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# Robust Generation of Analytical Redundancy Equations and Applications to Diagnosis

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## Abstract

This paper describes a procedure for generating analytical redundancy equations for process fault detection and isolation. The proposed procedure uses a direct approach by considering the relations between the inputs and the outputs of the process ; systems with known or partially known inputs are considered and it is pointed out that the same technique yields for these different situations.

## 1. Introduction

The design of fault tolerant control systems requires failures to be detected, identified and taken into account within acceptable time interval as not to affect excessively the system operation. Surveys on design methods for failure detection and isolation are given in the papers of Isermann (1984) and Gertler (1991). In the past decade, interest has been focused on the use of analytic redundancy equations rather than massive redundancy. In particular, analytical redundancy equations have been designed to form and process residuals ; these residuals are closed to zero if no failure occurs and differ significantly from zero when a failure occurs.

When certain inputs of the process cannot be measured, the state observation must be designed by eliminating the unknown inputs. This problem has received considerable attention in the literature. Several researchers have investigated state or input observers when some inputs are unknown ; among them, the reader is referred to Kurek (1983), El-Tohami (1983) or Miller (1984) for example. In the study of Park (1988) a closed-loop observer that can identify simultaneously states and inputs has been developed. A systematic investigation of unknown input observers was carried out recently by Wünnenberg (1990) using the Kronecker canonical form transformation. A more simple design procedure is given by Hou (1991) using only algebraic approach. In the present work, we introduce a generation scheme of analytical redundancy equations.

## 2. Decoupling properties

In practise, the process models are not perfectly known : parameter variations, unknown inputs and component faults significantly influence processes behavior. If these perturbations have not been included in the models, the model behavior differs from those of the actual process. Consequently, the residuals are as much sensitive to fault as to inherent uncertainties of the models. To separate the

faults from the model inaccuracy, we must use robust redundancy relations which are insensitive to undesired perturbations. We consider a linear system described by the following state space equation :

$$\lambda x(k) = A x(k) + B u(k) + \bar{B} f(k) \quad (1a)$$

$$y(k) = C x(k) + D u(k) + \bar{D} f(k) \quad (1b)$$

where  $f(k)$  is a perturbation vector,  $\lambda$  states for the  $s$  or  $z$  operator depending on the representation of the model in continuous or discrete form and  $\bar{B}$  and  $\bar{D}$  represent the perturbation matrices of appropriate dimensions. It is assumed in the following that  $C$  is a full rank matrix. Robust redundancy equations can be generated by the elimination of the state  $x(k)$  and the perturbation vector  $f(k)$ . Therefore, using (1), the equation to solve may be generally written :

$$(p^T(\lambda) \quad q^T(\lambda)) \begin{pmatrix} A - \lambda I & \bar{B} \\ C & \bar{D} \end{pmatrix} = (0 \quad 0) \quad (2)$$

where  $p(\lambda)$  and  $q(\lambda)$  are two vectors with compatible dimensions with the state matrices. If this condition holds, multiplying equation (1) with  $(p^T(\lambda) \quad q^T(\lambda))$ , yields the redundancy equations :

$$(p^T(\lambda) \quad q^T(\lambda)) \begin{pmatrix} 0 \\ I \end{pmatrix} y(k) - \begin{pmatrix} B \\ D \end{pmatrix} u(k) = 0 \quad (3)$$

If the system (2) has no solution, due for example to an excessive number of perturbations, then one may look for an approximate solution. In order to find the optimal approximation, a certain performance index that contains a measure of the effects of the disturbances must be defined. If the perturbation matrices do not exist, the preceding formalism is still valid and allows to generate the redundancy relations.

## 3. General solution of the decoupling problem

To solve equation (2), rewrite it in the equivalent form :

$$(p^T(\lambda) \quad q^T(\lambda)) \left( \begin{pmatrix} A & \bar{B} \\ C & \bar{D} \end{pmatrix} - \lambda \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \right) = (0 \quad 0) \quad (4)$$

Solving (4) corresponds to the well-known problem of the determination of the left nullspace of a binomial

matrix (Gantmacher, 1977). Taking the transpose of equation (4) yields :

$$\begin{pmatrix} A_0 & C_0 \\ B_0 & D_0 \end{pmatrix} - \lambda \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p(\lambda) \\ q(\lambda) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

