in a higher order process giving rise to the emission of photons with frequencies higher than the mechanical oscillation frequency.

Clearly, these specific temporal and spectral signatures of the emitted radiation may help to discriminate motion-induced radiation from potential stray effects in an experimental observation.

IV. ORDERS OF MAGNITUDE

We may now evaluate the orders of magnitude of motion induced radiation and discuss the feasibility of an experimental observation of this effect. As we are interested in the dissipative effect of vacuum, the number of thermal photons per mode in the input field needs to be smaller than 1 in the frequency range of interest. Low temperature requirements thus point to experiments using small mechanical structures with optical resonance frequencies as well as mechanical oscillation frequencies in the GHz range. In order to observe the dissipative effect of quantum vacuum and not of a thermal field, a

temperature of $T \sim 10\text{mK}$ is necessary [20]. At such a temperature, the finesse of a superconducting cavity can reach $10^9 - 10^{12}$ [16, 17]. A peak velocity ranging from $v \sim 2.5 \cdot 10^{-4}\text{m/s}$ for a finesse of 10^{12} to $v \sim 0.25\text{m/s}$ for a finesse of 10^9 would then be sufficient to obtain an effective phase shift η close to unity ($\eta \sim 0.9$). Under these conditions, the radiated flux of motion-induced photons created outside the cavity ranges from 0.001 photon/second for a finesse of 10^{12} to 1 photon per second for a finesse of 10^9 . Inside the cavity, the stationary photon number is in both case of the order of 1. This clearly illustrates that a very high finesse cavity needs much longer detection times as the mirrors transmission to the outside is extremely small. Nevertheless photon fluxes of these orders of magnitude are measurable by efficient photon-counting detection available in the GHz range. Alternatively, the field produced inside the cavity could be probed with the help of Rydberg atoms [16]. In both cases, the specific signatures of motion induced radiation are essential in order to be able to distinguish the motional radiation from stray effects.

It is important to emphasize that the peak velocity considered here is only a small fraction of the typical sound velocity in materials so that fundamental breaking limits do not oppose to these numbers. The observation of motional radiation in vacuum seems thus to be achievable by an experiment of this kind. The difficulty remains to find means for exerting a very large force to excite the motion of the cavity while keeping the optical part of the experiment at a very low temperature and largely unaffected by the stray fields induced by the excitation.

In order to have an efficient coupling between the mirrors' mechanical motion and the modes of the electromagnetic vacuum one has to suppose a mirror of macroscopic size. Let us assume in the following as an example a mirror with a surface $A = 25\text{mm}^2$. In this context the question arises whether or not it is possible to construct superconducting cavities with mirrors of such small surfaces. Otherwise, in the following one has to suppose that only a fraction of the mirrors surface oscillates.

A typical material for superconducting cavities is Niobium which has a density of $\rho = 10\text{g/cm}^3$. One needs a $1\mu\text{m}$ thick layer of Niobium in order to achieve a finesse of 10^9 . The mirrors volume is then $2.5 \cdot 10^{-4}\text{cm}^3$, leading to a mass of 2.5 mg. The Niobium layer would probably have to be fixed to a massive support, capable to evacuate the excitation heating. This support should be insensitive to vibrations due to radiation pressure and thus have a very different mechanical behavior than Niobium, as for example quartz.

For a cavity finesse of the order of $\mathcal{F} = 10^9$ a mirror's maximum speed of about $v=25\text{cm/s}$ gives an effective phase shift of 0.9. At 1GHz this mechanical velocity corresponds to a very small displacement $q_0 \sim \frac{v}{\Omega} \sim 10^{-11}\text{m}$, but to a large acceleration $a = 10^9\text{m/s}^2$. For even higher values for the cavity finesse, e.g. $\mathcal{F} \sim 10^{12}$, the mirrors maximum speed needs only to be about

$v = 2.5 \times 10^{-4}$ m/s, corresponding to an amplitude $q_0 \sim \frac{v}{\Omega} \sim 10^{-14}$ m and an acceleration of $a = 10^6$ m/s².

A possible excitation mechanism for the oscillation could be the piezoelectric effect, where one could benefit from a high mechanical quality factor of the support to excite the motion of the Niobium layer. Indeed mechanical quality factors of quartz for hypersonic acoustic waves ($\Omega \sim 10$ MHz) have been reported to be of the order of $10^6 - 10^8$ in the low temperature domain. However to my knowledge, it remains uncertain if the quality factors hold in the GHz domain. On the other hand several papers investigate the piezoelectric constant of quartz at Gigacycle frequencies [21–23]. There the excitation has been measured up to frequencies of 10 GHz.

The efficiency of the conversion of electromagnetic energy into acoustic energy for the piezoelectric effect is given by [22]

$$P_{\text{out}}^{\text{ac}} = P_{\text{in}}^{\text{em}} \frac{C^2 Q \lambda q}{2\pi V} \quad (19)$$

$$C^2 = \frac{4\pi d_{11}^2 c_{11}}{\epsilon}$$

C is the piezoelectric coupling factor which depends on the piezoelectric constant in a specific direction, here d_{11} , and on the elastic constant c_{11} . Q is the mechanical quality factor, λ the wavelength of the electromagnetic wave, q the crystal's thickness, V an appropriately chosen effective volume of the interaction and ϵ the dielectric constant of the medium. Because of the dependence on the Q value, the sensitivity might in practice be considerably improved by using superconductive cavities at low temperatures.

