3-colorability and contacts of segments

Patrice Ossona de Mendez

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P. Ossona de Mendez

CNRS UMR 0017, EHESS, 54 Boulevard Raspail, 75006, Paris, France.

Abstract. Using a result on planar graph representation [1], we shall prove that vertex 3-colorability is NP-complete for simple contact graphs of segments in 4 directions.

Let V be a finite set of segments in the plane. Associate a graph $G(V)$ called the *intersection graph* of V as follows : V is the vertex set of G and there is an edge connecting v_1 and v_2 if and only if the segments v_1 and v_2 intersect.

G. Ehrlich, S. Even and R. Tarjano [3] proved that vertex 3-colorability of intersection graphs of segments is NP-complete.

It has been conjectured that any planar graph is an intersection graph of segments and this has been proved for 3-colorable graphs [2], what gives another proof of the previous theorem.

In case where the segments in V never cross, but may only touch, the intersection graph of V is a *contact graph of segments*.

Contact graphs of segments are precisely those graphs G that satisfy :

– G is planar
– Any subgraph H of G, induced by $n_H \geq 2$ vertices, has a number of edges at most equal to $2.n_H - 3$

This class includes the planar graphs that are triangle free (3-colorable, according to Grötsch theorem), the outerplanar graphs (3 colorable, as proved by König), 3-regular graphs different from K_4 (3 colorable, according to Brooks' theorem in the cubic case) but also 4 chromatic graphs.

If the segments of V belongs to k directions, $G(V)$ is a *k-dir contact graph of segments*. Obviously, a 3-dir contact graph of segments is 3-colorable.

One may ask if the problem of finding the chromatic number is still NP-complete for contact graphs of segments, or even for 4-dir contact graphs of segments.

Theorem 1. *Vertex 3-colorability is NP-complete for 4-dir contact graphs of segments.*

Proof. As notice in [1], any planar graph G is the contact graph of T-shape. Moreover, the vertex 3-colorability of G is obviously equivalent to the vertex 3-colorability of the 4-dir contact graph of segments G' obtained by transforming each T-shape as shown in the figure. □

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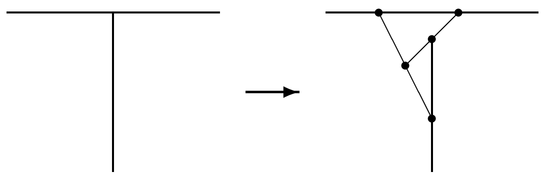

Fig. 1. Transformation of a T-shape contact graph into a 4-dir contact graph of segments, while preserving vertex 3-colorability

References

1. H. de Fraysseix, P. Ossona de Mendez, and P. Rosenstiehl. On triangle contact graphs. *Combinatorics, Probability and Computing*, 3:233–246, 1994.
2. H. de Fraysseix, P. Ossona de Mendez, and C. Thomassen. On graphs representable by contact graphs. in preparation.
3. G. Ehrlich, S. Even, and R.E. Tarjan. Intersection graphs of curves in the plane. *Journal of Combinatorial Theories, Series B*, 21:8–20, 1986.