The solution of equation (5) is obtained through a two-step algorithm. First, we solve (5) with  $\bar{D} = 0$  (i.e.  $D_0 = 0$ ). Second, we transform system (5) in order to satisfy this property. Then, the first step leads to solve the following system :

$$(\lambda I - A_0) p(\lambda) = C_0 q(\lambda) \quad (6a)$$

$$B_0 p(\lambda) = 0 \quad (6b)$$

Equation (6b) implies that  $p(\lambda)$  belongs to the kernel of  $B_0$ , so that the general solution of (6b) is expressed in term of an appropriate vector of constants :

$$p(\lambda) = \bar{B}_0 v(\lambda) \quad (7)$$

Substituting  $p(\lambda)$  in (6a) and pre-multiplying first by  $\bar{N}$  the left annihilator of  $C_0$  ( $\bar{N} C_0 = 0$ ) and second by  $\bar{C}_0$  the left inverse of  $C_0$  ( $\bar{C}_0 C_0 = I$ ) yields the two equations :

$$\bar{N} (\lambda I - A_0) \bar{B}_0 v(\lambda) = 0 \quad (8a)$$

$$\bar{C}_0 (\lambda I - A_0) \bar{B}_0 v(\lambda) = q(\lambda) \quad (8b)$$

Equation (8a) may be solved in respect to  $v(\lambda)$  (Gantmacher, 1977, p. 29-30), then  $p(\lambda)$  is deduced from (7) and  $q(\lambda)$  from (8b). At the second step, consider the complete system (5) in which  $D_0$  may always be expanded into :

$$D_0 = H \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} K^T \quad (9)$$

where  $H$  and  $K$  are two orthogonal matrices and  $R$  a regular matrix. With  $K^T q(\lambda) = \bar{q}(\lambda)$  and  $\bar{B}_0 = H^T B_0$ , we have from the second equation of system (5) :

$$\bar{B}_0 p(\lambda) + \begin{pmatrix} R & 0 \\ 0 & 0 \end{pmatrix} \bar{q}(\lambda) = 0 \quad (10)$$

which may be expanded into :

$$\bar{B}_{01} p(\lambda) + R \bar{q}_1(\lambda) = 0 \quad (11a)$$

$$\bar{B}_{02} p(\lambda) = 0 \quad (11b)$$

The solution  $\bar{q}_1(\lambda)$  of equation (11a) is then substituted in (6a) which gives :

$$(\lambda I - \bar{A}) p(\lambda) = C_{02} \bar{q}_2(\lambda) \quad (12)$$

$$\text{with : } \bar{A} = A_0 - C_{01} R^{-1} \bar{B}_{01} \quad (13)$$

Thus, the remaining equations (11b) and (12) have the same structure as the system (6) and therefore may be solved by using the same technique. Therefore, the residual generation consists in the extraction of the

redundancy relations from the system state equations. In fact, this extraction is possible by eliminating the unknown variables (state variables or perturbations). If the disturbance directions are unknown, the elimination affects only the state variables ( $\bar{B} = 0$ ,  $\bar{D} = 0$ ), otherwise a priori knowledge can be used to design the matrices  $\bar{B}$  and  $\bar{D}$ .

#### 4. Unknown inputs systems

Consider the linear system, with a  $p$  dimensional unknown input vector  $d(k)$ , described by :

$$\lambda x(k) = A x(k) + B u(k) + G d(k) \quad (14a)$$

$$y(k) = C x(k) + D u(k) + F d(k) \quad (14b)$$

where  $G$  and  $F$  are known constant matrices. Without loss of generality, we assume that :

$$\text{rank} \begin{pmatrix} G \\ F \end{pmatrix} = \text{dim}(d) \quad (15)$$

The classical conditions of observability (Kurek, 1983), are not necessary because we do not try to estimate system state. With obvious definition of the matrices  $M$ ,  $N$ ,  $\bar{A}$ ,  $Q$  and  $\bar{B}$ , the equations (14) can be expressed as :

$$M \lambda x(k) + N y(k) = \bar{A} x(k) + Q d(k) + \bar{B} u(k) \quad (16)$$

Assuming that equation (15) holds, there exist two orthogonal matrices  $H$  and  $K$  and a non-singular matrix  $R$  such that :