There exist two possibilities to couple an electromagnetic wave to a piezoelectric crystal, either with a re-entrant cavity or a direct coupling via an impedance adaptation system [24]. The re-entrant cavity allows to couple to a crystal with a large surface, that is a diameter of about $\phi \sim 5$ mm, but the coupling efficiency is only of the order of $\beta = -30$ dB. The impedance adaptation system allows for a better coupling of about -15 dB, but only to small surfaces corresponding to a diameter of the order of $\phi \sim 0.1$ mm. Such a surface seems to small in order to have sufficient coupling between the mechanical motion and the electromagnetic vacuum field modes. For this reason I will not consider this possibility in the following although the coupling is more efficient.

If one excites mechanical motion via piezoelectric effect on one end of a crystal, electromagnetic energy will be emitted through the inverse effect on the crystal's other end. This might cause a difficulty for the detection of motion-induced radiation. This difficulty can be circumvented either by an efficient screening with a metallic mirror, or by replacing the crystal with an Aluminium bulk coated with a quarter-wave thick layer of ZnO [24].

The orders of magnitude for the amplitude of the moving surface which can be reached with the re-entrant cavity are exposed in the following. Suppose an electromagnetic source of $P_{\text{in}} = 1$ W. The acoustic power is then

given by $P_{\text{ac}} = P_{\text{in}} 10^{\beta/10}$. This power is converted into mechanical motion of the surface with an amplitude q_0 at a frequency Ω

$$P_{\text{ac}} = \frac{1}{2} Z \Omega^2 q_0^2 A \quad (20)$$

Z is the acoustic impedance and A the moving surface. For a mirror of a surface $A = 25$ mm² with a coupling efficiency of about -30 dB, the acoustic input power is then $P_{\text{ac}} \approx 1$ mW. For an oscillation frequency of 1 GHz and with an acoustic impedance for Aluminium of $Z = 4 \times 10^7$ kg/m²/s one thus finds a maximum displacement $q_0 = \sqrt{\frac{2P_{\text{ac}}}{Z\Omega^2 A}} \sim 10^{-12}$ m achievable with the piezoelectric excitation. Remembering that for values of $\mathcal{F} = 10^9$ and $\mathcal{F} = 10^{12}$ of the cavity finesse, the necessary amplitudes in order to reach an effective phase shift $\eta \sim 0.9$ were respectively $q_0 = 10^{-11}$ m and $q_0 = 10^{-14}$ m, the piezoelectric effect therefore could be suitable to excite the necessary mechanical oscillation.

Finally, we also have to assure that the maximum excitation field applied to the superconducting mirror does not destroy the superconductivity. No matter how motion will be excited, the maximum field strength at the surface of the superconductor must not exceed $E_{\text{max}} = 25$ MV/m [17]. For higher field strengths Cooper pairs are broken up and the superconductor becomes normally conducting. As consistency check we may therefore calculate the maximum velocity which can be excited by such a field:

$$v_{\text{max}} = \Omega q_0 = \frac{\epsilon_0 E_{\text{max}}^2}{\rho d \Omega} \quad (21)$$

With $\epsilon_0 = 8.85 \times 10^{-12}$ As/Vm, a density $\rho = 10$ g/cm³, a layer thickness $d \approx 1$ μ m and an oscillation frequency of 1 GHz, the numerical value for the maximum velocity is found to be $v_{\text{max}} \sim 2 \times 10^{-4}$ m/s. This value is compatible only with the peak velocity for a cavity of finesse 10^{12} , but not for a cavity of lower finesse.

V. CONCLUSION

This paper has discussed motion-induced radiation from an oscillating high finesse cavity. The present results apply only for a one dimensional space. As is well known from the analysis of squeezing experiments, the transverse structure of the cavity modes does not change appreciably the results obtained from this simplified model. Each transverse mode is correctly described by a two-dimensional model as soon as the size of the mirrors is larger than the spot size associated with the mode. The two-dimensional model thus corresponds to a conservative estimate where one transverse mode is efficiently coupled to the moving mirrors. A more precise evaluation for a realistic configuration should take diffraction into account and would probably lead to a result obtained by multiplying the one-dimensional result by the Fresnel

number, i.e. the number of efficiently coupled transverse modes [25]. Precise calculations with three dimensional perfectly reflecting cavities have been performed recently for example in [26, 27].

To resume the possibilities of observing experimentally motion-induced radiation emitted by an oscillating high finesse cavity, the emitted photons may be detected outside the cavity by performing sensitive photon-counting detection of the radiated flux. Inside the cavity the state of the field could be probed with the help of Rydberg atoms. However, to excite the motion, a very large force would have to be applied onto the mirrors which would create spurious signals. To distinguish experimentally the motion-induced photons from spurious signals, the particular signatures of the effect such as pulse shaping and frequency up-conversion are therefore extremely important.

Generally, one may say that the challenge of this kind

of experiment does not come from one particular constraint, but from the fact that all of the above discussed conditions will have to be fulfilled simultaneously. The present paper shows that there is no fundamental objection to the realization of such an experiment, but that it is very difficult, to meet the ensemble of necessary conditions. However, as dissipative effects of vacuum fluctuations are a fundamental phenomenon related to important conceptual questions in physics, their observation would be worth the effort.

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