$$Q = H \begin{pmatrix} R \\ 0 \end{pmatrix} K^T \quad (17)$$

Introducing a new vector  $\bar{d}(k)$  defined by :

$$\bar{d}(k) = K^T d(k) \quad (18)$$

and pre-multiplying equation (16) with  $H^T$ , we can partitioned equation (16) as :

$$M_1 \lambda x(k) + N_1 y(k) = \bar{A}_1 x(k) + R \bar{d}(k) + \bar{B}_1 u(k) \quad (19a)$$

$$M_2 \lambda x(k) + N_2 y(k) = \bar{A}_2 x(k) + \bar{B}_2 u(k) \quad (19b)$$

The first equation (19a) may be used to estimate the unknown input  $\bar{d}(k)$  since  $R$  is regular. The second one (19b) may contain redundancies between the input  $u(k)$  and the output  $y(k)$  provide it is possible to eliminate the state  $x(k)$ . For that purpose (19b) is rewritten :

$$(\lambda M_2 - \bar{A}_2) x(k) = \bar{B}_2 u(k) - N_2 y(k) \quad (20)$$

Redundancy equations are then obtained by searching the left null space of the pencil  $(\lambda M_2 - \bar{A}_2)$  :

$$p^T(\lambda) (\lambda M_2 - \bar{A}_2) = 0 \quad (21)$$

The solution of equation (21) is the same as those of the equation (4). The redundancy equations can be described by :

$$p^T(\lambda) (\bar{B}_2 u(k) - N_2 y(k)) = 0 \quad (22)$$

The previously described method can be used to detect and localize faulty actuators. Consider a standard system where the  $i^{\text{th}}$  input has been isolated :

$$\lambda x(k) = A x(k) + B_i u_i(k) + \bar{B}_i \bar{u}_i(k) \quad (23a)$$

$$y(k) = C x(k) + D_i u_i(k) + \bar{D}_i \bar{u}_i(k) \quad (23b)$$

where  $B_i$  is the  $i^{\text{th}}$  column of  $B$  and  $\bar{B}_i$  is the  $n(r-1)$  matrix obtained from  $\underline{B}$  by deleting  $B_i$  (with the same definition for  $D_i$  and  $\bar{D}_i$ ). Let  $u_i(k)$  be the  $i^{\text{th}}$  entry of  $u(k)$  and  $\bar{u}_i(k)$  the  $(r-1)$  column vector obtained from  $u(k)$  by deleting  $u_i(k)$ . The structure of the system (23) is similar to the structure of the system (14). We can consider  $u_i(k)$  as an unknown input and generate the redundancy relations independently of this input. If we exchange the roles of  $u_i(k)$  and  $\bar{u}_i(k)$  which is therefore considered as unknown inputs, the redundancy equations depend on all but one input, therefore it make faulty actuators isolation easier. This approach should be compared with the dedicated observer approach (Frank, 1989).

### 5. Numerical example

Consider a third order system described by equation (14), where :

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad G = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the decomposition (17), we obtain :

$$H = \begin{pmatrix} H_0 & 0 \\ 0 & I \end{pmatrix} \text{ with } H_0 = \begin{pmatrix} a & -a & 0 \\ a & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where  $a = \sqrt{2}/2$  and  $K = 1$ .

The decomposition (19) allows to define the pencil :

$$\lambda M_2 - \bar{A}_2 = \begin{pmatrix} -a(\lambda-1) & a(\lambda-0.5) & 0 \\ 0 & 0 & \lambda-0.5 \\ -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

from which we find the left orthogonal matrix :

$$p^T(\lambda) = \begin{pmatrix} 1 & 0 & a(1-\lambda) & a(2\lambda-1.5) & -a(2\lambda-1.5) \\ 0 & 1 & 0 & 0 & \lambda-0.5 \end{pmatrix}$$

Applying (22), the redundancy equations are then expressed :

$$u(k) - (\lambda-0.5) y_3(k) = 0 \\ 2u(k) - 2(1-\lambda) y_1(k) - (4\lambda-3) y_2(k) + (4\lambda-3) y_3(k) = 0$$

### 6. Conclusion

The problem of analytical redundancy equation design has been considered. We have pointed out a systematic procedure to design these equations. The technique can be used for systems with unknown inputs.

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